Multiband retardation control using multi-twist retarders

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ABSTRACT

We introduce and demonstrate an approach to create highly chromatic retardation spectra across various wavelengths. The design approach is based on Multi-Twist Retarder (MTR) principle where multiple liquid crystal polymer layers are coated on top of each other on a single substrate. Previous MTRs have been applied to develop broadband achromatic retarders, but here we show that MTRs are quite flexible, and their retardation spectrum can be tuned to create arbitrary profiles. As a representative example, we show this tailorability by creating a retarder which produces approximately zero retardation in visible (500-900 nm) and half-wave retardation in near-infrared (1-2.7 \( \mu \)m) wavelength region. This would provide enhancement in remote sensing, telecom, and spectroscopy systems where it is advantageous to have an optical element which affects only one band, but is largely transparent otherwise.

Keywords: Multi-Twist Retarders, retardation control, half-wave retarders, liquid crystals, MTR, chromatic

1. INTRODUCTION

Precise retardation control is an important when dealing with measurement of elements or within a filtering setting such as with remote sensing, telecom, or spectroscopy systems. Previous attempts which allowed for multiple bands of different output retardation, both in the achromatic and chromatic schemes, resulted in complex elements that had complicated assembly requirements, such as Solc; Lyot-Ohman; and a hybrid of these two designs, Evans. These designs, called birefringent chain filters by their creators, can be thought of as multiband or narrowband controllers due to the passband like nature of the resulting transmission spectrum.\(^1\) These bands have a complex controlled effect on the output retardation.

New fabrication methods have been developed to simplify the element alignment and thus assembly process by coating birefringent photoalignable materials on a single substrate. Initially, these new fabrication methods were used to create passive achromatic elements.\(^2\) Here we show that it is possible to use these methods to develop chromatic Multi-Twist Retarders (MTR) with similar properties to the achromatic MTRs introduced previously. With chromatic MTRs, the Stokes vector at the output of the retarder can vary across wavelength, allowing for bands with different, precisely tunable retardation.

2. BACKGROUND

2.1 Traditional Approaches to Chromatic Retardation Control

In the past, birefringent chain filters were developed to allow narrow bandwidths of transmitted signal to pass through a system of waveplates, birefringent plates, and polarizers. These color filters fall into two main categories: Lyot-Ohman and Solc.\(^1,3\) With Solc-type filters, there are multiple retarders in series with a polarizer on both ends of the stack. The two types of Solc filters, shown in Fig.1(a-b), describe the methods to align the orientation angle throughout the system. Folded Solc filters have the optical axes of the retardation plates so that they vary \( \pm \rho \), where \( \rho \) is defined as \( \pi/4N \) and \( N \) is the number of plates in the system.\(^4\) Fan Solc filters have retardation plates optical axes that increase with each additional element and rotate around from 0 to 2\( \pi \), with all the elements in the system.\(^3,4\)

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Lyot-Ohman systems have a series of birefringent elements each separated by a polarizer. This separation of the birefringent stack makes it different than our groups work. A derivative of the Lyot-Ohman, the Evans filter, which has several sets of thick waveplates and quarter waveplates (QWP) separated by polarizers is shown in Fig. 1(c). Although the filter designs can have high transmission, 86% as shown by Evans, there is still the problem with the thickness of the design, alignment of each element in the stack, and small useable aperture. In addition, all these designs depend on the use of polarizers which act to reduce the signal output.

Another approach studied was the use of bi-layer polarization gratings for optical filters which had the advantage of removing the lossy polarizers in the stack, but still had the limitation that each component had to be aligned precisely.

