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On Distribution and Limits of Information Dissemination Latency and Speed In Mobile Cognitive Radio Networks

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***Abstract*—Dissemination latency and speed are central to the applications of *cognitive radio networks*, which have become an important component of current communication infrastructure. In this paper, we investigate the distributions and limits of information dissemination latency and speed in a cognitive radio network where licensed users (primary users) are static and cognitive radio users (secondary users) are mobile. We show that the dissemination latency depends on the stationary spatial distribution and *mobility capability* α (characterizing the region that a mobile secondary user can reach) of secondary users. Given any stationary spatial distribution, we find that there exists a critical value on α, below which the latency and speed are *heavy-tailed* and above which the right tails of their distribution are bounded by *Gamma* random variables. We further show that as the network grows to infinity, the latency asymptotically scales linearly with the "distance" (characterized by transmission hops or Euclidean distance) between the source and the destination. Our results are validated through simulations.**

I. Introduction

Today's spectrum is regulated by governmental agencies and is assigned to licensed users (also called *primary users*) on a long term basis in large geographical regions. A recent report from FCC reveals that under this static allocation, merely $5\% \sim 15\%$ of the spectrum is utilized on average [1]. This significant under-utilization has spurred a surge of interest in studying and optimizing the spectrum efficiency. Cognitive radio is an emerging technology aiming at improving the spectrum efficiency by allowing cognitive users (*secondary users*) to opportunistically utilize channels without interfering with the coexisting primary users.

Recently, there have been many works on the optimal design and performance analysis of cognitive radio networks [2]–[4]. In these works, many efforts have been put forward on detecting "spectrum opportunities" (e.g., unused spectrum) while restricting the potential interference to primary users, or on developing *sensing* or *spectrum sharing* strategies to maximize the throughput. While these works are very helpful for recognizing the potential of cognitive communications in maximizing spectrum utilization, it is not well understood what type of applications are suitable for cognitive communications. Can an application with a certain required QoS be supported by the cognitive users? Can cognitive radio networks provide real-time service for emergent applications, such as *military networks*? This motivates the *latency* and *information dissemination speed* studies in this paper.

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Particularly, latency is an important QoS metric in wireless networks, which is unfortunately not well understood in general, partially due to the difficulties in analyzing it. The work in [5] studied the packet latency in *fully connected* multi-hop wireless networks and showed that there exist bounds on the latency and these bounds are tight when the number of nodes are large enough. The work in [6] further showed that the asymptotic latency scales at least linearly with the transmission distance in the *percolated* wireless networks. Although these results have greatly advanced our understanding of the nature of latency, a few interesting questions remain unanswered. First, wireless nodes in these works are assumed to be static. However, in many cognitive communication applications (e.g. cognitive vehicular networks or cognitive military networks), secondary users usually need to move around to obtain "better channel opportunity" or "more security". What is the impact of mobility on latency? Moreover, the existing works only derive the asymptotic latency which assumes that the number of wireless nodes is infinite or approaches to infinity. However in reality, the number of nodes is finite. Then what is the dissemination latency for a network with finite users? We attempt to provide answers to these questions in this paper.

Specifically, we consider a mobile cognitive radio network where primary users are static and the secondary users are mobile. We show that dissemination latency and speed depend on the spatial distribution and *mobility capability* α (characterizing the region that a mobile user can reach) of secondary users. We find that given any spatial distribution of secondary users, there exists a critical value on α, below which the latency and speed follow *heavy-tailed* distributions and above which the right tails of their distribution are bounded by *Gamma* random variables. And as the network grows to infinity, the latency is asymptotically linear with respect to the Euclidean distance between the source and destination. More precisely, we prove that the message sent by the source reaches the destination at a fixed asymptotic speed.

The outline of this paper is as follows. Section II establishes the network and mobility models, and defines the concepts and notations to be used later on. We present the analysis and results for dissemination latency and speed in a finite network in Section III, and in a large-scale network in Section IV, respectively. We finally conclude in Section V.

