Stability of Hierarchical Mobile Ad Hoc Networks

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***Abstract*—In hierarchical mobile ad hoc networks, the architectual stability is a key factor in determining the network performance. There are many solutions proposed to construct stable clusters, none of which has however revealed the maximum stability attainable in the mobile environments. In this paper, we define two metrics to measure the stability of hierarchical networks: the cluster lifetime and the inter-cluster link lifetime. We model and analyze the maximum of these two lifetimes with consideration of node mobility. The analytical understanding of maximum stability provides a guideline for the clustering and routing protocol design to optimize network performance.**

I. INTRODUCTION

The mobile ad hoc networks have attracted much attention in the research community for their flexible deployment. Coming with the flexibility, there are however limitations that prevent these networks from efficient and optimal performance. The node mobility is one of such factors that have significant impact on the network performance. Node movement introduces network topology dynamics, which further incurs frequent communication interruptions.

In large-scale ad hoc networks, the hierarchical architecture has been proven effective in addressing the scalability problems. However, node mobility still poses a big challenge. In a hierarchical network, *clusters* are constructed from the nodes in vicinity and communications are supported by the connected clusters. When a node moves, it may be attached to different clusters at different times, resulting in frequent path rediscovery each time it changes the point of attachment. The cluster connectivity affects the path stability too. When an inter-cluster link fails, all the communication paths traversing the broken link have to be replaced. Ideally, the stability of the clusters and their connections should be maximized in order to optimize the network performance.

A lot of work has been done to construct stable clusters in mobile environments [1]–[5]. They improve the cluster lifetime effectively. However, as these clustering algorithms are heuristic based, the maximum stability of the hierarchical architecture remains unknown yet. We study in this paper the longest possible lifetimes of the clusters and the inter-cluster links under the constraint of node mobility. We first identify the conditions under which the longest lifetimes will take place. Then we mathematically model and derive the expected maximum lifetimes. These lifetime bounds provide a clearer idea of the achievable network stability in mobile environments than the previous heuristic approaches. The conditions under which these bounds are reached also suggest the strategies to optimize the network performance.

The rest of this paper is organized as follows. We discuss the existing work on the clustering algorithms in Section II. We formulate the lifetime problem in Section III and present our analysis in Section IV. Section V provides the numerical results. Finally, Section VI concludes this paper.

II. RELATED WORK

The clustering algorithms construct clusters by determining the clusterheads and their affiliated clustermembers. To determine the clusterheads, node characteristic values may be used, such as the Lowest-ID [6] and the Weight-Based [7] algorithms. These algorithms have the problem of frequent cluster changes in mobile environments.

Node mobility [2], [3] has also been considered in the way that the low mobility nodes are selected to serve as clusterheads. Other strategies like the Least Cluster Change [4], Cluster Contention Interval [1] and GDMAC algorithm [5] extend the cluster lifetime after the clusters are constructed.

Although the consideration of mobility and the avoidance of excessive cluster changes have enhanced the cluster stability, all the discussed work has followed heuristic approaches and therefore does not provide the understanding of the *maximum* attainable stability of the hierarchical structure, which is the foundation for optimizing the network performance.

III. PROBLEM FORMULATION

In this paper we focus on the mobility impact on the architectural stability of hierarchical mobile ad hoc networks. Specifically, we assume the following node mobility model.

A. Node Mobility Model

This mobility model is very similar to the Random Walk model [8] except a few modifications to represent more realistic moving behaviors. In this model, a node alternates in the moving and the pausing phases. In the moving phase, the node chooses a random direction from $[0, 2\pi]$ and a random speed from $[v_{min}, v_{max}]$. The direction and the speed are both uniformly distributed random variables. The node also chooses a random destination in its travel direction. The travel distance to the destination is uniformly distributed in $[0, d_{max}]$. If the node hits the network boundary, it bounces back into the network to finish its planned travel distance. When the node arrives at the destination, it stays in the pausing phase for an exponentially distributed time with mean τ_s and then starts another movement. This Random-Walk-like mobility model maintains uniform node spacial distribution over time.

