

On the Devolution of Large-scale Sensor Networks in the Presence of Random Failures

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Abstract—In battery-constrained large-scale sensor networks, nodes are prone to random failures due to various reasons, such as energy depletion and hostile environment. Random failures can substantially impact the communication connectivity, which in turn impairs the sensing coverage. Redeploying additional sensors is one effective way to maintain the connectivity; however, it may be infeasible and costly to replace failed sensors one by one. When should a redeployment be conducted is an interesting and important question in designing resilient sensor networks. In this paper, we tackle this problem by investigating the devolution process of large-scale sensor networks. We first define a new metric called the *first partition time*, which is the first time that a network starts to discompose to multiple isolated small components. Then we analyze the devolution process in a geometric random graph from a percolation-based connectivity perspective and obtain the condition under which the graph is not percolated. Finally, we find out that the lower bound of the first partition time depends on the node lifetime distribution and should be of the order between $\log(\log n)$ and $(\log n)^{1/\rho}$ for $\rho > 1$. This result provides a theoretical upper bound of the latest time that a redeployment has to be carried out.

I. INTRODUCTION

In large-scale wireless sensor networks, nodes are vulnerable to multiple failures caused by devastating environment, software or hardware malfunctioning, and energy depletion. These random failures can impact connectivity, which is the necessary condition for coverage since the transmission ranges of sensors are typically greater than their sensing ranges. As pointed out in [1], redeploying additional nodes is necessary to replace failed sensors and maintain a connected topology; however, for many unattended outdoor sensor network applications, such as military surveillance and environmental monitoring, it is usually inefficient and costly to replace failed sensors one by one. Therefore, it is important for us to know the proper time before the breakdown of network topology so that a batch of new sensors can be redeployed at once. This demands an in-depth understanding on the devolution process of wireless networks, especially the critical time when network partitioning happens due to random failures.

We notice that the connectivity of wireless networks has attracted growing interest and been studied extensively [2]–[10]. While the only work addressing the transition time in the devolution of sensor networks, to the best of our knowledge, was presented in [10]. In the paper, authors showed that the appearance of isolated nodes or parts occurs very sharply in

time and there exists a critical time, determined solely by sensor lifetime distribution, at which the number of emergent lacunae of a given (geometric) size is asymptotically Poisson, as the number of sensors grows. Nevertheless, in traditional connectivity studies, a network is disconnected as long as there exist isolated nodes, which is, however, not applicable for sensor networks. Sensor nodes are usually densely deployed hence the underlying network has high redundancy for sensing and communications [1], which implies that a few of dead nodes or isolated nodes should not be considered as the sign of network partitioning. Thus, a full connectivity requirement is unnecessary and impractical for sensor networks, and we need a new concept characterizing the connectivity change due to random failures.

In this paper, we are interested in the following questions: *for a large-scale wireless sensor network in the presence of random failures, when does the network first become fully partitioned?* Here a network is called *fully partitioned* if the network discomposes into many isolated parts so that even the largest component contains a small portion of sensors only [11]. To tackle this problem, we couple the devolution process in a wireless sensor network with a continuum percolation process [12], [13] in a geometric random graph [14]. By using the concept of *percolation probability*, we first define *first partition time* as the first time at which a network is fully partitioned, then we analyze the percolation condition under which a geometric random graph does not have a giant component. Through the analysis, we find out that the first partition time is dependent on the network size n , initial density λ_0 , transmission radius r , and individual node's lifetime distribution $S(t)$. Finally, when $S(t)$ is light-tailed (exponential), the lower bound of the first partition time is shown to be of the order $\log(\log n)$; while if $S(t)$ is heavy-tailed (Pareto), the order is $(\log n)^{1/\rho}$, where ρ is the shape parameter of Pareto distribution and $\rho > 1$.

The rest of this paper is organized as follows. In Section II, we define the first partition time and formulate the problem. In Section III, we overview the application of percolation theory in wireless networks. In Section IV, we derive the lower bound of first partition time, followed by conclusion in Section V.