II. System Assumptions and Notations

Before we investigate the dissemination latency and speed, it is necessary to understand how information disseminates in a

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mobile cognitive radio network. First, we describe the system model used in this paper.

A. System Model

We consider a mobile cognitive radio network $\mathcal{F}_{m,n}$ indexed by the number of primary users m and the number of secondary users n. Secondary users $\{v_1,\ldots,v_n\}$ are supposed to move over a bi-dimensional Torus surface $\mathcal{D}_n$ of size $L_n \times L_n$ (to avoid border effects) and simultaneously, m *static* primary users $\{u_1,\ldots,u_m\}$ are randomly distributed in $\mathcal{D}_n$. We assume that L_n scales with n as n^δ, with $0 \le \delta \le 1/2$. The extreme cases are: $\delta = 0$, for which the network area remains constant and node density increases linearly with n; $\delta = 1/2$, for which the network area increases linearly with n and node density remains constant. The variable δ determines how the network area behaves as the number of nodes increases. In this paper, we focus on the scenario that $L_n = \sqrt{\frac{n}{\lambda}}$ ($\delta = 1/2$). Our results can be easily extended to the cases with different L_n.

Let $V(t) = (v_1(t),\ldots,v_n(t))$ and $U(t) = (u_1,\ldots,u_m)$ denote the vector of locations, and $V(0)$ and $U(0)$ be the initial distribution, of secondary users and primary users respectively. We consider that the stationary spatial distribution of a secondary user is generally non-uniform over the space. A secondary user typically spends most of the time in a small region, and rarely visits the areas far away from it. We model this behavior by assuming that each secondary user v_i has a *home point*, located at v_i^h. Secondary users move "around" their home points according to independent stationary and ergodic processes, i.e., given any $\eta-$uple $(B_1, B_2, \ldots, B_\eta)$ of Lebesgue measurable subsets of $\mathcal{D}_n$, it results

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t \mathbb{I}_{(\cap_i v_i(\tau)\in B_i)}d\tau = \cap_i E[\mathbb{I}_{(\cap_i v_i(t)\in B_i)|\mathcal{F}_n}] \qquad \text{w.p.1},$$

where $\mathbb{I}$ represents the logical indicator function and $\mathcal{F}_n$ the Borel-field generated by $\{v_i^h\}_{i=1}^n$.

We assume that home points $\{v_i^h\}_{i=1}^n$ are uniformly and independently distributed in $\mathcal{D}_n$. We describe the spatial distribution of secondary user v_i around v_i^h by a non-increasing and direction-invariant function $\Psi_i(x) = \Psi(x - v_i^h)$. We assume that Ψ_i is non-zero in and only in a region characterized by a constant α; that is, $\Psi_i(x) = \Psi(x - v_i^h) > 0$ when $\|x - v_i^h\| < \alpha$ and $\Psi_i(x) = \Psi(x - v_i^h) = 0$ otherwise. α is called ***mobility capability*** since 2α represents the *moving diameter* of secondary users.

The class of mobile networks that we study are very general such that they account for a wide range of possible scenarios of realistic mobility processes. Notice that, the case of static nodes uniformly deployed over the area (Gupta-Kumar case [7]) can be obtained by setting $\Psi_i(x) = \delta(x - v_i^h)$. And the *i.i.d* mobility model in [8] corresponds to the case when $\Psi(x)$ is a constant function independent of x and $\alpha = \infty$; furthermore, when $\alpha < \infty$, we obtain the *constrained i.i.d* model used in [6]. For simplicity, we assume that the home points are uniformly distributed here. However, our analysis can be easily extended to the more general scenario where spatial inhomogeneity of home points is taken into account.

B. Interference Model

We assume that interference among simultaneous transmissions is described by the *protocol model* [7]. Particularly, denote R_I as the interference range of the primary users and r as the transmission range of secondary users. Secondary user v_i is allowed to transmit to v_j at time t, only if: i) there is no primary users in the neighborhood, i.e., $\|v_i(t) - u_k\| > R_I$ and $\|v_j(t) - u_k\| > R_I$ for any k; and ii) $\|v_i(t) - v_j(t)\| < r$, where $\|\cdot\|$ denotes the Euclidean distance.

