Incremental Cost Consensus Algorithm
in a Smart Grid Environment

Y3.F.C1 Distributed Control of FREEDM System

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Outline

• Background
• Motivations & Goal
• Technical Approach
• Incremental Cost Consensus Algorithm
  – Problem Formulation
  – Convergence Rate Analysis
• Future Plan
Background

System Demonstration:
- Intelligent Energy Management

Enabling Technology:
- Distributed Grid Intelligence (DGI)

Fundamental Technology:
- System Theory Modeling and Control (SMC)
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Motivation & Goal

Challenges for the Current Power Grid
• Lack of support for Distributed Generation and Renewable Energy
• Lack of flexibility and adaptability
• Vulnerability to Cyber attack, natural disasters and human errors
• $100 Billion annual loss due to power quality problems
• Aging Components

Solution:

*Take advantages from the new technologies -- make the grid smarter*

Project Goal

Design and implement high performance distributed controls to achieve real-time intelligent power allocation in FREEDM system.
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Central Control vs. Distributed Control

<table>
<thead>
<tr>
<th>Central Control</th>
<th>Distributed Control [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>System</td>
<td>Puppets and Puppeteer</td>
</tr>
</tbody>
</table>

- President Iain D. Couzin, Jens Krause, Nigel R. Franks and Simon A. Levin, "Effective leadership and decision-making in animal groups on the move", *Nature* 433, 513-516 (3 February 2005)
**Central Control vs. Distributed Control**

<table>
<thead>
<tr>
<th>Pros</th>
<th>Central Controlled System</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Control algorithm is relatively simple</td>
<td></td>
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<tr>
<td>• …</td>
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</table>

<table>
<thead>
<tr>
<th>Cons</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Computational limitation of central controller</td>
<td></td>
</tr>
<tr>
<td>• Communication limitation of central controller</td>
<td></td>
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<tr>
<td>• Single point of failure will affect the entire system</td>
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<tr>
<td>• …</td>
<td></td>
</tr>
</tbody>
</table>

| Usages | Normally more appropriate for systems with simple control |
What is consensus?

Consensus

A school of fish

Goal: swimming towards one same direction

Consensus

Chorus

Goal: Synchronize the melody

How can consensus be reached?

A sufficient condition for reach consensus: If there is a directed spanning tree* exists in the communication network, then consensus can be reached. **

*Spanning tree: a minimal set of edges that connect all nodes

Networked Control System

Picture from EPRI
Agent-based Distributed Control Network

Power Grid

Physical Layer

Cyber Layer

Future Renewable Electric Energy Delivery and Management Systems Center
Graph Theory Modeling

Adjacency matrix of a finite graph $G$ on $n$ vertices is the $n \times n$ matrix where the entry $a_{ij}$ is the number of edges from vertex $i$ to vertex $j$, $a_{ij} = 0$ represent that agent $i$ cannot receive information from agent $j$.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Adjacency matrix

$$diag(2,1,1) - A = L;$$

$$L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Laplacian matrix

$$D = \begin{bmatrix} 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 2 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

Row-stochastic matrix
First-order Consensus Algorithm

Consensus problem modeling

- Local information state $\xi_i$
- First-order system $\dot{\xi}_i = \xi_i, i = 1, \ldots, n$
- Consensus algorithm:

<table>
<thead>
<tr>
<th>Scalar From</th>
<th>Matrix Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous–time</td>
<td>$\dot{\xi}<em>i = -\sum</em>{j=1}^{n} a_{ij} (\xi_i - \xi_j), i = 1, \ldots, n$</td>
</tr>
<tr>
<td>Discrete-time</td>
<td>$\xi_i[k+1] = \sum_{j=1}^{n} d_{ij} \xi_j[k], i = 1, \ldots, n$</td>
</tr>
</tbody>
</table>

Where $L_n$ is the Laplacian matrix associated with A, and $D_n$ is Row-stochastic matrix associated with A.
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Decentralized Economic Dispatch

Assumptions:

• All the signals are “good”
  • No security issue

• No generation limitation (in this presentation)

• The cost functions are quadratic

• The power grid topology is fixed
Decentralized Economic Dispatch

Economic Dispatch Problem -- A constrained optimization problem

\[
\text{Min: Cost} = (561 + 7.92P_1 + 0.562P_1^2) + (310 + 7.85P_2 + 0.94P_2^2)
\]

s.t. : \(P_1 + P_2 = 500\)

Contour Graph

3D view
Economic Dispatch Problem -- A constrained optimization problem

Min: Cost = $f(P_1, P_2)$

s.t. : $g(P_1, P_2) = P_1 + P_2 - 500$

At the optimal point: $\nabla f(x, y) = \lambda \nabla g(x, y)$
Decentralize the Economic Dispatch Problem Using Consensus Network:

When using Lagrange multiplier method solving Economic Dispatch Problem, each generator will have the same Incremental Cost at the minimum cost point.
Incremental Cost Consensus

Mathematical Formulation:

Assume the fuel-cost curve of each generating unit is known and expressed in terms of the output power:

\[ C_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2, \quad i=1,2,...,m \]

where \( C_i(P_{Gi}) \) is the cost of generation for unit \( i \).

\( P_{Gi} \) is the output power of unit \( i \).