II. PROBLEM FORMULATION

In this work, we assume that all nodes of a wireless sensor networks are uniformly and randomly distributed on \mathbb{R}^2 and

confined into a square $B(s)$ with side length $s = \sqrt{n/\lambda_0} > 0$, where n is the (expected) number of nodes in $B(s)$ and λ_0 is the node density. When n is sufficiently large, the node deployment can be governed by a *homogeneous* Poisson point process with density λ_0 in the Euclidean plane, denoted by \mathcal{H}_{λ_0} . We further assume that all nodes have the same transmission radius r ($r > 0$). Then the communication graph of such a network is modeled by a *geometric random graph* [14], denoted by $G(\mathcal{H}_{\lambda_0,s}, r)$, where $\mathcal{H}_{\lambda_0,s} \triangleq \mathcal{H}_{\lambda_0} \cap B(s)$ with $\lambda_0 = n/s^2$ fixed as $n \rightarrow \infty$ and a link exists between a pair of nodes only if their distance is less than r .

To describe the impact of node failures on the devolution of wireless multi-hop networks, we introduce a random failure model to extend the basic geometric random graph model. In this model, each node is either *operational* or *failed* at any time and a failed node does not recover back to operational state. Let T_i ($1 \leq i \leq n$) denote the lifetime of node i before it is failed, then T_1, \dots, T_n are random variables, which are assumed independently and identically distributed (i.i.d.). The *complementary* cumulative distribution function (c.d.f.) of the node lifetime is called the *survival function*, denoted by $S(t) \triangleq Pr(T_i > t)$. The survival function $S(t)$ actually serves as the probability that a node is surviving at time t , which will be used extensively in our succeeding analysis.

In order to explain how the devolution process of a large-scale sensor network is related with a continuum percolation process on the geometric random graph, we need to introduce some percolation terminologies first. Let \mathcal{C}_0 be the component of a graph $G(\mathcal{H}_{\lambda,s}, r)$ containing the origin $\mathbf{0}$ in \mathbb{R}^2 , then the *percolation probability*, denoted by $p_\infty(\lambda)$, is the probability that \mathcal{C}_0 contains infinite nodes as $n \rightarrow \infty$, i.e., $p_\infty(\lambda) \triangleq Pr(|\mathcal{C}_0| = \infty)$. A fundamental result of continuum percolation is there exists a *critical density* λ_c . If $\lambda > \lambda_c$, $G(\mathcal{H}_{\lambda,s}, r)$ is in the *supercritical* phase with a giant component and $p_\infty(\lambda) > 0$; while if $\lambda < \lambda_c$ then $G(\mathcal{H}_{\lambda,s}, r)$ is in the *sub-critical* phase with no giant component and $p_\infty(\lambda) = 0$.

Now given a large-scale sensor network represented by $G(\mathcal{H}_{\lambda_0,s}, r)$ with each node associated with a survival function $S(t)$, according to the *Thinning theorem* [14], the set of operational nodes is also a Poisson process with density function $\lambda_1(t) \triangleq \lambda_0 S(t)$. As time goes, although more and more failures are present, as long as $\lambda_1(t)$ is high enough, most of remaining nodes are still connected in a giant component; while once $\lambda_1(t)$ drops below λ_c , the connectivity among remaining operational nodes breakdowns quickly and the network is fully partitioned. Thus, the percolation process is an analogy to the devolution process aforementioned. To understand the phase transition time, we formally define a new metric called *first partition time* as follows.

Definition 1: Let $G(\mathcal{H}_{\lambda_0,s}, r)$ be a geometric random graph, in which each point is independently associated with the same survival function $S(t)$. Given $\lambda_0 = \lambda_1(0) > \lambda_c$, the *first partition time* is defined by

$$t_p(n) \triangleq \inf\{t > 0 : p_\infty(\lambda_1(t)) = 0\}. \quad (1)$$

where $\lambda_1(t) \triangleq \lambda_0 S(t)$.

With the definition of $t_p(n)$, we formulate the question raised in Section I as follows.

Definition 2: Network Partition Time (NPT) Problem: For a geometric random graph $G(\mathcal{H}_{\lambda_0,s}, r)$ with a common survival function $S(t)$, suppose the graph is initially fully connected but devolves as time goes, find out a lower bound of the first partition time $t_p(n)$ with respect to $S(t)$.