C. Notations and Metrics

By assuming that every node communicates with a randomly chosen destination, and serves as relay for all the other S-D pairs, the trade-offs between throughput and delay in wireless networks under different mobilities has been extensively studied in related works [8], [9]. Instead of capacity-delay trade-offs, our target in this paper is to answer the question of how fast a message can be disseminated in mobile cognitive radio networks. Particularly, there is a *single message* to be delivered by a source secondary user v_s to a destination v_d. Intermediary secondary users can be used as relay nodes. The goal is to determine the distributions and the asymptotic characteristics of the dissemination latency and speed.

In a mobile environment, two nodes can exchange information and thus share a link if they can decode each other's signals at *some time*. Since different pairs of nodes can share links at different points in time, connectivity varies over time. Without considering the propagation delay, when the source v_s sends a message at time t_0, all the nodes in the connected component of v_s receive the message instantaneously. The destination v_d may not receive this message if it is not in this connected component. As time goes on, nodes move, and the message is passed from message-carrying nodes to other nodes whenever they share communication links. Thus, the message is disseminated through the network and v_d may receive this message at some point as this process continues.

Definition 1: Denote $\mathbb{L}(t)$ as the set of communication links at time t. Given the initial node distribution $V(0)$ and $U(0)$, let *the first hitting time* of secondary users v_i and v_j be

$$T_h(v_i, v_j) \triangleq \inf\{t \ge 0 : l_{ij} \in \mathbb{L}(t)\}$$

where l_{ij} denotes a communication link between v_i and v_j.

Let $\mathcal{A}(t)$ be the set of secondary users that have received the message sent at time 0 by time t. Define $\mathbb{T}_d$ as:

$$\mathbb{T}_d \triangleq \inf\{t \ge 0 : v_d \in \mathbb{A}(t)\}.$$

Then, $\mathbb{T}_d$ is the *latency* of information dissemination from source v_s to the randomly chosen destination v_d. It is easy to couple $\mathbb{T}_d$ as the first passage time in the weighted graph,

$$\mathbb{T}_d = \inf_{w(v_s,v_d)\in\mathbb{W}(v_s,v_d)} \{\sum_{l_{ij}\in w(v_s,v_d)} T_h(v_i, v_j)\},$$

where $w(v_s, v_d)$ is an arbitrary path joining v_s and v_d and $\mathbb{W}(v_s, v_d)$ denotes the set of all such paths. To accurately describe "how fast" a message can be disseminated, we next define the *dissemination speed*.

Definition 2: Denote $d^{(t)}(v_i, v_j)$ and $d_h(v_i, v_j)$ as the Euclidean distance between secondary users v_i and v_j at time t and the Euclidean distance between v_i^h and v_j^h respectively. Define $\mathbb{S}_d \triangleq \frac{\mathbb{T}_d}{d^{(0)}(v_s, v_d)}$. $\mathbb{S}_d$ can be interpreted as the dissemination latency between v_s and v_d when the initial distance between them is 1. $\mathbb{S}_d$ can be used to characterize the *dissemination speed*. In the rest of this paper, we focus on $\mathbb{S}_d$ and call it *speed* for simplicity.

III. The Distributions of Dissemination Latency $\mathbb{T}_d$ and Speed $\mathbb{S}_d$ in Finite Networks

In this section we are interested in the distributions of dissemination *latency* $\mathbb{T}_d$ and *speed* $\mathbb{S}_d$ in finite networks. In the following lemma, we first study the first hitting time between two neighboring secondary users v_i and v_j $T_h(v_i, v_j)$. Particularly, we prove that unless each secondary user v_i can move to any point in $\mathcal{D}_n$ (with $\alpha \approx \sqrt{\frac{2n}{\lambda}}$), the first hitting time $T_h(v_i, v_j)$ between any two secondary users v_i and v_j has a heavy tail and $E(T_h(v_i, v_j)) = \infty$.

