## Maxwell's Equations:

$$\vec{\nabla}\cdot\vec{D} = \rho_v \qquad \text{Gauss's law}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \text{Faraday's law}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \underbrace{\frac{\partial\vec{D}}{\partial t}}_{J_d} \qquad \text{Amper's law}$$

## Maxwell's Equations for time harmonics:

$$\vec{\nabla}\cdot\vec{\tilde{D}} = \tilde{\rho}_v$$

$$\vec{\nabla}\times\vec{\tilde{E}} = -j\omega\vec{\tilde{B}}$$

$$\vec{\nabla}\cdot\vec{\tilde{B}} = 0$$

$$\vec{\nabla}\times\vec{\tilde{H}} = \vec{\tilde{J}} + \underbrace{j\omega\vec{\tilde{D}}}_{\vec{J}_d}$$

## Maxwell's Equations in integral forms:

$$\oint_S \vec{D}\cdot d\vec{s} = Q$$

$$\oint_C \vec{E}\cdot d\vec{l} = -\int \frac{\partial\vec{B}}{\partial t}\cdot d\vec{s} \Rightarrow V_{emf} = -N\frac{d\phi}{dt} = -N\frac{d}{dt}\int\vec{B}\cdot d\vec{s}$$

$$\oint_C \vec{B}\cdot d\vec{l} = 0$$

$$\oint_C \vec{H}\cdot d\vec{l} = I + \underbrace{I_d}_{\text{Displacement current}} = \int\vec{J}\cdot d\vec{s} + \int\vec{J}_d\cdot d\vec{s} = \int\vec{J}\cdot d\vec{s} + \int\frac{\partial\vec{D}}{\partial t}\cdot d\vec{s}$$

## Maxwell's Equations for EM wave in charge free medium:

$$\vec{\nabla}\cdot\vec{\tilde{E}} = 0$$

$$\vec{\nabla}\times\vec{\tilde{E}} = -j\omega\mu\vec{\tilde{H}}$$

$$\vec{\nabla}\cdot\vec{\tilde{H}} = 0$$

$$\vec{\nabla}\times\vec{\tilde{H}} = j\omega\varepsilon_c\vec{\tilde{E}} \qquad \text{where } \varepsilon_c = \overset{\varepsilon}{\varepsilon'} - \underbrace{j\overset{\sigma/\omega}{\varepsilon''}}_{\text{loss term}}$$

**Coulomb's law:** $\vec{E} = \dfrac{q(\vec{R}-\vec{R}_1)}{4\pi\varepsilon|\vec{R}-\vec{R}_1|^3}$ ; $V = \dfrac{q}{4\pi\varepsilon|\vec{R}-\vec{R}_1|}$

For charge distributions:

Volume: $\vec{E} = \dfrac{1}{4\pi\varepsilon}\displaystyle\int_V \hat{R}\,\dfrac{\rho_v dv}{R^2}$ ; $V = \dfrac{1}{4\pi\varepsilon}\displaystyle\int_V \dfrac{\rho_v dv}{R}$

Surface: $\vec{E} = \dfrac{1}{4\pi\varepsilon}\displaystyle\int_S \hat{R}\,\dfrac{\rho_s ds}{R^2}$ ; $V = \dfrac{1}{4\pi\varepsilon}\displaystyle\int_S \dfrac{\rho_s ds}{R}$

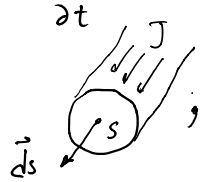
line: $\vec{E} = \dfrac{1}{4\pi\varepsilon}\displaystyle\int_l \hat{R}\,\dfrac{\rho_l dl}{R^2}$ ; $V = \dfrac{1}{4\pi\varepsilon}\displaystyle\int_l \dfrac{\rho_l dl}{R}$

**Voltage:** $V_b - V_a = -\displaystyle\int_a^b \vec{E}\cdot d\vec{l}$ ; $\vec{E} = -\vec{\nabla}V$

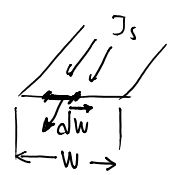
Poisson's equation: $\nabla^2 V = -\dfrac{\rho_v}{\varepsilon}$ Laplace equ: $\nabla^2 V = 0$ when $\rho_v = 0$

