On the Expected Connection Lifetime and Stochastic Resilience of Wireless Multi-hop Networks

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Abstract—To understand how node mobility and Byzantine node failures affect connectivity of wireless multi-hop networks, this paper investigates resilience of geometric random graphs to lifetime-based node failure and derives the expected connection time before an end-user is isolated from the graph. Different from previous analytical studies, which mainly focused on the so called critical transmission range, our study sheds light on the resilience analysis from the perspective of end-user's connection experiences. In the paper, we first introduce a simple but general node behavior model by a semi-Markov process. Then we apply the theory of renewal process to the degree of a generic node and analyze the stochastic property of node connection time. At last, we provide the probability that the node isolation event occurs within any end-user's lifetime and a close-form approximation of the network resilience. Our analysis and numeric simulation results indicate that networks with heavy-tailed lifetime (such as Weibull) distributions provides no improvement than those with light-tailed (e.g., Exponential) distributions in terms of longer expected connection lifetime for any end-user. Further, node mobility has more significant impact than lifetime does.

I. INTRODUCTION

Resilience of wireless multi-hop networks has attracted significant attention in research literature [1], [2], [3]. A classical problem in this line of study is to understand the connectivity of networks under random failures [4], [5] and node misbehaviors [6], [7]. To this end, many existing works assume independent node behaviors and random link or node failures and begin with an investigation into node isolations. For example, in [4], the connection resilience of ad hoc networks, with regard to random node failures, was analyzed as a fault tolerance measure. In [6], the problem of node isolation due to misbehaving neighbors was first studied, then the connectivity of entire network was approximated by using the probability of individual node isolation. These works all provide stochastic relationship between network resilience and a variety of impacts, such as node distribution, system size, connectivity requirement, and node failure probability; however, few works consider networking service availability from the perspective of end-users.

This work is motivated by considering the intrinsic behavior of end-users in wireless multi-hop networks. In these networks, especially mobile ad hoc networks, end-users (or nodes) may enter or depart networks at any time and any location; meanwhile, failures develop due to various reasons, such as node mobility, power depletion, or just voluntary user decision to leave. Thus, for each end-user, he/she may be more interested in whether he/she will encounter isolation events, i.e., the case of no active neighbors, during his/her lifetime in a network. In other words, after a node enters a network, as long as the node can enjoy a stable connection during its lifetime, the networking service provided by the network is "resilient" for this node. Therefore, we are interested to investigate: *What is the expected connection time before a node is isolated; what is the probability that a node outlives all of its neighbors; what is the resilience of a network in terms of the satisfaction of end-users to their connection experience?*

To cope with the questions raised above, we start our investigation with a simple but generic node behavior model in which a node transits its behavior between *active* and *failed* associated with arbitrary distributed transition times. Then we use renewal process to model the number of active neighbors of a node at any time and derive the expected connection time before a node is isolated. Next, we use the probability of a node outliving all its neighbors as the cornerstone to provide an approximation of the resilience of the entire network. Here "resilience" generally refers the probability of a network having no node isolation; while it also refers to the ability of an arriving user to stay connected to the rest of graph in this paper. Note that a similar methodology was used in [8] to address the resilience for peer-to-peer networks; however, contrary results are obtained for wireless multi-hop networks in this work. Specifically, numeric simulation results show that heavy-tailed lifetime distribution actually does not provide improvement compared with exponential lifetime distribution, and the node mobility has more significant impact on the expected connection time.

The rest of the paper is organized as follows: Section II introduces our node behavior model and the assumptions used; Section III derives the expected connection time of a generic node; Section IV provides the close-form approximation of network resilience; followed by future works and conclusions in Section V.