Lemma 1: Given $V(0)$, $U(0)$ and any two secondary users v_i and v_j, if $\alpha < \frac{\sqrt{\frac{2n}{\lambda}} - r}{2}$, the first hitting time between v_i and v_j $T_h(v_i, v_j)$ follows a *heavy tailed* distribution with $E(T_h(v_i, v_j)) = \infty$; otherwise, $E(T_h(v_i, v_j)) < \infty$ and $\mathbb{P}(T_h(v_i, v_j) > t) \le e^{-ct}$ for sufficiently large t. c is some positive constant.

Proof: Note that any two nodes v_i and v_j can communicate with each other directly at time t if and only if $d^{(t)}(v_i, v_j) \le r$ and $d^{(t)}(v_i, v_j) \ge d_h(v_i, v_j) - 2\alpha$ for all t. In our model, home points are randomly distributed in the whole network $\mathcal{D}_n$ and for any two secondary users v_i and v_j, $d_h(v_i, v_j)$ can be any value between 0 and $\sqrt{\frac{2n}{\lambda}}$. Thus, if $\alpha < \frac{\sqrt{\frac{2n}{\lambda}} - r}{2}$, we can find, with some positive probability, some secondary user v_i such that $d_h(v_i, v_k) - 2\alpha > r$ for all $k \ne i$, which implies $d^{(t)}(v_i, v_j) > r$ for any t. That is, there exist some probability $\epsilon > 0$ that $T_h(v_i, v_j) = \infty$, which implies $E(T_h(v_i, v_j)) = \infty$ and $T_h(v_i, v_j)$ follows a *heavy tailed* distribution.

On the other hand, when $\alpha > \frac{\sqrt{\frac{2n}{\lambda}} - r}{2}$, if we denote $\mathcal{E}_t$ as the event that there exists no communication link between v_i and v_j at any time point t and denote $\varepsilon = \mathbb{P}(\mathcal{E}_t)$, $0 < \varepsilon < 1$. Denote ρ as a *renewal* interval for secondary user v_i, i.e., for any $t > 0$, $\{v_i(t') : t' \le t\}$ and $\{v_i(t''+\rho) : t'' \ge t\}$ are independent; i.e., after ρ, v_i completely forgets where it was ρ ago. ρ depends on the specific mobility model. Observe that for the first hitting time $T_h(v_i, v_j)$, we have $\{T_h(v_i, v_j) > t\} \subset \cap_{s \in \mathcal{I}_t} \mathcal{E}_s$ for any subset $\mathcal{I}_t \subset (0, t]$. And if we choose $\mathcal{I}_t = \{\rho, 2\rho, \ldots, \lfloor \frac{t}{\rho} \rfloor \rho\}$, it follows that $\mathcal{E}_{t_i}$ are all independent for any $t_i \in \mathcal{I}_t$ since ρ is the *renewal* interval for v_i and v_j.

$$\mathbb{P}(T_h(v_i, v_j) > t) \le \mathbb{P}(\cap_{s \in \mathcal{I}_t} \mathcal{E}_s) = \prod_{s \in \mathcal{I}_t} \mathbb{P}(\mathcal{E}_s) = \varepsilon^{\lfloor \frac{t}{\rho} \rfloor} \sim e^{-ct},$$

where $c = -log\varepsilon/\rho$. This completes the proof. ■

Lemma 1 implies that two neighboring secondary users can communicate with each other within finite time with high probability only when $\alpha \ge \frac{\sqrt{\frac{2n}{\lambda}} - r}{2}$ and we are only interested in $\mathcal{T}_d$ and $\mathbb{S}_d$ in this case. Before proceeding, we introduce an important concept *coupling method*, which will be used throughout this paper.