**Current density:** $\vec{J} = \sigma\vec{E}$ , $\vec{J}_d = \dfrac{\partial\vec{D}}{\partial t}$

Current: $I = \displaystyle\int \vec{J}\cdot d\vec{s}$ for volume




; $I = \displaystyle\int \vec{J}_s\cdot d\vec{w}$ for surface




**Resistance:** $R = \dfrac{V}{I} = \dfrac{-\int\vec{E}\cdot d\vec{l}}{\int\vec{J}\cdot d\vec{s}} = \dfrac{-\int\vec{E}\cdot d\vec{l}}{\int_S \sigma\vec{E}\cdot d\vec{s}}$

**Capacitance:** $C = \dfrac{Q}{V} = \dfrac{\int\rho dv}{-\int\vec{E}\cdot d\vec{l}}$ ; We also have: $RC = \dfrac{\varepsilon}{\sigma}$

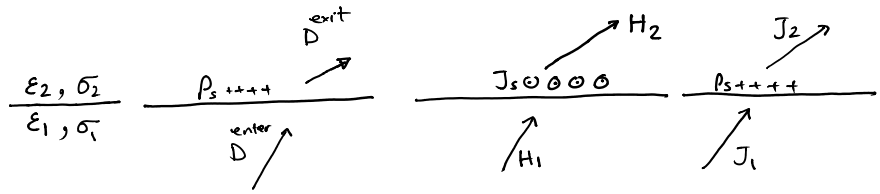
**Power:** $P = \displaystyle\int_V \vec{E}\cdot\vec{J}\,dv = \int_V \sigma|E|^2 dv$

Electrostatic Potential energy: $W_e = \frac{1}{2}\int_V \varepsilon E^2 \, dv$

Magnetostatic Potential energy: $W_m = \frac{1}{2}\int_V \mu H^2 \, dv$

Force: $\vec{F} = -\vec{\nabla} W_e$ when $Q$ is constant

## Boundary Conditions




For electric field: Tangent Component: $E_t^{ex} = E_t^{en} \Rightarrow \frac{D_t^{ex}}{\varepsilon_2} = \frac{D_t^{en}}{\varepsilon_1}$

Normal Component: $D_n^{ex} - D_n^{en} = \rho_s$

For magnetic field: Tangent component: $H_{1t} - H_{2t} = J_s$

Normal Component: $B_{1n} = B_{2n}$ or $\mu_1 H_{1n} = \mu_2 H_{2n}$

For current density: Tangent Component: $\frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2}$

Normal Component: $J_{1n} = J_{2n}$ & $J_{1n}\left(\frac{\varepsilon_1}{\sigma_1} - \frac{\varepsilon_2}{\sigma_2}\right) = \rho_s$

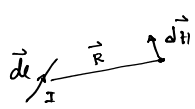
## Force by magnetic field:

$$\vec{F}_m = q\vec{V} \times \vec{B}$$

$$\vec{F}_m = I\int d\vec{l} \times \vec{B} \quad \text{if uniform } \vec{F}_m = I\vec{l} \times \vec{B}$$

## Biot-Savart Law:

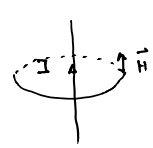
$$\vec{H} = \frac{I}{4\pi}\int \frac{d\vec{l} \times \hat{R}}{R^2}$$




