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The Speed Bounds on Event Reporting in Mobile Sensor Networks with Energy Constraints

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Abstract—**Ensuring timely delivery of event reports is a critical requirement in mobile sensor networks, as the effectiveness of area surveillance depends largely on the sensor network response delay. However, due to the sensor mobility and the energy limitation, fast report transportation is constrained by the intermittent wireless link connections between sensor nodes. We derive lower and upper analytical bounds to characterize quantitatively the event reporting speed by considering both the sensor mobility and the energy consumption. Our work evaluates the delay performance of mobile sensor networks to meet the prompt reporting requirement in these networks.**

I. INTRODUCTION

Mobile sensor networks [1], [2] are playing an increasingly important role in the network application scenarios that require continuous or periodic area surveillance. To monitor a vast area and detect the abnormalities occurring in this area, for example in the protection of a wildlife habitat, a large amount of sensors can be deployed and each sensor node takes the responsibility of observing a piece of assigned area. Compared to static sensors, mobile sensors have the added advantages of increased mornitoring flexibility and decreased surveillance cost. A moving sensor captures the events of abnormality more accurately as it may come closer to the event locations. In addition, the sensor movement effectively extends the horizon visible to each sensor and, as the result, less sensors are needed in the network to fulfill the same surveillance task.

When a sensor node detects an occurrence of abnormality, it generates an event report and sends the report to a sink node that is responsible for collecting, processing and reacting to the event reports. The event report is delivered to a sink node via multiple relays through other sensor nodes [3]–[5]. Especially in large sensor networks, the relay path may consist of a large number of hops. However, due to the sensor mobility, neighboring sensor nodes may not have an always-available communication link between them. When each sensor roams around in its allocated region, it meets a neighboring sensor only periodically. A report can be relayed only at the time of two sensors being in contact. At other times, a sensor has to buffer the report until it has the chance to forward the report. Therefore, a potential communication problem exists with the delay experienced by the report in the network.

The event reporting delay is a critical performance metric in a sensor network. Some events detected by a sensor node require fast reporting to the sink node such that prompt actions can be taken in response to the events. For example, a fire alarm report is always time-urgent in its delivery. The effectiveness and responsiveness of a mobile sensor network largely depend on how fast the network transports and delivers the event reports generated by the mobile sensors. Given the time criticality of event reporting in mobile sensor networks, we intend to characterize in this paper the speed of transporting an event report from its origin sensor node to the sink node.

Besides node mobility, the fastness of transporting a report is also determined by the transmission powers. When the distance between a transmitter node and a receiver node is fixed, higher transmission power results in larger wireless link capacity and hence a report can be transmitted over the wireless link faster. Nonetheless, the transmission powers cannot be arbitrarily high due to the limited energy supply to each sensor node. The battery-powered sensor nodes usually work in an energy efficient mode [6], [7] such that they can stay functional for the longest time. The sparing energy usage places a limit on the wireless link capacity as well as the distance over which a report can be communicated directly between neighboring nodes. Therefore, in addition to the sensor mobility, the energy constraint is a second factor in determining how fast an event report can be transported in a mobile sensor network.

We characterize the event reporting speed in mobile sensor networks by taking both the sensor mobility and the node transmission power into consideration. We model the sensor mobility by using a simple and representative probability distribution of the sensor locations over time and translate the energy preservation requirement of sensor nodes into the wireless link availability between neighboring sensors. Based on our mobility and link models, we derive both a lower bound and an upper bound on the event reporting speed when the sensors forward an event report to the sink node. Our derived bounds are functions of the sensor mobility and the per-hop node energy consumption, which present a quantified evaluation of the sensor network responsiveness to the event occurrences under the influence of node mobility and energy limitation. Our work therefore contributes to understanding the fundamental delay performance in mobile sensor networks.

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The rest of this paper is organized as follows. In Section II, we describe our sensor node mobility model and wireless link model, and also formulate the problem of event reporting speed. In Section III, we provide a quantitative relation that maps the per-hop communication energy consumption to the maximum wireless link distance. In Section IV, we present both a lower bound and an upper bound on the event reporting speed by considering the sensor mobility and their energy consumption. Finally, we conclude this paper in Section V.