Definition 3: If Z and Z' are random variables such that $\mathbb{P}(Z > z) \le \mathbb{P}(Z' > z)$, we say that Z is *sarcastically dominated* by Z' and write $Z \overset{\mathcal{D}}{<} Z'$; and if $Z \overset{\mathcal{D}}{<} Z'$, there exists a random variable $\hat{Z}'$ which has the same distribution of Z' such that $Z \le \hat{Z}'$ ($\hat{Z}'$ is called a *coupling* of Z' [10].)

We now present the main result on $\mathbb{T}_d$ and $\mathbb{S}_d$.

Theorem 2: Given $V(0)$, if $\alpha \ge \frac{\sqrt{\frac{2n}{\lambda}} - r}{2}$, the dissemination latency $\mathbb{T}_d$ is *stochastically dominated* by a *Gamma distribution*.

Proof: Lemma 1 shows that the first hitting time $T_h(v_i, v_j)$ between any v_i and v_j is *stochastically dominated* by $exponential(c)$. If we can show that if $T_h(v_i, v_j)$ follows the distribution $exponential(c)$, the dissemination latency $\mathbb{T}'_d$ is *stochastically dominated* by a *Gamma* distribution, then by *coupling*, we have $\mathbb{T}_d \le \mathbb{T}'_d$, which indicates $\mathbb{T}_d$ is *stochastically dominated* by a *Gamma* distribution.

Assume $T_h(v_i, v_j) \sim exponential(c)$. Denote by τ the number of copies of the message in the network at the time when the message is delivered to the destination v_d. The proof is based on modeling τ as an absorbing finite-state Markov chain . The Markov chain is in state $k = 1, 2, \ldots, n-1$ when $\tau = i$ and it is in state n when the message has been delivered to the destination v_d. The transition diagram of the Markov chain is given in Fig. 1. In our model, each secondary user which has a copy of the message is allowed to send it to a secondary user which does not have a copy. Therefore when there are k copies of the message in the network, a new copy is created at the rate $ck(n-1-i)$ (transition from k to $k+1$) and one of these k copies reaches the destination v_d at the rate kc, as depicted in 1. The chain jumps from state k to $k+1$ with probability $\frac{n-1-k}{n-k}$ and it jumps from k to n with probability $\frac{1}{n-k}$. The sojourn time S_k in state k is exponentially distributed with intensity $ck(n-k)$. And $S_1, S_2, \ldots, S_{n-1}$ are mutually independent random variables. Thus $\mathbb{P}(\tau = k) = \frac{1}{n-k} \prod_{j=1}^{k-1} \frac{n-1-j}{n-j} = \frac{1}{n-1}$. Conditioning $\mathbb{T}'_d$ on τ,

$$\mathbb{T}'_d | (\tau = k) = \sum_{j=1}^{k} S_j \text{ with probability } \frac{1}{n-1}. \qquad (1)$$

Through Eq. (1), we can obtain the *moment generating function* [11] of $\mathbb{T}'_d$,

$$E(e^{-\mu \mathbb{T}'_d}) = \frac{1}{n-1} \sum_{k=1}^{n-1} \prod_{j=1}^{k} \frac{cj(n-j)}{cj(n-j)+\mu}.$$

$\mathbb{T}'_d$ uniquely determines the distribution of $\mathbb{T}'_d$ by inverse transformation and this distribution is very complicated. Considering that each S_j follows an exponential distribution, we can find some $\gamma > 0$ such that

$$\lim_{t \to \infty} e^{\gamma t} \mathbb{P}(\mathbb{T}'_d > t) < \infty,$$

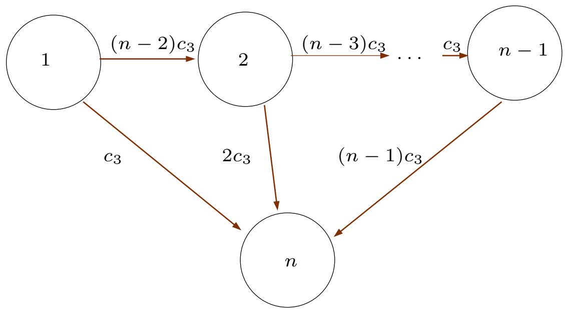

Fig. 1. Transition diagram of the Markov chain for the number of message copies at the time when the message is delivered to the destination.