$$\vec{H} = \frac{1}{4\pi}\int \frac{\vec{J} \times \hat{R}}{R^2} dv$$

$$\vec{H} = \frac{1}{4\pi}\int \frac{\vec{J}_s \times \hat{R}}{R^2} ds$$

For long wire: $\vec{H} = \frac{I}{2\pi r}\hat{\varphi}$



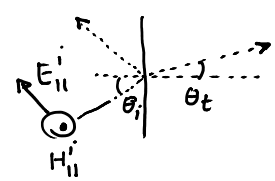
- Magnetic Flux: $\Phi = \int \vec{B}\cdot d\vec{s}$ (We, Weber) Magnetic Flux Linkage: $\Lambda = N\Phi$
- Inductance: $L = \frac{\Lambda}{I} = \frac{N\Phi}{I}$ (H, Henry)
- Phase velocity: $u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{n}$ where $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8$ m/s ; $n = \sqrt{\mu_r \varepsilon_r}$ ; For non-magn: $n = \sqrt{\varepsilon_r}$
- For an E (or H) wave in the form of $\vec{E} = \vec{E}_0 \cos(\omega t + \beta x + \phi)$: $\beta = \frac{\omega}{u_p}$ ($\beta$ is same as $k$).
- Wave equations in charge-free medium:

$$\begin{cases} \nabla^2 \tilde{\vec{E}} - \gamma^2 \tilde{\vec{E}} = 0 \\ \nabla^2 \tilde{\vec{H}} - \gamma^2 \tilde{\vec{H}} = 0 \end{cases}$$

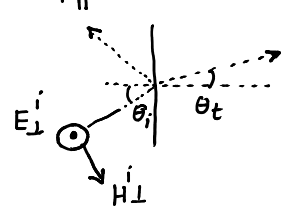
where $\gamma = -\omega^2 \mu \varepsilon_c$ and $\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$

For lossless medium $\sigma = 0 \rightarrow \varepsilon_c = \varepsilon \rightarrow \gamma = -k^2$ $k = \omega\sqrt{\mu\varepsilon}$

- For loss-less medium: $\tilde{\vec{H}} = \frac{1}{\eta}\hat{k}\times\tilde{\vec{E}}$ and $\tilde{\vec{E}} = -\eta\,\hat{k}\times\tilde{\vec{H}}$ ; $\hat{k}$ is direction of travel.
- Impedance: $\eta = \sqrt{\frac{\mu}{\varepsilon}}$
- Refection Coeff: $\Gamma = \frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ ; $\tau = 1 + \Gamma$ ; $S = \frac{1+|\Gamma|}{1-|\Gamma|}$
- Snell's Law: $\theta_i = \theta_r$ ; $n_1 \sin\theta_1 = n_2 \sin\theta_2$
- Critical Angle: $\sin\theta_c = \frac{n_1}{n_2}$
- Parallel Polarization: $\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$




- Perpendicular Polarization: $\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$




- Brewster Angle: $\sin\theta_{B_{\parallel}} = \sqrt{\frac{1-(\eta_2/\eta_1)^2}{1-(\varepsilon_1/\varepsilon_2)^2}}$ For non-magnetic materials $\frac{1}{1+\frac{\varepsilon_1}{\varepsilon}}$ or $\tan\theta_B = \frac{n_2}{n_1}$

## Vector Algebra:

- $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$ $\quad A \to B \to C \to A$
- $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$ "bac - cab" rule!
- Cartesian to cylindrical: $\hat{r} = \hat{x}\cos\varphi + \hat{y}\sin\varphi$ ; $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ ; $\hat{z} = \hat{z}$

  For vector $\vec{A}$: $A_r = A_x\cos\varphi + A_y\sin\varphi$ , $A_\phi = -A_x\sin\phi + A_y\cos\varphi$ ; $A_z = A_z$

- Cylindrical to cartesian: $\hat{x} = \hat{r}\cos\varphi - \hat{\phi}\sin\varphi$ ; $\hat{y} = \hat{r}\sin\varphi + \hat{\phi}\cos\varphi$ ; $\hat{z} = \hat{z}$

  For vector $\vec{A}$: $A_x = A_r\cos\varphi - A_\varphi\sin\varphi$ ; $A_y = A_r\sin\varphi + A_\varphi\cos\varphi$ , $A_z = A_z$

- Cartesian to spherical:

  $\hat{R} = \hat{x}\cos\varphi\sin\theta + \hat{y}\sin\varphi\sin\theta + \hat{z}\cos\theta$; $\hat{\theta} = \hat{x}\cos\theta\cos\varphi + \hat{y}\cos\theta\sin\varphi - \hat{z}\sin\theta$ ; $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$

- spherical to Cartesian:

  $\hat{x} = \hat{R}\sin\theta\cos\varphi + \hat{\theta}\cos\theta\cos\varphi - \hat{\phi}\sin\phi$; $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\varphi + \hat{\phi}\cos\varphi$; $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$

- Cartesian: $\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

  Cylindrical: $\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z}$

  spherical: $\vec{\nabla} = \hat{R}\frac{\partial}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial}{\partial \theta} + \hat{\phi}\,\cancel{R\sin\theta\,\partial\phi}$

- Gradient: $\vec{\nabla}T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$

- $\vec{\nabla}\cdot\vec{E} \triangleq \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$ $\qquad \int_V \vec{\nabla}\cdot\vec{E}\,dv = \int_S \vec{E}\cdot\vec{ds}$

- $\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$ $\qquad \int_S (\vec{\nabla}\times\vec{E})\cdot\vec{ds} = \oint_C \vec{E}\cdot\vec{dl}$

- Laplacian: $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

- Cylindrical: $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}$

- Spherical: $\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial r}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \varphi^2}$

## Transmission Line Formulas:

- V and I wave equations: $\frac{d^2\tilde{V}}{dz^2} - \gamma^2\tilde{V}(z) = 0$ ; $\frac{d^2\tilde{I}}{dz^2} - \gamma^2\tilde{I}(z) = 0$

- Propagation constant: $\gamma = (R' + j\omega L')(G' + j\omega C')$ $\quad R', L', C'$ values are per unit length.

  $\gamma = \alpha + j\beta$ $\quad \alpha$: attenuation constant ; $\beta$: phase constant $(= k)$

$$\begin{cases} V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} = \dfrac{V_0^+}{Z_0} e^{-\gamma z} - \dfrac{V_0^-}{Z_0} e^{\gamma z} \end{cases}$$

For lossless line: $\gamma = j\beta$

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

For lossless line: $Z_0 = \sqrt{\frac{L'}{C'}}$ ; $\beta = \omega\sqrt{L'C'}$ ; $\alpha = 0$

For lossless TEM line: $\beta = \omega\sqrt{\mu\varepsilon}$ $(= k)$ and $u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$ and if $\mu = \mu_0$: $\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$

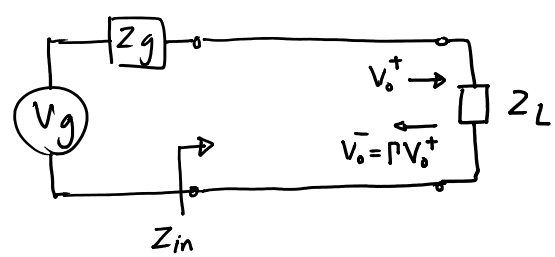
$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{-I_0^-}{I_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

For lossless $\gamma = j\beta$, and using $V_0^- = \Gamma V_0^+$, we can write:

$$\begin{cases} \tilde{V}(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right) \\ \tilde{I}(z) = \dfrac{V_0^+}{Z_0} \left(e^{-j\beta z} - \Gamma e^{j\beta z}\right) \end{cases}$$

Standing Wave Ratio: $S = \frac{|\tilde{V}|_{max}}{|\tilde{V}|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$

Input impedance: $Z_{in}(z) = Z_0 \dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$



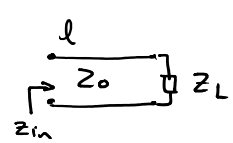

$$V_0^+ = \frac{\tilde{V}_g Z_{in}}{Z_{in} + Z_g} \frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}$$

At the load

Short circuit line: $Z_{in}^{sc} = jZ_0 \tan\beta l$

Open circuit line: $Z_{in}^{oc} = -jZ_0 \cot\beta l$

Quarter-wave Transformer: $Z_{in} = \frac{Z_0^2}{Z_L}$ $\quad l = \frac{\lambda}{4} + n\frac{\lambda}{2}$




Smith Chart: $z_L = \frac{1 + \Gamma}{1 - \Gamma}$ ; $y_L = \frac{1}{z_L} = \frac{1 - \Gamma}{1 + \Gamma}$