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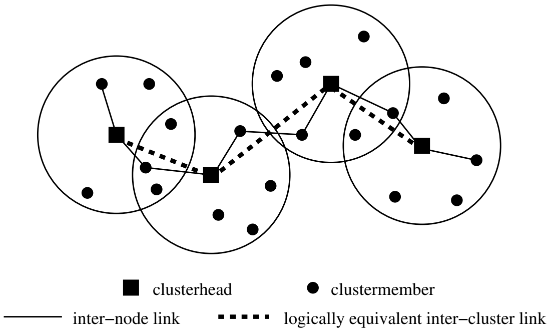

Fig. 1. The clustered structure and the communication path.

B. Network Model

The network consists of N uniformly distributed mobile nodes in an area of l^2 square meters. Every node moves independently and obeys the mobility model defined above. The communication range is r meters for every node. We assume link existence is solely determined by the distance between nodes and ignore the link disruptions due to wireless signal interferences and obstructions. A *cluster* is constructed by determining the clusterhead and its affiliated clustermembers. A clustermember is always connected directly to its clusterhead. Two clusters are neighbors if there exists at least one link that connects two nodes from the two clusters respectively. We illustrate the hierarchical network in Fig. 1.

C. Hierarchical Architecture Lifetime Bounds

Before we model and analyze the maximum lifetimes of the clusters and the inter-cluster links, we first identify the conditions for such maximum lifetimes to occur.

1) Maximum Cluster Lifetime: The lifetime of a cluster is determined by both the node mobility and the re-clustering criterion. A cluster breaks down when the composing nodes move apart. Besides, when certain conditions occur, for example two clusterheads come into contact or a clustermember hears a better clusterhead than its current one, additional re-clustering takes place. It is straightforward to see that if we defer re-clustering to the time of cluster breakdown due to node mobility, the cluster lifetime will achieve its maximum.

2) Maximum Inter-Cluster Link Lifetime: Since two clusterheads are connected if there exists at least one link that connects any two nodes from their respective clusters, the maximum inter-cluster link lifetime is determined by the continuous availability of such links.

IV. Modeling and Analysis of the Lifetime Bounds

Next, we model and analyze the maximum lifetimes of the hierarchical architecture based on our identified conditions.

A. Cluster Lifetime

The longest cluster lifetime is achieved when the clusterhead undertakes its role without interruption until all of its affiliated clustermembers have moved away. Since it is determined by the time when the last clustermember leaves, we investigate a clustermember's membership time first.

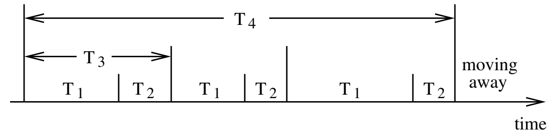

Fig. 2. The intervals of pauses and movements.

1) Cluster Membership Time: The longest membership time takes place when the clustermember stays affiliated to its clusterhead all the time until they move to r meters apart. This is measured by their neighboring time. We denote the longest membership time as the random variable T_m and discuss it in three cases corresponding to the initial mobility phases of the clusterhead and the clustermember at the time of cluster construction: i) both are stationary, ii) one is stationary and the other is moving, and iii) both are moving.

In the first case, for the ease of analysis we first assume that one node is fixed to its location such that it never moves. The other node pauses and moves in its vicinity. T_m in this case consists of an interval sequence of alternated pauses and movements until the movable node finally moves away from the fixed node. Fig. 2 depicts the interval sequence, in which T_1 and T_2 represent the random durations in the pausing and the moving phases respectively. T_1 is exponentially distributed with the mean $E(T_1) = \tau_s$, as specified in the mobility model. The mean of T_2 is determined as $E(T_2) = \tau_{1,I} = E(\frac{D_I}{V}) = E(D_I)E(\frac{1}{V})$, where $\tau_{1,I}$ denotes the mean time of *one* node moving *inside* the transmission area of the fixed node, D_I is the random travel distance *inside* the transmission area, and V is the random speed. We illustrate such a movement in Fig. 3(a). $E(D_I)$ is determined by

$$\begin{aligned} E(D_I) = & \frac{1}{(\pi r^2)^2}\int_0^{2\pi}\!\!\int_0^{r}\!\!\int_0^{2\pi}\!\!\int_0^{r} d(r_1,\theta_1,r_2,\theta_2)\, r_1 \mathrm{d}r_1 \mathrm{d}\theta_1\, r_2 \mathrm{d}r_2 \mathrm{d}\theta_2 \\ \approx & \frac{1}{(\pi r^2)^2}\sum_{\{r_1,\theta_1,r_2,\theta_2\}} d(r_1,\theta_1,r_2,\theta_2)\, r_1(\frac{r}{k})(\frac{2\pi}{k})r_2(\frac{r}{k})(\frac{2\pi}{k}) \\ = & \frac{4}{k^4 r^2}\sum_{\{r_1,\theta_1,r_2,\theta_2\}} d(r_1,\theta_1,r_2,\theta_2)\, r_1 r_2, \end{aligned} \quad (1)$$

where

$$d(r_1,\theta_1,r_2,\theta_2) = \sqrt{(r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1 - r_2\sin\theta_2)^2}. \quad (2)$$