Before we solve the NPT problem, we provide a brief overview on the percolation theory and its applications in wireless multi-hop networks in the next section.

III. PERCOLATION IN WIRELESS MULTI-HOP NETWORKS

Percolation theory, originally developed in physics, chemistry, and material science, has been used to describe the behavior of connected clusters in a (large) random graph for decades. The main concept of percolation theory is the existence of a *percolation threshold* (e.g., critical density), below which only finite clusters are present and above which an infinite cluster is possible to appear. The appearance of the infinite cluster in percolation theory corresponds to the emergence of the giant component in graph theory, which makes percolation theory a powerful mathematical tool in network connectivity analysis.

In the standard discrete percolation model, of the main concern is the existence of infinite “open paths” in the plane square lattice \mathbb{Z}^2 , where each edge of \mathbb{Z}^2 is either *open* with probability p and *closed* otherwise. The percolation threshold in this model is the *critical probability* defined by $p_c \triangleq \inf\{p : p_\infty(p) > 0\}$, which is actually equal to $\frac{1}{2}$ in two dimensions [13]. Different from the discrete percolation model, the continuum percolation model is induced by a homogeneous Poisson point process defined on the continuous space \mathbb{R}^2 [12], where the percolation threshold is just the critical density λ_c aforementioned in Section II. Nevertheless, the exact value for the critical density λ_c is still unknown so far, although some numeric bounds were obtained from rigorous mathematical proofs with poor estimation (e.g., $0.696 < \lambda_c < 3.372$) or from computer experiments with little theoretical justification [15].

We notice that the percolation theory, especially the continuum percolation model, has been used to analyze the connectivity, capacity, and latency of wireless networks recently [6]–[8], [11]. For example, the impact of signal interference on the connectivity of wireless ad hoc networks was studied in [6] and it was found that long-distance multi-hop communications are achievable only if the orthogonal factor is below a certain critical value. In [7], it was shown that if the attenuation function does not have a singularity at the origin and is uniformly bounded, then either the network becomes disconnected or the available rate per node decreases like $1/n$ based on the percolation condition obtained for large node densities. And in [8], the latency of message deliveries between any sensing node and a fixed sink was studied for wireless sensor networks by using an extension of first passage percolation theory, which was also used in [11] to evaluate the performance of a degree-dependent energy management algorithm. The above works demonstrate the applications of the percolation theory in wireless multi-hop networks; while none of them ever addresses our NPT-problem.

IV. ANALYSIS ON THE FIRST PARTITION TIME

In fact, a trivial solution to the NPT problem tells that: when $S(t) < \lambda_c/\lambda_0$, no giant component exists and thus the network is fully partitioned. However, as we mentioned in Section III, the exact value for λ_c is unknown, which makes the NPT problem an open and challenging problem in the continuous domain. In our approach, we first map the continuum percolation process onto a discrete lattice and obtain the percolation condition for the discrete lattice. Then, we carry out a reverse mapping back to the continuous plane so that the infinite edge cluster on the discrete plane implies a giant component on the continuous plane. At last, we obtain the continuum percolation condition involving the survival function $S(t)$, which enables us to provide the scaling law on $t_p(n)$ by giving $S(t)$ specific distributions.

A. Mapping and Open Edge Definition

We begin by constructing a square lattice, denoted by \mathcal{L} over the plane, with edge length d . Let \mathcal{L}' be the dual lattice of \mathcal{L} , constructed by putting a vertex in the center of every face (square) of \mathcal{L} , and an edge across every edge of \mathcal{L} . Since \mathcal{L} is a square lattice, \mathcal{L}' is simply the same lattice shifted by $d/2$ horizontally and vertically, as depicted in Fig. 1.

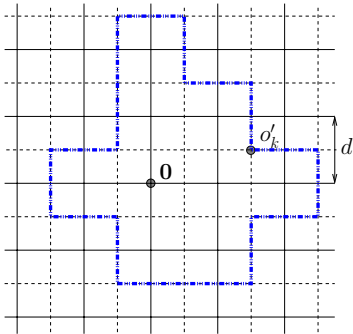


Fig. 1. Lattice \mathcal{L} (solid), its dual \mathcal{L}' (dashed), and a circuit (bold).