II. MODELS AND ASSUMPTIONS

In this section, we introduce our node behavior model and explain the assumptions used later in the paper. First, we model a wireless multi-hop network by a *Geometric Random Graph* [9], in which N nodes are independently and uniformly distributed in a metric space, and a link exists between two nodes if and only if their distance is less than a predefined

transmission radius. Nevertheless, we do not confine the system size N fixed during the whole network lifetime; instead, we allow it to vary with time according to any arrival/departure process as long as N stays sufficiently large. Note that in this paper, we assume that networks are well-connected and nodes moves independently with similar statistic properties.

To describe an individual end-user's behavior, we adopt the node behavior model proposed in [6] but simplify it to a twostate semi-Markov process in order to keep our derivations tractable. For each node in the network, it behaves either in a cooperative (C) state or in a failed (F) state. A node is considered to be cooperative if it works properly and obeys all routing and forwarding rules; otherwise, it is said to be failed. Most possible reasons for a node to become failed may include but not limited to: moving out of a network, depleting all its power, malfunctioning unexpectedly, or turning down due to voluntary end-user's decision. Meanwhile, we also allow node to recover from the failed state again into the cooperative state, by moving back in the network field, recharging/replacing batteries or just a restarting. Notice that the transition time T_{ii} $(i, j \in \{C, F\})$ between states may be distributed arbitrarily, a semi-Markov process is quite appropriate to describe the node behavior aforementioned. The state diagram is depicted in Fig. 1, where p_{ij} $(i, j \in \{C, F\})$ are the transition probabilities, and $F_{ii}(t)$ are the cumulative distribution functions (CDF) of the corresponding transition times.



Fig. 1. A semi-Markov node behavior model.

Note that in our model, it can be easily proved that $p_{cf} = p_{fc} = 1$, $T_{cf} = T_c$ (the sojourn time in state C), and $T_{fc} = T_f$ (the sojourn time in state F). In the following context, we call T_c and T_f as a node's *Lifetime* and *Recovery-time*, respectively. Although we allow end-user's lifetime distributed arbitrarily from heavy-tailed (such as Weibull) to Exponential, we require certain stationarity of lifetime T_c . This means that users joining the network at different times should have their lifetimes drawn from the same distribution $F_{T_c}(x)$.

It is worth pointing out that the lifetime of a node u is an *absolute* concept, that is, no matter where node u is or goes to, as long as it is in the network and provides cooperative networking services for other nodes, it is said to be *alive*. However, from the perspective of any generic node u, even if a neighbor, say v, keeps alive in the cooperative state, node v will be treated as a failure when it moves out of u's transmission range. Thus, node u's connection time is not only related with each neighbor's lifetime T_c but also related with the *link-time* between u and v. In this paper, we define the link-time between a pair of node u and v as the maximum time that u and v are in each other's transmission ranges, which is similar to the definition presented in [10]. For a generic node u, we denote its link-time with its neighbor i by $LT_i^{\{u\}}$ and may omit $\{u\}$ if it is explicit in the context.

III. EXPECTED CONNECTION TIME ANALYSIS

In this section, we address the first question prompted in Section I, that is: what is the expected *connection time*, denoted by TC, of a node before it is isolated by its neighbors.

A. Definition of Connection Time

To derive the expectation value of TC, denoted by E[TC], for a generic node u, we consider the following simple scenario without losing generality. We assume that a node uarrives the network at time $t = t_0$ and it has d_0 cooperative neighbors initially. As time goes, some neighbors of u may still be cooperative (or may be recovered back to cooperative state from a previous failure) while others may be failed (or have moved out of u's transmission range); in addition, it is possible for u to have some new neighbors (due to arrivals in u's transmission range). Nevertheless, it is reasonable to confine the maximum number of neighbors (both cooperative and failed) to a number d. Thus, at any time t > 0, neighbor i $(1 \le i \le d)$ can be considered to be on if it is cooperative or to be off if it is failed. In other words, the neighbor failure/recovery procedure can be represented by an on/off process $X_i(t)$:

$$X_i(t) = \begin{cases} 1, & \text{neighbor } i \text{ is cooperative at } t \\ 0, & \text{neighbor } i \text{ is failed at } t. \end{cases}$$

We illustrate the evolution of d neighbor on/off processes $X_1(t), \dots, X_d(t)$ in Fig. 2.