Because no closed-form solution exists for the integral in (1), we approximate it by the numerical computation that divides the domain of each variable into k subsets and sums up the approximate integration result in each subset combination. The speed V is uniformly distributed and we compute $E(\frac{1}{V})$ as

$$E(\frac{1}{V}) = \int_{v_{min}}^{v_{max}} \frac{1}{v}\cdot\frac{1}{v_{max}-v_{min}}\,\mathrm{d}v = \frac{\ln(v_{max}) - \ln(v_{min})}{v_{max}-v_{min}}. \quad (3)$$

We define $T_3 = T_1 + T_2$. For mathematical tractability, T_3 is approximated by an exponential distribution. Its mean is $E(T_3) = E(T_1) + E(T_2) = \tau_s + \tau_{1,I}$. As there is a probability of $P_O = \frac{d_{max}^2 - r^2}{d_{max}^2}$ in each moving phase that the movable node travels beyond the reach of the fixed node,

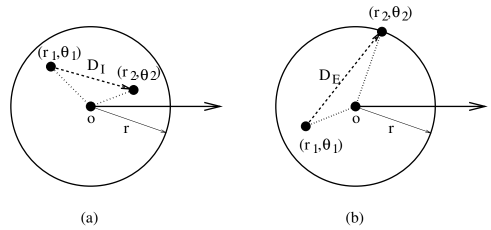

Fig. 3. The movements inside and beyond the coverage area of another node.

the total neighboring time T_4 is an exponentially distributed random variable with the mean $E(T_4) = \frac{E(T_3)}{P_O}$. Then we remove the assumption of one node fixed and consider the independent movements of both nodes. Let us denote the two nodes as A and B, and denote the neighboring time when A is fixed as $T_{4,A}$ and that when B is fixed as $T_{4,B}$. We have $T_m = \min(T_{4,A}, T_{4,B})$ and determine its mean as

$$E_{s,s}(T_m) = \frac{E(T_4)}{2} = \frac{\tau_s + \tau_{1,I}}{2P_O}, \quad (4)$$

where $E_{s,s}(T_m)$ denotes the mean in the case of both nodes being initially stationary.

In the second case, one node is stationary and the other is moving initially. With probability $P_I = 1 - P_O = \frac{r^2}{d_{max}^2}$, the moving node stops inside the transmission area of the stationary node. The mean of this duration is $\tau_{1,I}$. After that, the rest neighboring time is exactly what we have discussed in case 1 where the mean duration is $E_{s,s}(T_m)$. With probability P_O, the moving node has a destination outside the coverage of the stationary node. Let us denote the time before the two nodes become r meters apart as T_5, its mean is determined by $E(T_5) = \tau_{1,E} = E(\frac{D_E}{V}) = E(D_E)E(\frac{1}{V})$, where $\tau_{1,E}$ denotes the mean time of *one* node moving to the *edge* of the transmission area of the stationary node. The random travel distance to the *edge*, denoted as D_E, is shown in Fig. 3(b). Its mean is determined by

$$\begin{aligned} E(D_E) &= \frac{1}{2\pi \cdot \pi r^2}\int_0^{2\pi}\int_0^{2\pi}\int_0^{r} d(r_1,\theta_1,\theta_2)\, r_1 \mathrm{d}r_1 \mathrm{d}\theta_1\, \mathrm{d}\theta_2, \\ &\approx \frac{2}{k^3 r}\sum_{\{r_1,\theta_1,\theta_2\}} d(r_1,\theta_1,\theta_2)\, r_1, \end{aligned} \quad (5)$$

where

$$d(r_1,\theta_1,\theta_2) = \sqrt{(r_1\cos\theta_1 - r\cos\theta_2)^2 + (r_1\sin\theta_1 - r\sin\theta_2)^2}. \quad (6)$$

