HW4

Note Title 2/4/2008

Problem 2

The impedance matching happens when:

$$\frac{m_2}{m_1} = \frac{E - V_0}{E} \Rightarrow E = \frac{V_0}{1 - m_2/m_1}$$

$V_0 = 1.5\,eV$ & $m_1/m_2 = 3 \Rightarrow$

$$E = \frac{1.5}{1 - 1/3} = 2.25\,eV$$

For the transmission flux coefficient at $E \to \infty$

notice that: $E - V = \frac{\hbar^2 k^2}{2m} \to k = \frac{\sqrt{2m(E-V)}}{\hbar}$

$$\left.\frac{k_2}{k_1}\right|_{E\to\infty} = \left.\frac{\sqrt{2m_2(E-V_2)}}{\sqrt{2m_1(E-V_1)}}\right|_{E\to\infty} = \sqrt{\frac{m_2}{m_1}}$$

$$\left.\frac{v_2}{v_1}\right|_{E\to\infty} = \left.\frac{m_1}{m_2}\,\frac{k_2}{k_1}\right|_{E\to\infty} = \frac{m_1}{m_2}\sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{m_1}{m_2}}$$

$$\left.\text{Trans}\right|_{E\to\infty} = \left.\frac{v_2}{v_1}\,\frac{4}{\left(1+\frac{v_2}{v_1}\right)^2}\right|_{E\to\infty} = \sqrt{\frac{m_1}{m_2}}\,\frac{4}{\left(1+\sqrt{\frac{m_1}{m_2}}\right)^2}$$

For $\frac{m_1}{m_2} = 3$ this gives:

Trans = 0.928

Reflection = $1 - 0.928 = 0.072$

This suprising as it says that an electron with infinite energy is reflected by a finite potential step!