Fig. 2. The illustration of neighbor on/off processes.

By using the notation in (1), the number of cooperative neighbors of node u, called as u's *cooperative degree* (different from the traditional node degree) at time t is a random process, denoted by Y(t), and it is equal to:

$$Y(t) = \sum_{i=1}^{d} X_i(t).$$
 (1)

Similar to the definition in [6], a node is isolated at such time t_1 ($t_1 > t_0$) when all of its neighbors are in the failed state (see Fig. 2). Thus, we can formally define the connection time TC as follows

Definition 1: For a generic node u arriving at time t_0 , u's connection time $TC^{\{u\}}$ is defined as the *first hitting time to* 0 of u's degree Y(t), i.e.,

$$TC^{\{u\}} = \inf(t > t_0 : Y(t) = 0 | Y(t_0) = d_0),$$
 (2)

where d_0 is the initial degree of node u at time t_0 .

In the later context, without losing generality, we set $t_0 = 0$ and $d_0 = d$ to make our derivation tractable and omit $\{u\}$ in $TC^{\{u\}}$ for clarity.

B. Derivation of Expected Connection Time

We derive the expected connection time and use a similar method that was provided in [8]. Consider that Y(t) (defined in (1)) can be viewed as an *alternating renewal process* [11], in which each on stage corresponding to Y(t) > 0 and each off stage to Y(t) = 0. The evolution of the alternating renewal process Y(t) is illustrated in Fig. 3, in which $TC^{\{j\}}$ (connection time) is the duration of the *j*-th off stage.



Fig. 3. The illustration of the alternating renewal process Y(t).

To determine the expectation value of the first on stage duration, i.e., $E[TC^{\{1\}}]$, we first argue that $TC^{\{1\}}$ has the same distribution as all remaining connection times $TC^{\{j\}}$ $(j \ge 2)$. It is obvious that $TC^{\{1\}}$ is different from the other $TC^{\{j\}}$ in that the stage starts from $Y(t) = d_0 \ (1 \le d_0 \le d)$ while other stages start from Y(t) = 1. Nevertheless, when a node arrives the network, it still needs a period of time to "discover" its cooperative neighbors through some neighbor discovery mechanisms, such as directional antenna [12] and HELLO message [13]. Although the time for neighbor discovery may be quite short, compared with the connection time, it is still reasonable to view the stage of $TC^{\{1\}}$ starting from Y(t) = 1. Thus, we have $TC^{\{1\}}$ and other $TC^{\{j\}}$ (j > 1) the same distribution asymptotically. By applying the steady state distribution to Y(t) (Corollary in [11], p.p. 297), we have the probability of Y(t) > 0 at any time t as follows:

$$\lim_{t \to \infty} P(Y(t) > 0) \approx \frac{E[TC]}{E[TC] + E[TI]}.$$
(3)

Notice that the probability above can also be expressed as the probability that at least one cooperative neighbor is still alive, i.e.,

$$\lim_{t \to \infty} P(Y(t) > 0) = 1 - p_f^d, \tag{4}$$

where p_f is the probability of node u finding a neighbor in the failed state. Note that this probability is different from the *limiting probability* of any node being failed (refer to [6]). To explain this discrepancy, we consider that when node u arrives the network at time t_0 , one of its cooperative neighbors, say node i ($1 \le i \le d$), has being cooperative in the network for some time. We suppose that u outlives i, then the connection time *between* u and i (before the first disconnection) is dependent on the minimum between the residual lifetime of i and link-time between u and i (i.e., $LT_i^{\{u\}}$). Let RL_i denote the residual lifetime of neighbor i, T_{on} denote the duration of the process $X_i(t)$ being in the on state, then $T_{on} = \min\{RL_i, LT_i\}$. Similarly, we can also view $X_i(t)$ as an alternating renewal process, then we have p_f expressed by