Numerical approximation is used in (5) similar to (1). Summarizing both possibilities, the mean membership time in the case of initially one node stationary and the other moving is

$$E_{s,m}(T_m) = P_I(\tau_{1,I} + E_{s,s}(T_m)) + P_O\tau_{1,E}. \quad (7)$$

In the third case, both nodes are moving initially. With probability P_I, they stop within each other's transmission area. Denoting the time to one node stopping as T_6, we have $E(T_6) = \tau_{2,I} = E(\frac{D_I}{V_R}) = E(D_I)E(\frac{1}{V_R})$, where $\tau_{2,I}$ denotes the mean time of *two* nodes moving *inside* each other's transmission area, and V_R is their relative speed. Fig. 4 depicts the formation of V_R, from which we obtain

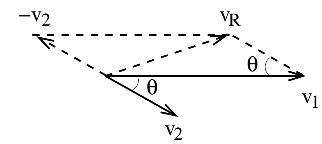

Fig. 4. The relative speed V_R.

$$\begin{aligned} E(\frac{1}{V_R}) &= \frac{1}{\pi(v_{max} - v_{min})^2}\int_{v_{min}}^{v_{max}}\int_{v_{min}}^{v_{max}}\int_0^{\pi} \frac{1}{V_R}\ \mathrm{d}\theta \mathrm{d}v_1 \mathrm{d}v_2 \\ &\approx \frac{1}{k^3}\sum_{\{\theta,v_1,v_2\}} \frac{1}{V_R}, \end{aligned} \quad (8)$$

where

$$V_R = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}. \quad (9)$$

Numerical integral approximation is used in (8) as before. $E(D_I)$ is obtained from (1). After one node has come to stop, the time to the other node stopping is described by T_2 and $E(T_2) = \tau_{1,I}$. After both nodes become stationary, the rest of their neighboring time is again the case 1. With probability P_O, the two nodes will move apart. There are two possibilities of their mobility phases at the time of being apart: both moving or one moving and the other stationary. Let us denote the respective probabilities as γ and $1-\gamma$. In the former case, we denote the random neighboring time as T_7 and obtain $E(T_7) = \tau_{2,E} = E(\frac{D_E}{V_R}) = E(D_E)E(\frac{1}{V_R})$, where $\tau_{2,E}$ denotes the mean time of *two* nodes moving to the *edge* of each other's transmission area. $E(D_E)$ and $E(\frac{1}{V_R})$ are obtained from (5) and (8) respectively. In the latter case, one node stops inside the transmission area of the other node first and then the other node continues to move away. The time to one node stopping is described by T_6 with mean $\tau_{2,I}$ and the time to the other node moving away is described by T_5 with mean $\tau_{1,E}$. γ is determined as follows. The probability of one node stopping before the two nodes become apart is

$$\begin{aligned} Q &\approx P\Big\{\frac{W \cdot X}{V} < \tau_{2,E}\Big\} \\ &= \iiint_{\frac{w \cdot x}{v} < \tau_{2,E}} \frac{1}{d_{max}(v_{max} - v_{min})} \mathrm{d}w\mathrm{d}x\mathrm{d}v \\ &\approx \frac{1}{k^3}\sum_{\{w,x,v\}} 1_{\{\frac{w\cdot x}{v} < \tau_{2,E}\}}(w,x,v) \end{aligned} \quad (10)$$

where W is uniformly distributed in $[0,1]$, X is the random travel distance, $W \cdot X$ is the residual travel distance at the time of cluster formation, and $1_{\{\frac{w\cdot x}{v} < \tau_{2,E}\}}(w,x,v)$ is an indicator function. Numerical approximation is used to compute Q. Then the probability of both nodes moving at the time they are apart is $(1-Q)^2$ and the probability of one moving and one stationary is $2Q(1-Q)$. Normalizing them, we obtain

$$\gamma = \frac{(1-Q)^2}{(1-Q)^2 + 2Q(1-Q)},\ \ 1-\gamma = \frac{2Q(1-Q)}{(1-Q)^2 + 2Q(1-Q)}. \quad (11)$$

Considering both P_I and P_O, the mean membership time is

$$\begin{aligned} E_{m,m}(T_m) = &\ P_I(\tau_{2,I} + \tau_{1,I} + E_{s,s}(T_m)) \\ &+ P_O(\gamma\tau_{2,E} + (1-\gamma)(\tau_{2,I} + \tau_{1,E})) \end{aligned} \quad (12)$$

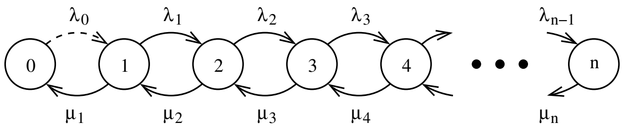

Fig. 5. The Markov transition diagram.

where $E_{m,m}(T_m)$ denotes the mean of T_m in the case that both nodes are moving initially.