![Figure 1. Two Types of Solc Filters: (a) Folded and (b) Fan and (c) Evans filter with QWPs aligned to 0°. We see that each retardation controller is a unique set of individual polarizers and waveplates.](image)

### 2.2 Multi-Twist Retarders

MTRs are passive birefringent retarders created with doped liquid crystals coated onto a single prepared substrate. The substrate itself is prepared with Light Polymerizable Polymer (LPP) ROP-108 (Rolic Technologies Ltd.) to align the liquid crystals to the appropriate angle $\phi_0$. Each liquid crystal layer has the parameters of twist $\phi_m$ and thickness $d_m$, which allow for the generation of distinct transmission profiles as illustrated in Fig. 2. The parameter $M$ specifies the number of individual transform matrices that are part of the full MTR transformation matrix. A higher $M$ equates to more layers each with individual twists and thicknesses. The total number of defined parameters for the system is $2M + 1$. The twist of the layer is generated by doping nematic liquid crystal with small amounts of chiral liquid crystal in solvent at specific concentrations. For this test, we used the experimentally verified Liquid Crystal Polymer (LCP) RMS03-001c (Merck Chemicals Ltd.) with a birefringence profile of $\Delta n(\lambda) = 0.128 + 8340/\lambda^2$. Doping of the LCP was accomplished using CB15 (Merck Chemicals Ltd.) and C45S (LC Matter Corp).

Each chromatic design begins with a set of transformation matrices originally design for twisted nematic cells. One transformation matrix, $T_m$ describes an individual layer of the full design with the individual parameters for each layer of normalized phase retardation $\zeta_m$, magnitude of normalized retardation and twist $\chi_m$, and mean of the twist $\bar{\phi}$. Each of these components have the properties:

$$\zeta_m = \Gamma_m(\lambda)/2 = \pi\Delta n(\lambda)d_m/\lambda$$

(1)
χ_m = \sqrt{\bar{\chi}_m^2 + \phi_m^2} \tag{2}

\bar{\phi} = 1/m \sum_{i=0}^{m} \phi_i + \pi/2 \tag{3}

The series of transform matrices become a full MTR design when multiplied together, \( \mathbf{T}_{MTR} = \mathbf{T}_m \cdots \mathbf{T}_2 \cdot \mathbf{T}_1 \).

Figure 2. A chromatic MTR as viewed from (a) its side and (b) top view. In (a), each subsequent layer is aligned to the twist beneath it. From the top, the light passes through and follows the combined twist through the MTR.

3. HIGHLY CHROMATIC MTR THEORY

3.1 Design Approach

We use traditional linear algebra to represent the output Stokes vector such that \( \mathbf{S}^o(\lambda) = \mathbf{T}_{MTR} \cdot \mathbf{S}^i(\lambda) \), where \( \mathbf{S}^o(\lambda) \) is the Stokes vector at the output. With the chromatic MTR system, \( \mathbf{S}^o \) is a function of wavelength, thus complex chromatic retardation schemes can be achieved.

A specific MTR cost function \( f = 1 - \mathbf{S}^o(\lambda) \cdot \mathbf{S}^i(\lambda) \) is used in designing the proper parameters of twist and thickness for a MTR. This cost function when applied to the solutions of the \( \mathbf{S}^o(\lambda) \) helps determine how accurate the \( \mathbf{S}^o \) is to the target Stokes profile \( \mathbf{S}^t \) by taking the mean of the Stokes vector deviation from the target across all wavelengths. A good solution has been achieved when the \( f \) is minimized. To test the chromatic MTR theory, a retarder was designed and built for \( S_3 \) input that is produces zero retardation in visible (VIS, 500-900 nm) and half-wave retardation in near-infrared (NIR, 1-2.7 \( \mu \)m). This retarder converts from one circular polarization to another across the wavelengths of interest classifying it as a circular to circular halfwave retarder as seen in Fig.3. These retarders can be useful because their coatings could be applied to more complex patterned optics, such as polarization grating or q-plates which act on \( S_3 \) input. Three different designs for this circular to circular retarder were generated using the cost function for three different \( M \) values. The resulting designs are listed in Table 1.
Table 1. The twist and thicknesses of 3 different circular to circular chromatic half waveplate retarder solutions found with various different $M$'s.