The objective is to minimize total cost of operation:

\[ C_T = \sum C_i(P_{Gi}). \]

Subject to constrains: \( \Sigma P_{Di} - \Sigma P_{Gi} = 0; \)

From the conventional economic dispatch we know:

\[ IC_i = \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} = \lambda_i \]

\[ \lambda_i [k+1] = \Sigma d_{ij} \lambda_j[k], \]

where \( d_{ij} \) is the \((i,j)\) entry of row-stochastic matrix \( D_n \).

The consensus algorithm for the leader (mediator/ coordinator) generator becomes:

\[ \lambda_i [k+1] = \Sigma d_{ij} \lambda_j[k] + \varepsilon \Delta P, \]

where \( \varepsilon \) is a scalar which controls the convergence speed.

\[ \Delta P = \Sigma P_{Di} - \Sigma P_{Gi}. \]
Flow Chart:

Start

Set $\lambda$

Consensus Algorithm

Exceed Generation Limit?

Yes

Set $P_i = P_{i,\text{max}}$ or $P_i = P_{i,\text{min}}$

No

Calculate $P_i$
Incremental Cost Consensus

Flow Chart:

1. Start
2. Set $\lambda$
3. Consensus Algorithm
4. Exceed Generation Limit?
   - Yes: Set $P_i = \bar{P}_i, \text{or } P_i = \underline{P}_i$
   - No: Calculate $P_i$
5. Calculate $\Delta P = P_{\text{load}} - \Sigma P_i$
6. $|\Delta P| < \text{Tolerance?}$
   - Yes: End
   - No: Increase or decrease $\lambda$ base on $\text{sign}(\Delta P)$
Simulation Results

Using a fully connected 3-bus system with initial conditions:

\[ P_D = 850 \text{MW}, \quad P_{G1}(0) = 450 \text{MW}, \]
\[ P_{G2}(0) = 300 \text{MW}, \]
\[ P_{G3}(0) = 100 \text{MW}, \]

\[ C_1(P_{G1}) = 561 + 7.92P_{G1} + 0.001562P_{G1}^2 \text{$/hr}$
\[ C_2(P_{G2}) = 310 + 7.85P_{G2} + 0.00194P_{G2}^2 \text{$/hr}$
\[ C_3(P_{G3}) = 78 + 7.79P_{G3} + 0.001482P_{G3}^2 \text{$/hr}$

When IC Consensus algorithm reach the steady state, the final IC we obtained is equal to the $\lambda$ which calculated by using the Lagrange multiplier method.

Increase $P_D$ to 950MW at 5 second
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Convergence Rate Analysis

The convergence rate of Incremental Cost Consensus (ICC) algorithm can be affected by following configurations:

• General configurations (which also apply to conventional EDP):
  – Inertia of synchronous generators
  – Power grid topology
  – System sampling rate
  – Signal transmission delay

• Feature configurations (which only valid when using ICC):
  – Communication topology
  – Location of leader
  – Weighting of the edges of communication network
Communication Topology

5 Unit Star connection ICs

5 Unit ICs with random connection
The Location of Leader

Centrality indices have been tested:

- $C_D$: Degree centrality (Nieminen 1974)
- $C_B$: Betweenness centrality (Anthonisse 1971, Freeman 1979)
- $C_C$: Closeness centrality (Sabidussi 1966)
- $C_E$: Eigenvector centrality (Bonacich 1972)
- $C_S$: Subgraph centrality (Estrada and Rodriguez-Velazquez 2005)

Node centralities value calculated by different indices: (the larger the better)

<table>
<thead>
<tr>
<th>Node</th>
<th>$C_D$</th>
<th>$C_B$</th>
<th>$C_C$</th>
<th>$C_E$</th>
<th>$C_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0.14</td>
<td>0.41</td>
<td>3.03</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.54</td>
<td>4.33</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.6</td>
<td>0.2</td>
<td>0.47</td>
<td>3.82</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0.13</td>
<td>0.18</td>
<td>1.67</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.54</td>
<td>4.33</td>
</tr>
</tbody>
</table>

The ranking of five nodes based on different centrality measure are:

- $C_B$: $G3 > G2 = G5 > G1 = G4$
- $C_E$ and $C_S$: $G2 = G5 > G3 > G1 > G4$
The Location of Leader

Consensus algorithm simulation results by selecting different node as leader:

Convergence rate from simulation:
G2 = G5 > G3 > G1 > G4
The convergence time is consistent with $C_E$ and $C_S$'s result.
Thus, we suggest use $C_E$ or $C_S$ for leader election.
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Threats and Future Work

• Detailed Greenhub distributed control modeling and simulation
  – Extend to full Greenhub scale
  – Include both communication network and power grid and their interactions
  – Use dynamic topology to simulate “Plug-and-Play” scenario
• Intelligent distributed control algorithms for FREEDM Greenhub
  – Effectively select leaders in the consensus algorithms to guarantee fastest convergence rate
  – Adjust appropriate weightings during consensus updating
• Analyze the robustness of algorithms
  – Package Loss
  – Link failure
  – Node failure
• Investigate the bandwidth limitation issue
  – Develop and implement adaptive sampling strategies
  – Develop and implement distributed bandwidth allocation algorithms
• Investigate network delay effects on the Greenhub distributed control
  – Develop corresponding network delay compensation algorithms
Related Publications


FREEDM website / Members Only/ Research Groups / Distributed Grid Intelligence / Distributed Control of FREEDM System
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