To define an open edge, we need to use the *crossing* property [12]–[14] to describe the connection among multiple points.

Definition 3: In a 2-D plane, let $X_v = (x_v, y_v)$ be the position of a point v . For a 2-D box $B \triangleq [0, l_1] \times [0, l_2]$, if there exist a series of points v_1, v_2, \dots, v_m within B such that $\forall 1 \leq i < j \leq m$ $x_{v_i} < x_{v_j}$, $0 < x_{v_1} < r$, $l_1 - x_{v_m} < r$, and $\|X_{v_{i+1}} - X_{v_i}\| \leq r$, then B has a connected path from left to right called *LR-crossing*. If the conditions above are satisfied in B when x and l_1 are substituted by y and l_2 , respectively, B has a connected path from top to bottom called *TB-crossing*.

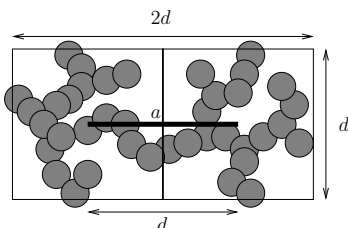


Fig. 2. A horizontal edge a that fulfills the LR-crossing and TB-crossing.

Now for every horizontal edge a of \mathcal{L} , let (x_a, y_a) be the coordinates of the point in the center of a . We use a similar way used in [16] to define the vicinity of a and introduce an event E_a that occurs if the following three events occur,

- 1) $LR_a \triangleq \{ \text{there is an LR-crossing in the rectangle } B_a \triangleq [x_a - d, x_a + d] \times [y_a - \frac{d}{2}, y_a + \frac{d}{2}] \}$,
- 2) $TB_a^L \triangleq \{ \text{there is a TB-crossing in the "left" square } B_a^L \triangleq [x_a - d, x_a] \times [y_a - \frac{d}{2}, y_a + \frac{d}{2}] \}$, and
- 3) $TB_a^R \triangleq \{ \text{there is a TB-crossing in the "right" square } B_a^R \triangleq [x_a, x_a + d] \times [y_a - \frac{d}{2}, y_a + \frac{d}{2}] \}$.

The occurrence of the event E_a is illustrated in Fig. 2 where balls represent Poisson points. E_a can be defined similarly for vertical edges by exchanging the notations of x_a and y_a in the conditions above.

Now we can define *open edges* as follows

Definition 4: In \mathcal{L} , an edge a is *open* iff the event E_a occurs in its associated rectangle B_a ; and *closed* otherwise. In \mathcal{L}' , an edge a' is open if there is an open edge of \mathcal{L} crossing a' ; otherwise, a' is closed.

This open edge definition bridges the discrete and continuous planes in that a cluster comprised of *adjacent* open edges in \mathcal{L} corresponds to a unique component (comprised of operational nodes) in $G(\mathcal{H}_{\lambda_0, s}, r)$. This rationale guarantees the validity of our mapping approach.

Lemma 1: Given the mapping and open edge defined above, if there exists an infinite open edge cluster in \mathcal{L} , then there exists a giant component in $G(\mathcal{H}_{\lambda_0, 0}, r)$.

Sketch of proof: For two adjacent edges (of the same direction), a and b , in \mathcal{L} , suppose they are associated with rectangles B_a and B_b , respectively, then B_a and B_b intersect in the same square S_{ab} , i.e., $S_{ab} = B_a \cap B_b$. If a and b are open, there exists at least one TB-crossing \mathcal{P}_s in S_{ab} . Let \mathcal{P}_a and \mathcal{P}_b be the LR-crossings in B_a and B_b , respectively, then both of them must intersect with the same TB-crossing in S_{ab} . This implies an LR-crossing of the rectangle $B_a \cup B_b$, formed by \mathcal{P}_a , \mathcal{P}_b , and \mathcal{P}_s . For perpendicular adjacent edges, similar rationale also applies. Further, the union of the rectangles of all edges in \mathcal{L} actually covers the whole area of the graph in the continuous plane, then the result follows. ■

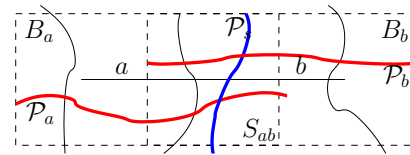


Fig. 3. A long horizontal crossing formed by two adjacent open edges.