<table>
<thead>
<tr>
<th>Design</th>
<th>$\phi_0$ (°)</th>
<th>$\phi_1$ (°)</th>
<th>$d_1$ (µm)</th>
<th>$\phi_2$ (°)</th>
<th>$d_2$ (µm)</th>
<th>$\phi_3$ (°)</th>
<th>$d_3$ (µm)</th>
<th>$\phi_4$ (°)</th>
<th>$d_4$ (µm)</th>
</tr>
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<tbody>
<tr>
<td>2TR</td>
<td>0</td>
<td>-59.7</td>
<td>4.84</td>
<td>59.7</td>
<td>4.84</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3TR</td>
<td>0</td>
<td>-21.9</td>
<td>6.56</td>
<td>95.0</td>
<td>1.16</td>
<td>-103</td>
<td>10.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4TR</td>
<td>0</td>
<td>75.4</td>
<td>1.04</td>
<td>47.6</td>
<td>6.15</td>
<td>-72.2</td>
<td>3.57</td>
<td>64.1</td>
<td>1.34</td>
</tr>
</tbody>
</table>

Figure 3. Circular to Circular Retarder with 3 bands of interest: zero wave, where the $S_3$ input is preserved, transition region, and a half wave region, where the $S_3$ input switches handedness. The 10% and 90% lines indicate the retarder should have high contrast to distinguish between the zero and half wave bands.

3.2 Impact of $M$

Generally, as the number of $M$ is increased the design matches the specification more precisely. The designs for a circular to circular chromatic half waveplate retarder with $M=2$, 3, and 4 are shown with $S_3$ input in Fig.4. Both the 2TR and 3TR fail at making accurate matches to the desired target profile. This is most profound in the VIS region where both of these designs do not match the shape of the target profile and also introduce additional peaks. We selected the 4TR version of the circular to circular chromatic halfwave retarder for fabrication because it quickly changed between the zero wave and half wave bands and had the best contrast between the VIS and NIR bands.
Figure 4. $M=2,3,$ and 4 designs are compared with $S_3$ input. (a) Transmission with parallel polarizers, $T_\parallel$ (b) Transmission with crossed polarizers $T_\perp$ (c) Effect on $S_3$, and (d) Retardation. The $M=4$ design matches all the target spectra most accurately.
4. CIRCULAR TO CIRCULAR CHROMATIC HALFWAVE RETARDER

4.1 Fabrication
Fabrication of the halfwave retarder is accomplished in the same way as for achromatic MTR, where an alignment layer is applied to a substrate and polymerized. Coatings of differently doped liquid crystal are applied to the aligned substrate and polymerized between layers. The recipe and mixtures needed to create the final part are listed in Tables 2 and 3 respectively.

Table 2. LCP mixtures for the circular to circular chromatic halfwave retarder.

<table>
<thead>
<tr>
<th>Mix</th>
<th>Materials</th>
<th>wt:wt Ratio</th>
<th>Net Chiral: LCP-Solids: Solvent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw LCP</td>
<td>Set by Manufacturer</td>
<td>Set by Manufacturer</td>
<td>0: 0.3: 0.7</td>
</tr>
<tr>
<td>A</td>
<td>CB15(+) : PGMEA</td>
<td>0.02 : 0.98</td>
<td>0.02 : 0 : 0.98</td>
</tr>
<tr>
<td>B</td>
<td>C45S(-) : PGMEA</td>
<td>0.02 : 0.98</td>
<td>0.02 : 0 : 0.98</td>
</tr>
<tr>
<td>C</td>
<td>Mix-A : Raw LCP</td>
<td>0.3 : 1</td>
<td>0.00462: 0.231: 0.765</td>
</tr>
<tr>
<td>D</td>
<td>Mix-A : Raw LCP</td>
<td>0.02 : 1</td>
<td>0.000392: 0.294: 0.705</td>
</tr>
<tr>
<td>E</td>
<td>Mix-B : Raw LCP</td>
<td>0.022 : 1</td>
<td>0.000431: 0.294: 0.706</td>
</tr>
<tr>
<td>F</td>
<td>Mix-A : Raw LCP</td>
<td>0.17 : 1</td>
<td>0.00291: 0.256: 0.741</td>
</tr>
</tbody>
</table>

Table 3. The recipe for the circular to circular chromatic halfwave retarder.