II. PROBLEM FORMULATION

We study the impact of node mobility and the communication energy limitation on the event reporting speed in mobile sensor networks. The following assumptions are made in our node mobility and energy consumption models.

A. Sensor Mobility

We assume that the mobile sensor network covers a large area with many sensor nodes deployed. Each sensor undertakes the surveillance task in an allocated region, which is assumed to be in a square shape with length H in each side. The surveillance regions of different sensors do not overlap such that the total number of sensors deployed is minimized. However, to guarantee full surveillance, adjacent regions join each other to provide seamless network coverage.

A sensor moves randomly in its allocated region. As locating a sensor precisely will render the analysis of event reporting speed intractable, we model the sensor mobility at a coarse-grained level. We divide the surveillance region allocated to a sensor into many small square-shaped cells, each with side length C. We assume $C = 1$ without loss of generality. At any time, the sensor must be located in one of the cells. We use the cell currently visited by the sensor as the sensor location while ignoring the location differences inside the cell. An example of the surveillance regions and the location cells is illustrated in Fig. 1.

When a sensor moves around in its surveillance region, it generates event reports if it observes abnormality occurrences. The sensor buffers all the reports until it meets another sensor in a neighboring region through which the reports can be routed to the sink node. In order to discover whether the neighboring sensor is within the communication range, each sensor sends beacon messages and listens to detect the presence of neighboring sensors. For energy saving reason, each sensor attempts to discover neighbors only periodically and the interval between consecutive attempts is not very short. We denote the neighbor discovery interval as T. Hence, in a slotted time scale with slot length T, a sensor may appear randomly in one of the cells within its surveillance region in any time slot. We assume that the randomness follows a uniform distribution in this paper. Two examples of the sensor mobility are depicted in Fig. 1.

B. Link Availability

Theoretically, any two sensors in the network can communicate directly, no matter how far they are separated from each other. The Shannon link capacity states

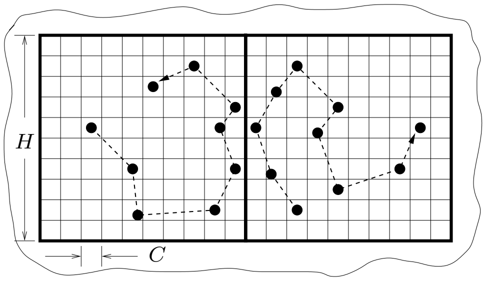

Fig. 1. An example of the surveillance regions, location cells, and sensor mobilities. For clarity, only two sensors are shown.

$$W_{ij} = B \log_2(1 + \frac{P_i x_{ij}^{-\alpha}}{N}), \tag{1}$$

where W_{ij} is the capacity of the wireless link between sensors v_i and v_j, B is the spectrum bandwidth, P_i is the transmitting power of v_i, x_{ij} is the distance between v_i and v_j (measured in the unit of path loss reference distance D), α ($\alpha \geq 2$) is the path loss exponent, and N is the noise and interference in the network (assumed to be a constant in this paper). We assume $D = 1$ without loss of generality. No matter how large x_{ij} is, v_j is able to receive a report correctly as long as v_i sends its signal slowly enough such that the transmission rate does not exceed W_{ij}. However, a transmission hop over a long distance is not feasible in reality due to the excessively large transmission delay. In accordance with the delay requirement, we hence assume that the report transmission over any wireless link should be completed within a time window τ.

Besides the delay requirement, each sensor node also has an energy budget allowed to use for a report transmission. Theoretically speaking, a sensor may send a report directly to another node located arbitrarily far away if it transmits with arbitrarily high power, which is however not energy efficient. To use the limited battery energy sparingly, we assume that each sensor node does not expend more than e amount of energy for the transmission of a report.