Fig. 3 gives an illustration for the formation of the LR-crossing described in the proof above. *Lemma 1* implies that if the lattice \mathcal{L} is percolated then the continuum percolation also occurs in the graph $G(\mathcal{H}_{\lambda_0, 0}, r)$; and vice versa. Thus, the percolation condition obtained in \mathcal{L} can be applied for continuum percolation, which will be explained next.

B. Non-Percolation Condition for Discrete Lattices

As we mentioned in Section III, in the discrete percolation theory, the open or close state of every edge is independent

from others [17]. In our discrete lattice mapping, the state of an edge is, however, dependent on how Poisson points surrounding the edge are connected, which implies at least adjacent edges are not independent. Therefore, we cannot directly use the critical probability (i.e., $p_c = \frac{1}{2}$) and we need to derive an alternative percolation condition for our mapping, which is based on the following fact.

Lemma 2: Given a lattice \mathcal{L} containing the origin $\mathbf{0}$, let $\sigma(m)$ be the number of paths with length m (i.e., comprising m edges) that start at $\mathbf{0}$, then $\sigma(m) \leq 4 \cdot 3^{m-1}$.

By using the fact above, we have

Lemma 3: For the given lattice \mathcal{L} constructed above, let p be the probability that an edge is open, if $p < \frac{1}{9}$, then $p_\infty = 0$.

Sketch of proof: The largest open edge cluster is finite iff no infinite open path (comprised of open edges) exists. Let \mathcal{P}_m be any path having a length m and beginning at the origin in \mathcal{L} , then $Pr(\mathcal{P}_m \text{ is open}) = Pr(\text{all } m \text{ edges are open})$. Based on the open edge definition in Section IV-A, the states of non-adjacent edges are independent, so at least $m/2$ edges among m edges of \mathcal{P}_m have independent states, which implies $Pr(\mathcal{P}_m \text{ is open}) \leq p^{m/2}$. Thus the probability that there exists an open path of length m is given by

$$Pr(\exists \text{ open path } \mathcal{P}_m) \leq p^{\frac{m}{2}} \sigma(m) = \frac{4}{3} (9p)^{\frac{m}{2}}. \quad (2)$$

If $9p$ is strictly less than 1, i.e., $p < \frac{1}{9}$, then (2) converges to 0 as $m \rightarrow \infty$, which implies no infinite open path existing in \mathcal{L} and thus $p_\infty = 0$. This finishes the proof. ■

Lemma 3 provides us a useful tool to study the percolation on the continuous plane. Based on the definition of open edge, we know that $p = Pr(E_a)$, so if $Pr(E_a) < \frac{1}{9}$ the graph is fully partitioned. By deriving an upper bound of $Pr(E_a)$, we can obtain the non-percolation condition with respect to the survival function $S(t)$, presented right next.

C. Non-Percolation Condition for Continuum Space

The critical condition under which no giant component exists in the graph $G(\mathcal{H}_{\lambda_0, s}, r)$ is concluded as follows.

Theorem 1: Given a graph $G(\mathcal{H}_{\lambda_0, s}, r)$ with each node associated with a common survival function $S(t)$, If $S(t)$ satisfies the following condition,

$$S(t) < \frac{\ln \sqrt{3} - \ln(\sqrt{3} - 1)}{cr \lambda_0 \log n}, \quad (3)$$

where $0 < c < \infty$ is a finite positive number independent of n , then $G(\mathcal{H}_{\lambda_0, s}, r)$ is in the sub-critical phase.