$$p_f = \lim_{t \to \infty} P(X(t) = 1) \approx \frac{E[T_f]}{E[T_{on}] + E[T_f]},$$
 (5)

where $E[T_{on}]$ and $E[T_f]$ are the expected time connecting to any neighbor and expected recovery-time, respectively. Then by substituting (5) into (4), and equating (4) and (3), E[TC]is solved as:

$$E[TC] \approx E[TI] \cdot \left(\left(1 + \frac{E[T_{on}]}{E[T_f]} \right)^d - 1 \right).$$
 (6)

Next, we derive $E[T_{on}]$. Let $F_{RL}(t)$ and $F_{LT}(t)$ be the distributions of *i*'s residual lifetime RL_i and link-time LT_i , respectively, then $E[T_{on}]$ is:

$$E[T_{on}] = \int_0^\infty (1 - F_{RL}(t))(1 - F_{LT}(t))dt.$$
(7)

We can further represent the CDF of the residual lifetime $F_{RL}(t)$ as follows, by applying basic renewal process theory (*Theorem-6.12* [11] p.p. 285):

$$F_{RL}(t) = P(RL_i < t) = \frac{1}{E[T_c]} \int_0^t \left(1 - F_{T_c}(x)\right) dx, \quad (8)$$

where $E[T_c]$ is just the expected lifetime. By substituting (8) into (7), we obtain $E[T_{on}]$ as a function of $E[T_c]$, $F_{T_c}(t)$, and $F_{LT}(t)$:

$$E[T_{on}] = \int_0^\infty (1 - F_{LT}(t)) \left(1 - \frac{\int_0^t (1 - F_{T_c}(x)) dx}{E[T_c]}\right) dt.$$
(9)

Next, we derive E[TI]. Let the first instant of the stage of Y(t) = 0 starts at time t_1 , as shown in Fig. 2, we know that there are d - 1 already-failed neighbors being in recovery and one neighbor just failed at time t_1 . Thus, TI is the minimum time for the last neighbor to recover or for any one of the ongoing recoveries to finish. Let RR_i $(1 \le i \le d - 1)$ denote the *Residual Recoverytime* of any already-failed neighbor *i*, then TI can be expressed as $TI = \min\{RR_1, RR_2, \cdots, RR_{d-1}, T_f\}$. Further, if let $RR' = \min\{RR_1, RR_2, \cdots, RR_{d-1}\}$, then TI = $\min\{RR', T_f\}$ (refer to Fig. 2). Let $F_{RR'}(t)$ and $F_{T_f}(t)$ be the distributions of RR' and T_f , respectively, then E[TI] is:

$$E[TI] = \int_0^\infty (1 - F_{T_f}(t))(1 - F_{RR'}(t))dt.$$
(10)

We can further express $F_{RR'}(t)$ by

$$F_{RR'}(t) = P(RR' < t) = 1 - (1 - P(RR_i < t))^{d-1}$$

= $1 - \left(1 - \frac{\int_0^t (1 - F_{T_f}(x)) dx}{E[T_f]}\right)^{d-1},$ (11)

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where $E[T_f]$ is the expected recovery-time. By substituting (11) into (10), we have

$$E[TI] = \int_0^{E[T_f]} \left(1 - \frac{z}{E[T_f]}\right)^{d-1} dz = \frac{E[T_f]}{d}, \quad (12)$$

where $z = \int_{0}^{t} (1 - F_{T_{f}}(x)) dx$.

Finally, we obtain the following result.

Theorem 1: In a wireless multi-hop network modeled by the geometric random graph (defined in Section II), the expected connection time of any generic node is:

$$E[TC] \approx \frac{E[T_f]}{d} \cdot \left(\left(1 + \frac{E[T_{on}]}{E[T_f]} \right)^d - 1 \right), \qquad (13)$$

where $E[T_{on}]$ is given by (9).