The delay requirement τ and the energy budget e together limit the availability of a wireless link between two sensors. A wireless link is usable if two conditions are satisfied simultaneously: first, the report transmission delay does not exceed τ, and second, the energy consumed by the transmitter node does not exceed e. If either condition is not met, a sensor cannot transmit the report directly to another sensor.

C. Event Reporting Speed

When a sensor node roams inside its surveillance region, it periodically observes a viable wireless link to communicate with a neighboring sensor, which happens when the two sensors move close to each other. A sensor forwards a report to the neighbor only when they come into contact and the wireless link between them is usable. When the wireless link is not ready, the report is withheld until the link becomes available. We assume each report has a constant length of L

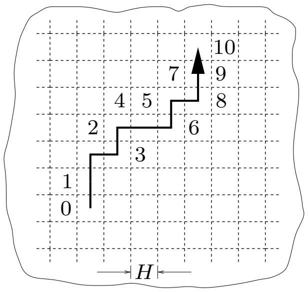

Fig. 2. The event reporting speed is defined as $\nu = \lim_{m\to\infty} \frac{mH}{\sum_{i=1}^{m} t_i}$, where t_i is the packet holding time before transmission on the i-th hop. In this example, $m = 10$.

bits during its transportation and define the long-term *event reporting speed* as

$$\nu = \lim_{m\to\infty} \frac{mH}{\sum_{i=1}^{m} t_i}, \quad (2)$$

where m is the number of hops travelled by the report since departure from its source and t_i is the report holding time at a relay node v_{i-1} before being forwarded to the next relay node v_i. The definition of ν is illustrated in Fig. 2.

Note that $\tau \ll t_i$ in reality, so we ignore the report transmission delay on each wireless link in the definition of ν. In addition, we assume that the dimension of a surveillance region is much larger than the length of a usable wireless link such that a sensor expects to meet one of its horizontal or vertical neighbors earlier than one of the diagonal neighbors and, as the result, a report travels in a zigzag path consisting of the horizontal and vertical hops only.

By rewriting Eq. (2), we can focus on one hop of report relay to study the long-term speed, as shown below,

$$\nu = \frac{H}{\lim_{m\to\infty} \frac{\sum_{i=1}^{m} t_i}{m}} = \frac{H}{\mathrm{E}[t_i]}. \quad (3)$$

In the next, we will determine $\mathrm{E}[t_i]$ under the impact of sensor mobility and link availability and then characterize ν by deriving both a lower bound and an upper bound on ν.

III. DETERMINATION OF LINK AVAILABILITY

Subject to the delay requirement τ and the energy budget e, the wireless link between any two sensors cannot exceed a maximum length. Otherwise, either of the two requirements will be violated and the wireless link is not usable. We next present an investigation of the maximum link distance under various energy budget requirements.

From the requirement that a report transmission over a wireless link should be finished within time τ, we have

$$\frac{L}{B\log_2(1+\frac{P_i x_{ij}^{-\alpha}}{N})} \le \tau \quad (4)$$

for each hop of transmission (the transmission from v_i to v_j is used to represent an arbitrary hop in the relay), and further obtain the transmission power requirement as

$$P_i \ge (2^{\frac{L}{B\tau}} - 1) N x_{ij}^{\alpha}. \quad (5)$$

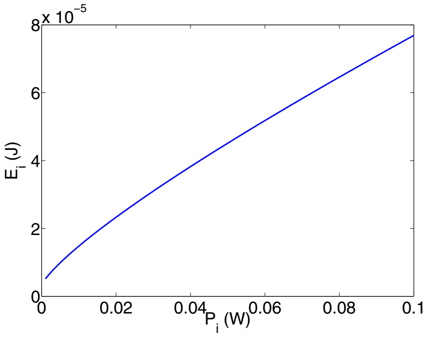

Fig. 3. The transmission energy consumption is a monotonically increasing function of the transmission power, $L = 1024$ bits, $B = 200$ kHz, $x_{ij} = 10$, $\alpha = 2$, $N = 10^{-5}$ W.