Summarizing all the three cases, we can write

$$E(T_m) = P_{s,s}E_{s,s}(T_m) + P_{s,m}E_{s,m}(T_m) + P_{m,m}E_{m,m}(T_m), \tag{13}$$

where $P_{s,s}$, $P_{s,m}$, $P_{m,m}$ are the probabilities of the respective cases. We know from the mobility model that the mean pausing and moving durations are τ_s and $\tau_m = E(\frac{D}{V}) = E(D)E(\frac{1}{V}) = \frac{d_{max}}{2}E(\frac{1}{V})$ respectively. Thus, the probabilities of a node's mobility phases are $P_s = \frac{\tau_s}{\tau_s+\tau_m}$ and $P_m = \frac{\tau_m}{\tau_s+\tau_m}$, from which we have $P_{s,s} = P_s^2$, $P_{s,m} = 2P_sP_m$, $P_{m,m} = P_m^2$.

2) Cluster Lifetime: Let T_h denote the longest cluster lifetime. We use the Markov model shown in Fig. 5 to study its mean value, where a state represents the number of clustermembers in the cluster. We assume clustermembers come and leave in Poisson processes. Transitions take place when nodes join and leave. By denoting S_{init} as the initial state, we see that T_h is the transition time from S_{init} to S_0.

On state S_0 the clusterhead will re-cluster to merge into another cluster, so the clustermember joining rate $\lambda_0 = 0$. The joining rate λ_j on the other states S_j is state dependent with finite node population. To simplify the model, we truncate the Markov chain at state S_n, where $n \ll N$. In this truncated model $\lambda_j = \lambda$ $(1 \le j \le n-1)$. We observe from simulations that the cluster size always fluctuates within a few multiples of its initial value, so we choose $S_n = 5S_{init}$. λ is determined as follows. Denoting N_m and N_h as the total number of clustermembers and clusterheads in the network respectively,

$$\lambda_j = \lambda = \left(\frac{N_m}{E(T_m)} + \frac{N_h}{E(T_h)}\right) \cdot \frac{\pi r^2}{l^2} \cdot \beta_1 \cdot \beta_2 \quad (1 \le j \le n-1), \tag{14}$$

where $\frac{N_m}{E(T_m)} + \frac{N_h}{E(T_h)}$ accounts for the networkwide arrival rate of nodes seeking a cluster to join, $\frac{\pi r^2}{l^2}$ is the geographical factor considering the percentage that takes place in the clusterhead's transmission area, β_1 is the mobility factor, and β_2 is the clusterhead selection factor. β_1 is concerned with the fact that a node is able to join the clusterhead only when they have relative movement: a relative stationary node inside the clusterhead's neighborhood is likely to be an affiliated clustermember already and a relative stationary node out of the reach of the clusterhead does not have the chance to join this clusterhead. β_2 considers the chance for a node to join a particular clusterhead given the possibilities that it may hear several candidate clusterheads in its neighborhood and it may also become a clusterhead itself. We determine them as $\beta_1 = P_m + P_sP_m$ and $\beta_2 = \frac{1}{\frac{\pi r^2}{l^2}N_h+1}$. The state dependent clustermember departure rate is

$$\mu_j = j\mu, \quad \mu = \frac{1}{E(T_m)} \quad (1 \le j \le n). \tag{15}$$

On average S_{init} has $\frac{N_m}{N_h}$ number of clustermembers.

