HW 3

Note Title 2/4/2008

Problem 1

ⓐ $\psi_1 = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{\pi}{L}\left(x+\frac{L}{2}\right)\right) e^{-i\omega_1 t}$

$$j = -\frac{ie\hbar}{2m}\left(\psi^* \frac{\partial\psi}{\partial x} - \psi \frac{\partial\psi^*}{\partial x}\right)$$

$$= -\frac{ie\hbar}{2m}\frac{2}{L}\left(\sin\left(\frac{\pi}{L}\left(x+\frac{L}{2}\right)\right) e^{i\omega t} \frac{\pi}{L}\cos\left(\frac{\pi}{L}\left(x+\frac{L}{2}\right)\right) e^{-i\omega t}\right.$$

$$\left. - \sin\left(\frac{\pi}{L}\left(x+\frac{L}{2}\right)\right) e^{-i\omega t} \frac{\pi}{L}\cos\left(\frac{\pi}{L}\left(x+\frac{L}{2}\right)\right) e^{i\omega t}\right)$$

$= 0$ => there is not current flow in a single standing wave.

ⓑ $\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}\left(x+\frac{L}{2}\right)\right) e^{-i\omega_1 t}$

$$\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi}{L}\left(x+\frac{L}{2}\right)\right) e^{-i\omega_2 t}$$

$$\psi = \frac{1}{\sqrt{L}}\left(\sin\left(\frac{\pi}{2}\left(x+\frac{L}{2}\right)\right) e^{-i\omega_1 t} + \sin\left(\frac{2\pi}{L}\left(x+\frac{L}{2}\right)\right) e^{-i\omega_2 t}\right)$$

Substitute in $j = -\frac{ie\hbar}{2m}\left(\psi^* \frac{\partial\psi}{\partial x} - \psi \frac{\partial\psi^*}{\partial x}\right)$ =>

$$j = -\frac{2e\pi\hbar}{mL^2}\left[\cos\frac{\pi x}{L}\cos\frac{2\pi x}{L} + \frac{1}{2}\sin\frac{\pi x}{L}\sin\frac{2\pi x}{L}\right]\sin(\omega_2-\omega_1)t$$

Hence, superposition of standing waves can carry current.

<u>Problem 2</u> $\psi = e^{i(kx-\omega t)}$

$$j = -\frac{ie\hbar}{2m}\left(e^{-i(kx-\omega t)}(ik)e^{i(kx-\omega t)} - e^{i(kx-\omega t)}(-ik)e^{-i(kx-\omega t)}\right)$$

$$= -\frac{ie\hbar}{2m}(2ik) = \frac{e\hbar k}{m}$$

remember $p = \hbar k \Rightarrow j = \frac{ep}{m} = \frac{emv}{m} = ev$

where v is the electron velocity.