When v_i transmits a report to v_j directly, Eq. (5) determines the minimum transmission power needed at v_i in order to meet the transmission delay requirement. We next show that this minimum transmission power maps to a minimum amount of energy consumption required at v_i.

The energy consumed by v_i for a transmission is determined by both the transmitting power and the duration of transmission, as given by

$$E_i = \frac{P_i L}{B\log_2(1+\frac{P_i x_{ij}^{-\alpha}}{N})}. \quad (6)$$

In order to determine the minimum energy consumption, we compute the derivative of E_i with respect to P_i as

$$\frac{\mathrm{d}E_i}{\mathrm{d}P_i} = \frac{L}{B\log_2^2(1+y)}\Big(\log_2(1+y) - \frac{y}{(\ln 2)(1+y)}\Big), \quad (7)$$

where $y = \frac{P_i x_{ij}^{-\alpha}}{N} > 0$. We observe that $\frac{L}{B\log_2^2(1+y)} > 0$ and the sign of $\frac{\mathrm{d}E_i}{\mathrm{d}P_i}$ is determined by $\log_2(1+y) - \frac{y}{(\ln 2)(1+y)}$. In order to determine which one is larger, $\log_2(1+y)$ or $\frac{y}{(\ln 2)(1+y)}$, we make the following comparisons.

- Letting $y = 0$, we obtain $\log_2(1+y) = 0$, $\frac{y}{(\ln 2)(1+y)} = 0$, and $\log_2(1+y) = \frac{y}{(\ln 2)(1+y)}$.
- Taking the derivatives of $\log_2(1+y)$ and $\frac{y}{(\ln 2)(1+y)}$ with respect to y, we have $\frac{\mathrm{d}\log_2(1+y)}{\mathrm{d}y} = \frac{1}{(\ln 2)(1+y)}$, $\frac{\mathrm{d}\frac{y}{(\ln 2)(1+y)}}{\mathrm{d}y} = \frac{1}{(\ln 2)(1+y)^2}$, and $\frac{\mathrm{d}\log_2(1+y)}{\mathrm{d}y} > \frac{\mathrm{d}\frac{y}{(\ln 2)(1+y)}}{\mathrm{d}y}$ for all $y > 0$.

The comparison above shows $\log_2(1+y) > \frac{y}{(\ln 2)(1+y)}$ for all $y > 0$ and hence $\frac{\mathrm{d}E_i}{\mathrm{d}P_i} > 0$. We plot an example curve of E_i versus P_i in Fig. 3 to demonstrate this monotonically increasing relation between E_i and P_i.

Plugging Eq. (5) into Eq. (6), we obtain the minimum required transmission energy for v_i to forward a report to v_j at x_{ij} distance away as

$$E_i \ge (2^{\frac{L}{B\tau}} - 1)\tau N x_{ij}^{\alpha}. \quad (8)$$

Since v_i cannot use more than e amount of energy, that is $E_i \le e$, we further have

$$x_{ij} \le \Big(\frac{e}{(2^{\frac{L}{B\tau}} - 1)\tau N}\Big)^{\frac{1}{\alpha}}, \quad (9)$$

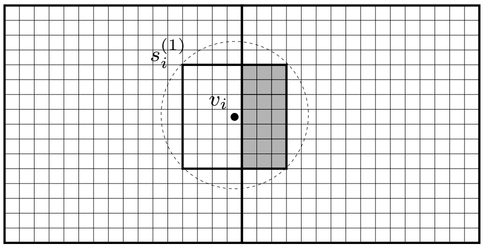

Fig. 4. Determination of a lower bound on the probability for a usable wireless link to exist between two neighboring sensors. If the sensor v_j (not shown) in the right region is located in the gray area, which is the overlap between $s_i^{(1)}$ and the right region, it must be able to communicate with the sensor v_i in the left region. The radius of the circle is r.