Defining t_j as the mean transition time from S_j to S_0,

$$\begin{cases} t_0 = 0, \\ t_j = \frac{1}{\lambda_j+\mu_j} + \frac{\mu_j}{\lambda_j+\mu_j}t_{j-1} + \frac{\lambda_j}{\lambda_j+\mu_j}t_{j+1} \quad (1 \le j \le n-1), \\ t_n = \frac{1}{\mu_n} + t_{n-1}. \end{cases} \tag{16}$$

The transition time from S_j to S_0 is the sum of the sojourn time on S_j and the transition time from the successive state (S_{j-1} or S_{j+1}) to S_0. The mean sojourn time on S_j is determined as $\frac{1}{\lambda_j+\mu_j}$. After that, a left transition occurs with probability $\frac{\mu_j}{\lambda_j+\mu_j}$ or a right transition occurs with probability $\frac{\lambda_j}{\lambda_j+\mu_j}$. At the boundary state S_n, only the left transition is possible because there are no joining nodes. Combining (14), (15) and (16), the solutions of t_j $(0 \le j \le n)$ can be obtained. Then $E(T_h) = t_{init}$. Note that (14) requires the knowledge of $E(T_h)$. In the computations, $E(T_h)$ is first assigned an initial value and then computed recursively until it converges.

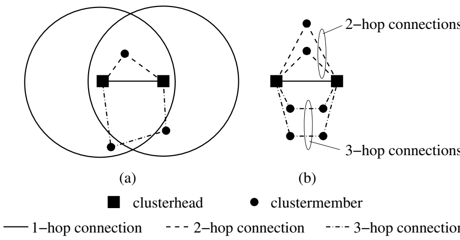

Fig. 6. The connection types and the logical link.

B. Inter-Cluster Link Lifetime

Fig. 6(a) illustrates all the connection types between neighbor clusterheads. We define the inter-cluster *Logical Link* to be the set of all these connections, as shown in Fig. 6(b). The logical link exists as long as *any* of the connection types exists. The longest inter-cluster link lifetime is then the lifetime of the logical link. The mean cluster membership time $E(T_m)$ can be generalized as the mean neighboring time between any two neighbor nodes. Thus we assume the inter-node links contained in the logical link have identical and exponential lifetime distributions with mean $\frac{1}{\mu} = E(T_m)$.

The lifetime of the logical link is determined by the initial composition of the link, the new connection arrivals and the failures of existing connections. Due to the unmanageable difficulty in determining the new connection arrival process, we approximate the lifetime with the time to the failures of all the initial connections, which serves as a lower bound on the real lifetime. Let n_i $(i = 1, 2, 3)$ denote the initial number of i-hop connections, $\mathcal{F} = \{F_x^{(j)}\}$ $(j = 1, 2, \cdots, \sum_{i=1}^{3} n_i)$ denote a permutation of the failure sequence of the $\sum_{i=1}^{3} n_i$ connections where $F_x^{(j)} \in \{F_1, F_2, F_3\}$ denotes that the j-th failure happens on an x-hop connection, $n_i^{(j)}$ denote the number of remaining i-hop connections after the $(j-1)$-th

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TABLE I
MOBILITY PATTERNS

Pattern	τ_s (min)	$[v_{min}, v_{max}]$ (m/s)	d_{max} (m)
MP1	8	$[1, 3]$	1000
MP2	6	$[1, 5]$	1000
MP3	4	$[1, 7]$	1000
MP4	2	$[1, 9]$	1000

but before the j-th failure in the sequence $\mathcal{F}$, and $1_i(F_x^{(j)})$ denote the indicator functions such that

$$1_i(F_x^{(j)}) = \begin{cases} 1 & x = i \\ 0 & x \neq i \end{cases} \qquad (i = 1, 2, 3). \tag{17}$$

The mean lifetime of the logical link is written as

$$E(T_l) = \sum_{\{\mathcal{F}\}} P(\mathcal{F}) T(\mathcal{F}), \tag{18}$$

where $P(\mathcal{F})$ is the probability of $\mathcal{F}$, $T(\mathcal{F})$ is the mean lifetime given $\mathcal{F}$, and $\{\mathcal{F}\}$ has cardinality $\binom{n_1+n_2+n_3}{n_1}\binom{n_2+n_3}{n_2}\binom{n_3}{n_3} = \frac{(n_1+n_2+n_3)!}{n_1!n_2!n_3!}$. $P(\mathcal{F})$ and $T(\mathcal{F})$ are determined by

$$P(\mathcal{F}) = \prod_{j=1}^{n_1+n_2+n_3} P(F_x^{(j)}) = \prod_{j=1}^{n_1+n_2+n_3} \frac{\sum_{i=1}^{3} i \cdot n_i^{(j)} \cdot 1_i(F_x^{(j)})}{\sum_{i=1}^{3} i \cdot n_i^{(j)}}, \tag{19}$$

$$T(\mathcal{F}) = \sum_{j=1}^{n_1+n_2+n_3} T(F_x^{(j)}) = \sum_{j=1}^{n_1+n_2+n_3} \frac{1}{\sum_{i=1}^{3} i \cdot n_i^{(j)} \cdot \mu}. \tag{20}$$