Next, we will use numerical simulations to reveal the impacts of node mobility and node lifetime distribution on the expected connection time.

C. Numerical Simulation Results

From (13), we know that the expected connection time is related with the link-time distribution $F_{LT}(t)$, node lifetime distribution $F_{T_c}(t)$, expected lifetime $E[T_c]$, expected recovery-time $E[T_f]$, and average node degree d. We will demonstrate the impacts of these parameters on E[TC].

In our numerical simulations, we use exponential function for $F_{LT}(t)$, i.e., $F_{LT}(t) = 1 - e^{-\lambda t}$, where $1/\lambda$ is the expected link-time. For $F_{T_c}(t)$, we use exponential function first, i.e., $F_{T_c}(t) = 1 - e^{-\mu t}$, where $E[T_c] = 1/\mu$. In addition, we also use a two-parameter Weibull function to model $F_{T_c}(t)$, i.e., $F_{T_c}(t) = 1 - e^{-(t/\beta)^{\alpha}}$ with $E[T_c] = \beta \cdot \Gamma(1 + 1/\alpha)$ where $\Gamma(\cdot)$ is the Gamma function. Based on the settings above, we have E[TC] simplified immediately to the following special case for exponential link-time and lifetime:

$$E[TC] \approx \frac{E[T_f]}{d} \cdot \left(\left(1 + \frac{1}{E[T_f](\lambda + \mu)} \right)^d - 1 \right), \quad (14)$$

where λ and μ are as mentioned above.

In Fig. 4(a), we show the impact of the expected link-time, ranging from 10s to 100s, on the expected connection time E[TC]. It is clear that E[TC] increases dramatically when the expected link-time increases. Further, by increasing the expected lifetime $E[T_c]$ from 300s to 1200s, it is shown that E[TC] can be increased correspondingly; however, when the expected link-time is small (< 50s), the increase is trivial. This indicates that node mobility impacts E[TC] more than node lifetime and becomes a dominant factor especially when link-times are short. Fig. 4(b) shows the same results but in the log-log scale to show the linearity.

In Fig. 5(a) and 5(b), we show the positive impact of $E[T_c]$ on E[TC] and the negative impact of $E[T_f]$ on E[TC]. It is intuitively shown that the slower neighbors recover from the failed state, the shorter a node is connected with the network. An interesting result here is that the impact of a smaller degree is almost similar as that of a longer recovery-time.

Now, we use Weibull lifetime function and show the impact of the expected link-time on the expected connection time



Fig. 4. Impact of expected link-time, $d = 10, E[T_f] = 300s$.



Fig. 5. Impact of expected lifetime, $\lambda = 0.01$.

in Fig. 6(a) and 6(b). In the Weibull function, $\alpha = 2$ and $\beta = E[T_c]/\Gamma(1 + 1/\alpha)$. The result indicates that there is no improvement gained if the lifetime is heavy-tailed distributed. And the exponential lifetime can actually provide better upper bound for E[TC].



Fig. 6. Impact of expected link-time, $d = 10, E[T_f] = 300s$.

IV. STOCHASTIC ANALYSIS ON RESILIENCE

In the previous section, we studied the expected connection time for any generic node, which provides individual user's the expectation on how long they can have a stable connection before the first disconnection. In this section, we further investigate the probability that a node is isolated and the network resilience in terms of the global connectivity.

A. Probability of Node Isolation

In this paper, the node isolation event is the case that a generic node u outlives all its neighbors, which is equivalent to the fact of u's lifetime $T_c^{\{u\}}$ being greater than u's connection time TC. Thus, the probability of node isolation, denoted by γ , is formally defined as follows

Definition 2: For a generic node u, the probability of u being isolated is $\gamma = P(TC < T_c^{\{u\}})$.