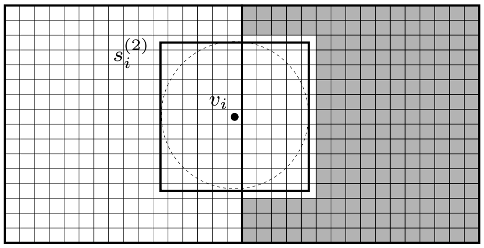

Fig. 5. Determination of an upper bound on the probability for a usable wireless link to exist between two neighboring sensors. If the sensor v_j (not shown) in the right region is located in the gray area, which is outside the overlap between $s_i^{(2)}$ and the right region, it is not able to communicate with the sensor v_i in the left region. The radius of the circle is r.

which shows that the energy budget e maps to the maximum wireless link distance $(\frac{e}{(2^{\frac{L}{B\tau}}-1)\tau N})^{\frac{1}{\alpha}}$ over which v_i may communicate with v_j directly.

Our result in Eq. (9) determines the wireless link availability between two neighboring sensors. As the sensors move randomly in their respective surveillance regions, the wireless link between them is usable when their distance does not exceed $r = (\frac{e}{(2^{\frac{L}{B\tau}}-1)\tau N})^{\frac{1}{\alpha}}$. Next, we characterize the event reporting speed based on this energy-distance mapping.

IV. BOUNDS ON EVENT REPORTING SPEED

When a sensor moves in its allocated surveillance region, it is connected to a neighboring sensor periodically. The sensor withholds the report until the wireless link to the next-hop neighboring sensor is usable and then forwards the report. Known from Eq. (3), the event reporting speed is determined by the expected time for a sensor to meet its next-hop neighbor. Note that we assume $H \gg r \gg 1$. As the result, a sensor has a much higher chance to meet one of the horizonal or vertical neighbors than one of the diagonal neighbors and therefore we only consider the horizontal and vertical wireless links. We next provide both a lower bound and an upper bound on the event reporting speed through an investigation on the expected report holding time at an arbitrary relay sensor.

A. Lower Bound

A report is transported slowly if it has to be withheld by a relay sensor for a long time. We find the lower bound on the event reporting speed by determining an upper bound on the expected report holding time, which is in turn determined by a lower bound on the probability for two neighboring sensors to be within the communication range of each other.

With energy budget e, two sensors v_i and v_j are in communication range when their distance does not exceed r. To simplify the analysis, we only look at a square area, denoted as $s_i^{(1)}$, that is centered at v_i and encompassed by a circle with radius r. If v_j is in $s_i^{(1)}$, v_i is able to forward the report to v_j, as illustrated in Fig. 4.

Depending on the location of v_i in the left region, $s_i^{(1)}$ overlaps the right region with an area of variable size. We suppose v_i is in a cell at distance k from the boundary of the two regions ($k = 1$ in Fig. 4) and determine the probability of v_j falling into $s_i^{(1)}$ as follows. With probability $\frac{H-2\lceil r\rceil}{H^2}$, v_i resides in a cell that is sufficiently far away from the region corners such that $s_i^{(1)}$ does not protrude outside the upper and lower region boundaries and the overlapping area between $s_i^{(1)}$ and the right region is at least $(2\lfloor\frac{r}{\sqrt{2}}\rfloor - 1)(\lfloor\frac{r}{\sqrt{2}}\rfloor - k)$, which indicates at least a probability of $\frac{1}{H^2}(2\lfloor\frac{r}{\sqrt{2}}\rfloor - 1)(\lfloor\frac{r}{\sqrt{2}}\rfloor - k)$ for v_j to stay in $s_i^{(1)}$. Considering all the possibilities of k, we obtain a lower bound on the probability of v_i and v_j being in communication range as

$$\begin{aligned} P &\geq \sum_{k=1}^{\lfloor\frac{r}{\sqrt{2}}\rfloor-1} \frac{H-2\lceil r\rceil}{H^2} \times \frac{1}{H^2}\Big(2\lfloor\frac{r}{\sqrt{2}}\rfloor - 1\Big)\Big(\lfloor\frac{r}{\sqrt{2}}\rfloor - k\Big) \\ &= \frac{H-2\lceil r\rceil}{H^4}\Big(2\lfloor\frac{r}{\sqrt{2}}\rfloor - 1\Big)\sum_{k=1}^{\lfloor\frac{r}{\sqrt{2}}\rfloor-1} \lfloor\frac{r}{\sqrt{2}}\rfloor - k \\ &\geq \frac{(H-2(r+1))(r-2\sqrt{2})^3}{2\sqrt{2}H^4}. \end{aligned} \tag{10}$$