In (19), $\sum_{i=1}^{3} i \cdot n_i^{(j)}$ is the total number of inter-node links before the j-th connection failure, $\sum_{i=1}^{3} i \cdot n_i^{(j)} \cdot 1_i(F_x^{(j)})$ is the number of inter-node links of which any break will result in $F_x^{(j)}$, their ratio determines $P(F_x^{(j)})$, and $P(\mathcal{F})$ is the multiplication of all the $P(F_x^{(j)})$'s. In (20), $\frac{1}{\sum_{i=1}^{3} i \cdot n_i^{(j)} \cdot \mu}$ is the mean time between the $(j-1)$-th and the j-th connection failures, and $T(\mathcal{F})$ is the sum of all these intervals.

V. NUMERICAL RESULTS

Due to problem complexity, analytical determination of N_h, N_m and n_i $(i = 1, 2, 3)$ are difficult, so their average values are obtained from simulations and then applied to the analysis. We configure an example network as $N = 240$, $l = 2000$m, $r = 250$m, and specify four mobility patterns by tuning the node moving speed and pause time as shown in Table I. Besides, we have also implemented the Lowest-ID and the GDMAC ($K = 3$, $H = 32$ as in [5]) algorithms in NS-2 [9] as comparison.

The mean cluster membership times from the analysis and the simulations are shown in Fig. 7. The Lowest-ID has a quite short cluster membership time as compared to the theoretical bound. The GDMAC improves over the Lowest-ID, but there is still a noticeable gap from the analytical bound. Similar observation on the mean cluster lifetime is shown in Fig. 8. We determine N_h and N_m by letting each node stay in its role (clusterhead/clustermember) as long as its cluster is still valid. The measured averages are $N_h = 56$ and $N_m = 184$ across the four mobility patterns with very slight variations (± 2). The transition times t_j are then determined and t_3 is chosen to be t_{init}, as S_3 is the state closest to the mean cluster size $\frac{184}{56} = 3.28$. We see in Fig. 8 that the cluster lifetime of the Lowest-ID is significantly shorter than the analytical bound and the GDMAC does not reach this bound yet.

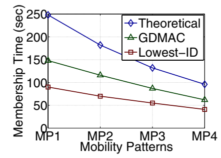

Fig. 7. $E(T_m)$.

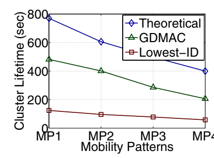

Fig. 8. $E(T_h)$.

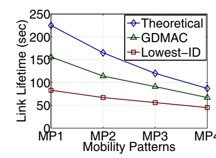

Fig. 9. $E(T_l)$.

With $N_h = 56$ and $N_m = 184$, we measure the average number of connections between neighbor clusterheads and obtain $n_1 = 0.23$ (± 0.03), $n_2 = 1.85$ (± 0.1), $n_3 = 1.05$ (± 0.15) across the four mobility patterns. As they are not integers, we approximate them as the following:

$$n_1 = \begin{cases} 1 & w.p.\ 0.23 \\ 0 & w.p.\ 0.77 \end{cases}, \quad n_2 = \begin{cases} 2 & w.p.\ 0.85 \\ 1 & w.p.\ 0.15 \end{cases}, \quad n_3 = \begin{cases} 2 & w.p.\ 0.05 \\ 1 & w.p.\ 0.95 \end{cases}.$$

The mean inter-cluster link lifetime is then determined from (18). Fig. 9 plots the results from analysis and simulations. Again we observe the similar instability of Lowest-ID and the gap between GDMAC and the analytical bound.

VI. CONCLUSIONS

Node mobility has a significant impact on the stability of hierarchical architectures in ad hoc networks. We have presented in this paper the mathematical modeling and analysis of the network stability in terms of the cluster lifetime and the inter-cluster link lifetime, using a Random-Walk-like mobility model. Our analysis demonstrates that the network stability can be maximized by two strategies: keeping the existing clusters unchanged to the largest extent and maximizing the connectivity among the existing clusters. These findings provide helpful guidelines for the clustering and routing protocol design to optimize the performance of mobile ad hoc networks.

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