which implies that $\mathbb{T}'_d$ is *stochastically dominated* by a Gamma distribution $Gamma(1, \gamma)$. Thus the result. ■

In this section, we have studied dissemination latency $\mathbb{T}_d$ and speed $\mathbb{S}_d$ in finite mobile networks. We show that $\mathbb{T}_d$ depends on the spatial distribution and the *mobility capability* α of secondary users. Given any spatial distribution, we prove the critical value on α, crossing over which $\mathbb{T}_d$ turns from *heavy-tailed* distribution to *Gamma* distribution. Dissemination latency is one of the key metrics in wireless networks and central to routing algorithms. Our results thus provide guidelines on the design of mobility models with *Gamma* (light-tailed) dissemination latency, new protocols as well as their performance analysis. In the next section, we will analyze the characteristics of $\mathbb{T}_d$ as the network grows to infinity.

IV. Asymptotic Analysis of Dissemination Latency $\mathbb{T}_d$ and Speed $\mathbb{S}_d$

To analyze asymptotic latency as the network grows large, we progressively increase the number of secondary users n. We are interested in establishing scaling behavior of $\mathbb{T}_d$. In Section III, we have shown that when the *mobility capabilities* α of secondary users is large enough, $\mathbb{T}_d$ and $\mathbb{S}_d$ are *stochastically dominated* by Gamma distribution. In this section, we show that by ignoring propagation delay, dissemination latency $\mathbb{T}_d$ scales linearly with $d_h(v_s, v_d)$. The main tool used in proving this result is the following Subadditive Ergodic Theorem.

Theorem 3: [[12], Liggett's subadditive ergodic theorem] Let $\{\mathcal{Z}_{h,q}\}$ be a collection of random variables indexed by integers satisfying $0 \leq h < q$. Suppose $\{\mathcal{Z}_{h,q}\}$ has the following properties: (i) $\mathcal{Z}_{0,q} \leq \mathcal{Z}_{0,h} + \mathcal{Z}_{h,q}$, (ii) For each q, $\mathbb{E}(|\mathcal{Z}_{0,q}|) < \infty$ and $\mathbb{E}(\mathcal{Z}_{0,q}) \geq cq$ for some constant $c > -\infty$. (iii) The distribution of $\{\mathcal{Z}_{h,h+k:k\geq 1}\}$ does not depend on h. (iv) For each $k \geq 1$, $\{\mathcal{Z}_{qk,(q+1)k} : q \geq 0\}$ is a stationary sequence. Then: (a) $\zeta = \lim_{q\to\infty} \mathbb{E}(\mathcal{Z}_{0,q})/q = inf_{q\geq 1} E(\mathcal{Z}_{0,q})/q$. (b) $\mathcal{Z} = \lim_{q\to\infty} \mathcal{Z}_{0,q}/q$ exists a.s. (c) $\mathbb{E}(\mathcal{Z}) = \zeta$. Furthermore, (v) If $k \geq 1$, $\{\mathcal{Z}_{qk,(q+1)k} : q \geq 0\}$ are ergodic, then (d) $\mathcal{Z} = \zeta$ a.s.

A. Asymptotic Latency Analysis

An underlying assumption in the previous analysis is the *full connectivity* of the network, that is, there exists a path (may be dynamic because of the mobility) between any two secondary users. To ensure the *full connectivity* of a finite network under our mobility model, the required *mobility capability* $\alpha > \frac{\sqrt{\frac{2n}{\lambda}} - r}{2}$. As n goes to infinity, this requires *infinitely large* mobility capability α or *infinitely large* transmission range r, which are overly restrictive and impossible to achieve.

To address the asymptotic latency, we consider a notion of connectivity based on the *percolation phenomenon* [13], which has been used to analyze the performance of large-scale random networks [4], [6]. The main result of percolation theory is that there exists a finite, positive value of the transmission range, or equivalently of the node spatial density, above which the network is percolated (super-critical) and below which it is not percolated (sub-critical). When the network is percolated, there exists a large connected component of nodes spanning almost the entire network (called the *giant component* in [13]), and when the network is not percolated, the network consists only of small isolated components of nodes.