Proof: Let SLR_a^L and SLR_a^R denote the events that there is an LR-crossing in B_a^L and B_a^R , respectively, where B_a^L and B_a^R are defined in Section IV-A. Then the occurrence of event LR_a guarantees the occurrences of both events SLR_a^L and SLR_a^R , and thus

$$\begin{aligned} Pr(E_a) &\leq Pr(SLR_a^L \cap TB_a^L \cap SLR_a^R \cap TB_a^R) \\ &= Pr(SLR_a^L \cap TB_a^L) Pr(SLR_a^R \cap TB_a^R). \end{aligned} \quad (4)$$

The last equality in (4) is due to the fact that events $SLR_a^L \cap TB_a^L$ and $SLR_a^R \cap TB_a^R$ occur in disjoint sets B_a^L and B_a^R , i.e., they are independent events. We further assume that the

points used for the LR-crossing are different than those used for the TB-crossing in B_a^L (and B_a^R), then by the *BK inequality* (*Theorem 2.3* [12]), we have

$$Pr(E_a) \leq (Pr(SLR_a^L) Pr(TB_a^L))^2 = Pr(SLR_a^L)^4. \quad (5)$$

To calculate $Pr(SLR_a^L)$, we study the occurrence of the complementary event of SLR_a^L , denoted by \overline{SLR}_a^L , i.e., $\overline{SLR}_a^L \triangleq \{ \text{no LR-crossing exists in } B_a^L \text{ (or } B_a^R) \}$. Suppose that there is a *band* with width length r crossing vertically through B_a^L , then the intersection of the band and B_a^L forms a rectangular with length d and width r , denoted by B_r . Let $SLR^c \triangleq \{ \text{no surviving nodes located in } B_r \}$, then \overline{SLR}_a^L surely occurs when SLR^c occurs, which is illustrated in Fig. 4. Since SLR^c is only one cause for the occurrence of \overline{SLR}_a^L , it is obvious that $Pr(SLR^c) < Pr(\overline{SLR}_a^L)$, which yields

$$Pr(E_a) < (1 - Pr(SLR^c))^4. \quad (6)$$

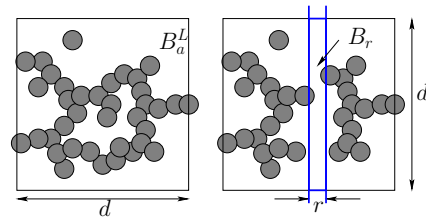


Fig. 4. An illustration of the event that intercepts an LR-crossing.

As aforementioned, the point process of (remaining) operational nodes is a Poisson process with density function $\lambda_1(t) = \lambda_0 S(t)$, then we have $Pr(SLR^c) = \exp(-\lambda_0 S(t) dr)$. To move forward, the challenge here is confining the side length d (of the lattice \mathcal{L}) to a proper order. On one hand, d cannot be a constant number or in the order of $O(1)$; otherwise the event E_a could not be guaranteed to occur with a high probability. On the other hand, d cannot be too large or even in the order of $O(\sqrt{n})$; otherwise the problem of finding the probability of an open edge will be equivalent to the problem of finding the probability that there exists a giant component in graph. Notice that the percolation conditions for the discrete plane, given in *Lemma 3*, are obtained under the condition that there are an infinite number of edges in the lattice \mathcal{L} (\mathcal{L}'). This implies that $\lim_{n \rightarrow \infty} \frac{\sqrt{n/\lambda_0}}{d} \rightarrow \infty$, i.e., $d = o(s)$. Thus, we conjecture that d should be in the order of $\log n$; more specifically, we define d as $d \triangleq c \ln(n)$, where $0 < c < \infty$ is a finite positive number independent of n . Then (6) can be written as

$$Pr(E_a) < (1 - \exp(-cr \lambda_0 S(t) \ln n))^4. \quad (7)$$

Since $p \triangleq Pr(E_a)$, if the following inequality holds

$$(1 - \exp(-cr \lambda_0 S(t) \ln n))^4 < \frac{1}{9}, \quad (8)$$

then $p < \frac{1}{9}$ and thus no infinite open edge cluster exists in the discrete lattice by *Lemma 3*. Finally, we can obtain the condition given in (3) from (8) easily by elementary derivations. Applying *Lemma 1*, when $S(t)$ satisfies (3), the original graph on the continuous plane is in the sub-critical phase. The proof completes. ■

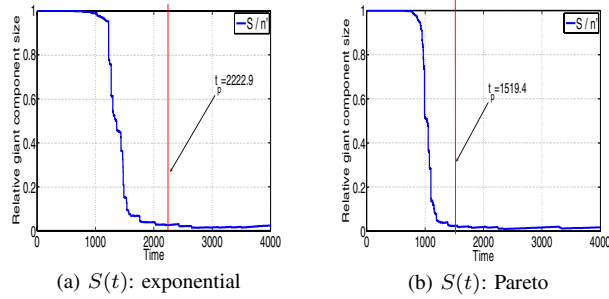


Fig. 5. The lower bounds of the first partition time.