As we assume $H \gg r \gg 1$, we have $H - 2(r+1) \geq \frac{H}{2}$ and $r - 2\sqrt{2} > 0$, and subsequently

$$P \geq \frac{(r-2\sqrt{2})^3}{4\sqrt{2}H^3}. \tag{11}$$

The expected report holding time is then upper bounded by

$$\mathrm{E}[t_i] = \frac{T}{P} \leq \frac{4\sqrt{2}H^3T}{(r-2\sqrt{2})^3}, \tag{12}$$

and the event reporting speed is lower bounded by

$$\nu = \frac{H}{\mathrm{E}[t_i]} \geq \frac{(r-2\sqrt{2})^3}{4\sqrt{2}H^2T}. \tag{13}$$

B. Upper Bound

Similar to our analysis for the lower bound, the upper bound on the event reporting speed is determined by the lower bound on the report holding time and in turn by an upper bound on the probability of two sensor nodes being in communication range. We next investigate this probability upper bound.

Instead of looking at the square $s_i^{(1)}$, we now study a larger square $s_i^{(2)}$ that encloses the circle centered at v_i with radius r, as illustrated in Fig. 5. Obviously, if v_j is located outside $s_i^{(2)}$, v_i cannot forward the report to v_j. When v_i is k cells

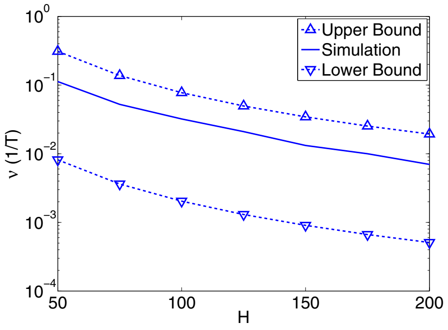

Fig. 6. The event reporting speed with various sensor mobilities.

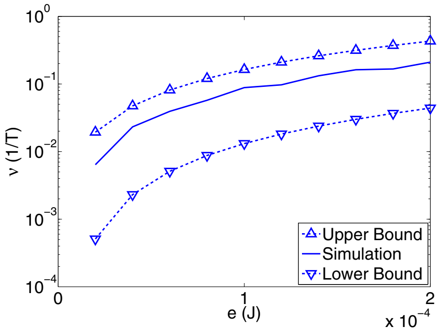

Fig. 7. The event reporting speed with various sensor energy budgets.

away from the boundary between two regions ($k = 1$ in Fig. 5), which happens with a probability $\frac{1}{H}$, the overlapping area between $s_i^{(2)}$ and the right region is at most $(2\lceil r\rceil + 1)(\lceil r\rceil - k + 1)$. Hence, with probability at least $1 - \frac{(2\lceil r\rceil+1)(\lceil r\rceil-k+1)}{H^2}$, v_j is out of direct reach of v_i. By considering all the location possibilities of v_i, we bound the probability of v_i and v_j not being in communication range as

$$\begin{aligned}\bar{P} &\geq \frac{H-\lceil r\rceil}{H} + \sum_{k=1}^{\lceil r\rceil}\frac{1}{H}\times\left(1-\frac{(2\lceil r\rceil+1)(\lceil r\rceil-k+1)}{H^2}\right)\\ &= 1-\frac{\lceil r\rceil(\lceil r\rceil+1)(2\lceil r\rceil+1)}{2H^3}\\ &\geq 1-\frac{(r+1)(r+2)(2r+3)}{2H^3},\end{aligned} \tag{14}$$

and therefore the probability of v_i and v_j being in communication range as

$$P \leq \frac{(r+1)(r+2)(2r+3)}{2H^3}. \tag{15}$$

The expected report holding time is then lower bounded by

$$\mathrm{E}[t_i] = \frac{T}{P} \geq \frac{2H^3T}{(r+1)(r+2)(2r+3)}, \tag{16}$$

and the event reporting speed is upper bounded by

$$\nu = \frac{H}{\mathrm{E}[t_i]} \leq \frac{(r+1)(r+2)(2r+3)}{2H^2T}. \tag{17}$$