In this subsection, we focus on the scenario that α and r are finite and instead of *fully connected*, the network is percolated. Instead of the *entire network*, we are interested in how fast information disseminates in the *giant component*. Note that if Ψ is a *Uniform* distribution, our mobility is equivalent to the *constrained i.i.d mobility* defined in [6]. And [6] has studied the message dissemination in the giant component of a percolated wireless network under *constrained i.i.d mobility* and has the following main result (see Theorem 5 in [6]).

Lemma 4: For a percolated wireless network under *constrained i.i.d mobility* G, denote $\mathcal{C}(G)$ as the giant component. For any two nodes $v_s, v_d \in \mathcal{C}(G)$, there exists a constant $0 < \zeta < \infty$ such that

$$\mathbb{P}\Big(\lim_{d(v_s,v_d)\to\infty} \frac{\mathcal{T}(v_s, v_d)}{d(v_s, v_d)} = \zeta\Big) = 1, \quad (2)$$

where $\mathcal{T}(v_s, v_d)$ and $d(v_s, v_d)$ denotes the dissemination latency and Euclidean distance between v_s and v_d.

There exist two main differences between the network model in [6] and the cognitive radio network under mobility considered in this work. First, instead of *Uniform*, the stationary distribution Ψ of v_i around its home point v_i^h is arbitrary. Second, the critical values on node density for percolation under these two models are completely different considering the fact that in cognitive radio networks, secondary users can communicate with each other only when not interfering with primary users.

Note that the proof technique for Lemma 4 (see [6]) does not require the *uniform* distribution of nodes around their home points. Indeed, it only requires that the expected first hitting time between any two neighboring nodes v_i and v_j is finite, i.e., $E(T_h(v_i, v_j)) < \infty$. By similar proof to Lemma 2 in [6], we can show that $E(T_h(v_i, v_j)) < \infty$ for any non-zero Ψ. In terms of the critical density of node spatial density, Ren *et al.* [4] have studied *percolation phenomenon* in a static cognitive radio network and characterize the critical density of secondary users as a decreasing function of the spatial density of primary users. By mapping a mobile wireless network to a static wireless network with dynamic links, [6] has characterized the

(a) Percolation regions.

(b) Asymptotic speed.

Fig. 2. Dissemination speeds and percolation conditions.

critical spatial density of nodes for a mobile wireless network. Combining techniques in [6] and [4], we can derive a critical density $\lambda_c(\lambda\frac{m}{n})$ of secondary users for a mobile cognitive radio network, where λ and $\lambda\frac{m}{n}$ denote the spatial density of secondary and primary users respectively and $\lambda_c(\lambda\frac{m}{n})$ is a decreasing function. When $\lambda > \lambda_c(\lambda\frac{m}{n})$, the mobile cognitive radio network is percolated (see Fig. 2(a)). For any two nodes in the *giant component*, we have the following result on the dissemination latency. The techniques for the proof are similar to those for Lemma 4. Due to space limitations, the proof is omitted here.

Theorem 5: Given $V(0)$, $U(0)$ and critical density $\lambda_c(\lambda\frac{m}{n})$, if $0 < \alpha < \infty$ and $\lambda > \lambda_c(\lambda\frac{m}{n})$, for any two nodes v_s and v_d in the *giant component*, there is a finite strictly positive constant κ_R such that

$$\mathbb{P}\Big(\lim_{d_h(v_s,v_d)\to\infty}\frac{\mathbb{T}_d}{d_h(v_s,v_d)}=\kappa_R\Big)=1.$$

B. Discussion and Simulation Results

In this section, we have studied the asymptotic dissemination latency in large-scale mobile cognitive radio networks. We find that it is overly restrictive and impossible to ensure the *full connectivity* due to the completely random locations of the secondary users. Thus we relax the requirement *full connectivity* and consider a *percolated* cognitive radio network, which consists of a *giant component* well dispersed through the whole network. While information disseminating in this *giant component*, we showed that the latency still asymptotically scales linearly with the “distance” between the sender and receiver.