D. Lower Bound of the First Partition Time

In the last subsection, we have obtained the condition under which a large-scale wireless sensor network starts to be partitioned fully, we can use the condition given in (3) to derive the lower bound of the first partition time $t_p(n)$ once the survival function $S(t)$ is given.

From reliability engineering, we know that many lifetime distributions (e.g., exponential, log-normal, Pareto, Weibull) are either light-tailed or heavy-tailed according to the decay speed of their tails. Since the exponential distribution is the only distribution to have a constant failure rate and applies naturally to model memoryless lifetime, it is used to represent light-tailed survival functions; while the Pareto distribution is used to represent heavy-tailed survival functions when node lifetime is power law or with very large variance.

Now we present the lower bounds of $t_p(n)$ with respect to the exponential and Pareto survival functions as follows.

Corollary 1: When $S(t)$ is light-tailed exponential with mean $1/\alpha$, i.e., $S(t) = e^{-\alpha t}$, the lower bound of $t_p(n)$ is,

$$t_p(n) = \frac{1}{\alpha} \ln(\ln(n)) + c_1 \sim \Theta(\log \log n), \quad (9)$$

where $c_1 = \frac{1}{\alpha} (\ln(cr\lambda_0) - \ln(\ln \sqrt{3}) - \ln(\sqrt{3} - 1))$ and $c > 0$ is a finite constant. When $S(t)$ is heavy-tailed Pareto with mean $\eta\rho/(\rho - 1)$ and $\rho > 1$, i.e., $S(t) = (t/\eta)^{-\rho}$, the lower bound of $t_p(n)$ is given by

$$t_p(n) = c_2 (\ln n)^{1/\rho} \sim \Theta((\log n)^{1/\rho}), \quad (10)$$

where $c_2 = \eta(cr\lambda_0/(\ln \sqrt{3} - \ln(\sqrt{3} - 1)))^{1/\rho}$.

By substituting $S(t) = e^{-\alpha t}$ and $S(t) = (t/\eta)^{-\rho}$ into (3), (9) and (10) can be obtained easily. Up to now, we have solved the NPT-Problem and quantified the first partition time.

To confirm the correctness of our analysis, we conduct a numeric simulation in which the relative giant component size is used to examine the devolution process of a network. The parameters used in the simulation are as follows: $n = 10000$, $r = 150$ m, $\lambda_0 = 0.00025$, $\alpha = 0.001$, $\eta = 500$, and $\rho = 2$. The simulation results are shown in Fig. 5 with theoretical values of $t_p(n)$ annotated. From the figure, we can see that the relative giant component is very low after $t_p(n)$, indicating a fully partition network, and our analytical results provide good approximations to the lower bound of the critical time.

V. CONCLUSIONS

In this paper, we studied the devolution process of large-scale sensor networks represented by a geometric random graph of n nodes in which each node is associated with a common survival function $S(t)$. By coupling with a continuum percolation process on the geometric random graph, we obtained the condition involving the survival function under which the graph is in the sub-critical phase. We found that the first time that the graph starts to discompose into many isolated finite components, defined as the *first partition time*, only depends on the network parameters (node density λ_0 , transmission radius r , network size n , and survival function $S(t)$). Further, by analyzing exponential and Pareto survival functions, the lower bound of the first partition time was shown to be of the order between $\log \log n$ and $(\log n)^{1/\rho}$ for $\rho > 1$. This result can be used as a theoretical guideline for scheduling the redeployment of additional nodes to maintain the connectivity of sensor networks, and will serve as the basis for our further understanding on the phase transition phenomena in the network devolution.

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