C. Simulations and Observations

We have simulated the event reporting speed to verify our analytical bounds. In our simulations, we set $L = 1024$ bits, $B = 200$ kHz, $\alpha = 2$, $N = 10^{-5}$ W, and $\tau = 10^{-3}$ second. We first evaluate our speed bounds with various sensor mobilities by changing the values of H while keeping $e = 2 \times 10^{-5}$ J constantly. A total of 100 hops are simulated to obtain the event reporting speed. The simulation results are plotted in Fig. 6. We observe that the event reporting speed slows down as the size of surveillance region increases, which agrees with our intuitive understanding, and the simulated speed is between our derived bounds. We also evaluate our speed bounds with various sensor node energy budgets by changing the values of e while keeping $H = 200$ constantly. The speed is also computed by measuring the average delay in 100 hops. The results are presented in Fig. 7. We observe that the event reporting speed is an increasing function of the sensor energy budget and the simulation result is bounded well between our analytical results. As each sensor uses more energy on report transmissions, a report can be forwarded to the next sensor at a larger distance, which effectively shortens the report holding time and thus improves the reporting speed.

V. CONCLUSION

Mobile sensor networks are an important application of the large-scale wireless networks to accomplish area surveillance tasks. One of the fundamental performance issues in mobile sensor networks is the promptness of event report delivery. We have investigated in this paper the event reporting speed by considering two characteristic constraints, the sensor mobility and the energy limitation, both of which place limit on the achievable speed in transporting an event report. By modeling the sensor mobility as a probability distribution of node locations and mapping the energy budget to the link availability between sensors, we have derived both a lower bound and an upper bound on the event reporting speed. Our results provide a quantitative evaluation of the network responsiveness to event occurrences, which helps us understand and design delay-satisfactory mobile sensor networks.

REFERENCES

[1] J. Gummeson, D. Ganesan, P. Shenoy, and M. Corner, "An Adaptive Link Layer for Range Diversity in Multi-radio Mobile Sensor Networks", *Proc. of IEEE INFOCOM*, April 2009.
[2] J.W. Ho, M. Wright, and S. Das, "Fast Detection of Replica Node Attacks in Mobile Sensor Networks Using Sequential Analysis", *Proc. of IEEE INFOCOM*, April 2009.
[3] Y. Wu and Y. Li, "Construction Algorithms for k-Connected m-Dominating Sets in Wireless Sensor Networks", *Proc. of ACM MobiHoc*, May 2008.
[4] W. Cheng, K. Xing, X. Cheng, X. Lu, Z. Lu, J. Su, B. Wang, and Y. Liu, "Route Recovery in Vertex-Disjoint Multipath Routing for Many-To-One Sensor Networks", *Proc. of ACM MobiHoc*, May 2008.
[5] G. Campobello, A. Leonardi, and S. Palazzo, "A Novel Reliable and Energy-Saving Forwarding Technique for Wireless Sensor Networks", *Proc. of ACM MobiHoc*, May 2009.
[6] Y. Sun, S. Du, O. Gurewitz, and D.B. Johnson, "DW-MAC: A Low Latency, Energy Efficient Demand-Wakeup MAC Protocol for Wireless Sensor Networks", *Proc. of ACM MobiHoc*, May 2008.
[7] Y. Zhuang, J. Pan, and L. Cai, "Minimizing Energy Consumption with Probabilistic Distance Models in Wireless Sensor Networks", *Proc. of IEEE INFOCOM*, March 2010.