We have performed a series of simulations to validate our theoretical results concerning the asymptotic latency. In these simulations, time is partitioned into unit slots. Each secondary user is *i.i.d* and uniformly distributed around its home point. The transmission range r of each secondary user and interference range R_I of primary users are set $r = 0.1$ (km) and $R_I = 0.3$ (km). Fig. 2(a) shows the percolation regions (see the shaded region) for a cognitive radio network in terms of the spatial density of secondary users and primary users, and mobility capability α. Form Fig. 2(a), we observe that the critical density of secondary users for percolation scales inversely with the spatial density of primary users and mobility is beneficial to percolation. This verifies our discussion on percolation in Section IV-A. Fig. 2(b) shows that dissemination latency in a *percolated* cognitive radio network asymptotically scales linearly with the Euclidean distance between the source and destination, which is in good accordance with our theoretical results in Theorem 5. We also observe in Fig. 2(b) that mobility can accelerate the information dissemination in percolated cognitive radio networks.

V. Conclusions

In this paper we have studied the latency and speed for information dissemination in *finite* as well as *large-scale* mobile cognitive radio networks. We have proved that in finite networks, there exists a critical value on *mobility capability* of secondary users, below which the distributions of the latency and speed are *heavy-tailed* and above which the right tails of their distributions are bounded by *gamma* random variables. And we further show that the latency scales linearly with the distance between the source and destination as the network grows to infinity. Our theoretical results provide guidelines on mobility modeling, performance analysis and protocol design in cognitive radio networks.

References

[1] F. C. Commission, “Spectrum Policy Task Force Report, FCC 02-155.” NOV 2002.

[2] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, “NeXt Generation/Dynamic Spectrum Access/Cognitive Radio Wireless Networks: A survey,” *Computer Networks*, May 2006.

[3] N. B. Chang and M. Liu, “Optimal Channel Probing and Transmission Scheduling for Opportunistic Spectrum Access,” in *Proceedings of ACM MOBICOM*, September 2007.

[4] W. Ren, Q. Zhao, and A. Swami, “Connectivity of Cognitive Radio Networks: Proximity vs. Opportunity,” in *Proceedings of ACM MOBICOM Workshop on Cognitive Radio Networks*, September 2009.

[5] Y. Xu and W. Wang, “The Speed of Information Propagation in Large Wireless Networks,” in *Proceedings of IEEE INFOCOM*, April 2008.

[6] Z. Kong and E. M. Yeh, “On the Latency for Information Dissemination in Mobile Wireless Networks,” in *Proceedings of ACM MOBIHOC*, May 2008.

[7] P. Gupta and P. R. Kumar, “The Capacity of Wireless Networks,” in *IEEE Transactions on Information Theory*, vol. 46, no. 2, March 2000, pp. 388–404.

[8] M. J. Neely and E. Modiano, “Capacity and Delay Tradeoffs for Ad-hoc Mobile Networks,” in *IEEE Transactions on Information Theory*, vol. 51, June 2005, pp. 1917–1936.

[9] X. Lin, G. Sharma, and N. B. Shroff, “Degenerate Delay-capacity Tradeoffs in Ad-hoc Networks with Brownian Mobility,” in *IEEE Transactions on Information Theory*, vol. 52, June 2006, pp. 2777–2784.

[10] T. Lindavll, *Lectures on the Coupling Method*. John Wiley & Sons, Inc.

[11] G. Casella and R. L. Berger, *Statistical Inference*, 2nd ed. Wadsworth Group, 2002.

[12] T. Liggett, “An Improved Subadditive Ergodic Theorem,” in *The Annals of Probability*, vol. 13, no. 4, 1985, pp. 1279–1285.

[13] R. Meester and R. Roy, *Continuum Percolation*. New York: Cambridge University Press, 1996.