If we know the CDF of TC, say $F_{TC}(t)$, we can use the following equation to calculate γ :

$$P(TC < T_c^{\{u\}}) = \int_0^\infty F_{TC}(t) f_{T_c}(t) dt, \qquad (15)$$

where $f_{T_c}(t)$ is the PDF (probability density function) of T_c (node lifetime). However, the exact distribution of TC is generally difficult to represent in a close-form since it depends on the evolution of the cooperative degree, i.e., random process Y(t) defined in (1). Fortunately, we have already touched the isolation probability γ in Section III-B. Notice that γ defined above is actually equivalent to the probability that the cooperative node degree becomes 0, then we have

$$\gamma = \lim_{t \to \infty} P(Y(t) = 0) \approx \left(\frac{E[T_f]}{E[T_{on}] + E[T_f]}\right)^d, \quad (16)$$

where d is the average node degree (not cooperative degree).

In addition, if we substitute (16) into (13), we can obtain

$$E(TC) \approx \frac{E[T_f]}{d} \left(\left(\frac{1}{\gamma}\right)^d - 1 \right), \tag{17}$$

which yields:

$$\gamma \approx \left(\frac{E[T_f]}{E[T_f] + d \cdot E[TC]}\right)^{\frac{1}{d}}.$$
(18)

Equation (18) provides new insight into the relationship between the node isolation probability and expected connection time. It can be easily predicted that as E[TC] increasing, the isolation probability γ decreases in logarithm.

B. Analysis on Network Resilience

In this subsection, we utilize the earlier derived metric γ to characterize the evolution of wireless multi-hop networks under random failures. In particular, we address the question: what is the probability that a wireless multi-hop network survives N end-users from potential node isolations during their entire lifetimes? We call this probability as network resilience and denote it by Ψ in the later context.

To address the question aforementioned, we take a close look at some useful results in random graph theory, especially the studies on geometric random graph. It has been proved that with high probability, the network becomes k-connected when the minimum node degree in the communication (geometric random) graph becomes k [9] (*Theorem-6.1.2*, p.p. 64). In other words, a wireless multi-hop network is connected if and only if every node has its cooperative degree (Y(t)) greater than 0, conditional on a sufficiently large system size N. Thus, the resilience of a network can be approximated by

$$\Psi \approx \lim_{t \to \infty} (1 - P(Y(t) = 0))^N.$$
⁽¹⁹⁾

Since $\lim_{t\to\infty} P(Y(t) = 0)$ is the isolation probability γ , by substituting (18) into (19), we finally have

$$\Psi \approx \left(1 - \left(\frac{E[T_f]}{E[T_f] + d \cdot E[TC]}\right)^{\frac{1}{d}}\right)^N.$$
 (20)

From (20), it is clear that the ratio between the expected recovery-time and expected connection time (i.e., $E[T_f]/E[TC]$) determines the value of resilience given fixed (maximum) node degree d and system size N. Specifically, suppose $E[T_f] \ll E[TC]$, then for any number $0 < \psi < 1$ we can obtain from (20) that the ratio should satisfy $\frac{E[T_f]}{E[TC]} < d \cdot (1 - \psi^{\frac{1}{N}})^d$ so that the resilience is no less than the given number, i.e., $\Psi > \psi$, asymptotically.

V. CONCLUSIONS AND FUTURE WORKS

In this work, we analyzed the stochastic property of node connection time and provided a close-form approximation. In addition, we derived the isolate probability and the network resilience as functions of the expected connection time. Our analysis indicates that networks with heavy-tailed lifetime (such as Weibull) distributions provides no improvement than those with light-tailed (e.g., Exponential) distributions in terms of longer expected connection lifetime for any enduser. Instead, node mobility has more significant impact than lifetime does. In our future works, we will take potential node misbehaviors into consideration to investigate their impact on the connection time and network resilience. Since this work is conducted on flat wireless multi-hop networks with uniform node distribution, we will also consider whether end-users and network can achieve better resilience in the networks with clusters or scale-free property.

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