<table>
<thead>
<tr>
<th>Design</th>
<th>Layer 1</th>
<th>Layer 2</th>
<th>Layer 3</th>
<th>Layer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4TR</td>
<td>Mix-C, 1160 rpm</td>
<td>Mix-D, 585 rpm</td>
<td>Mix-E, 700 rpm</td>
<td>Mix-F, 965 rpm</td>
</tr>
</tbody>
</table>

4.2 Results
For the first measurement, the final waveplate was measured using an in-house Full-Stokes Polarimetry system with $S_1$ input polarization with a wavelength band of 425 to 800 nm. From the Stokes output of the retarder, the unique parameters of twist and thickness were fitted to the data. This fit, displayed in Table 4, was used as the best model of the data so that it is possible to model what would be like with a $S_3$ input polarization, since broadband creation of $S_3$ input was not feasible. The fit, along with the data and the design for $S_1$ are shown in Fig. 5.

Table 4. The fit found for the circular to circular retarder from in-house Full-Stokes Polarimetry system.

<table>
<thead>
<tr>
<th>Fit</th>
<th>$\phi_0$ (°)</th>
<th>$\phi_1$ (°)</th>
<th>$d_1$ (µm)</th>
<th>$\phi_2$ (°)</th>
<th>$d_2$ (µm)</th>
<th>$\phi_3$ (°)</th>
<th>$d_3$ (µm)</th>
<th>$\phi_4$ (°)</th>
<th>$d_4$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4TR</td>
<td>0</td>
<td>83.6</td>
<td>1.23</td>
<td>55.9</td>
<td>5.62</td>
<td>-85.7</td>
<td>3.82</td>
<td>62.8</td>
<td>1.26</td>
</tr>
</tbody>
</table>

A Varian Cary 5E was used to get a broadband transmission spectrum view of the retarder to show that this retarder has a controlled broadband effect with the $S_1$ input available in the lab, shown in Fig. 6. The transmission captured was between parallel linear polarizers. The design and fit are overlayed onto this data. With the Cary 5E data, we see the fit is more accurate in the VIS than in NIR. This fit discrepancy likely is due to the fit being calculated for 425 to 800 nm and not the entire 500 to 2700 nm spectrum. Across the entire spectrum, the data matches the general shape of the design, but does not reach the same level of contrast.

Modeling the fit using $S_3$ input, seen in Fig. 7, shows a VIS region that does not convert the $S_3$ input and a NIR region that is approximately halfwave. In NIR, the contrast is lower than the design and transmission data from Cary 5E suggest. This could be due to the error in fit model seen for $S_1$ input.
Figure 5. Final retarder measured and fit with Full-Stokes Polarimetry system with linear source and a 425 to 800 nm wavelength band. The resulting fit was used to model the retarder’s response to $S_3$ input.

Figure 6. Broadband transmission measurement of retarder with Varian Cary 5E, a $S_1$ input tool.
Figure 7. Design and fit of circular to circular chromatic halfwave retarder are compared with $S_3$ input. (a) Transmission with parallel polarizers $T_{//}$; (b) Transmission with crossed polarizers $T_{\perp}$; (c) Effect on $S_3$, and (d) Retardation.
5. CONCLUSION

We have developed and demonstrated chromatic MTRs that have specific bands with different chromatically controlled retardations. Previously MTRs were designed to augment the retardation achromatically, now we can generate chromatic designs that can precisely tailor the retardation spectrum of a waveplate across the wavelengths of interest. To show this in practice a representative sample, a circular to circular halfwave retarder, was created to test the chromatic theory. This circular to circular retarder shows it is possible to create bands of different distinct retardation with MTRs. The chromatic branch of MTRs opens up a variety of new elements where the chromatic coating could be applied to complex patterned optical elements that work with $S_3$ input, i.e. polarization gratings and $q$-plates, for additional control of the retardation spectrum and bandwidth properties.

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REFERENCES