Information Delivery in Large Wireless Networks with Minimum Energy Expense

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***Abstract*—Energy efficient communication is a critical research problem in large-scale multihop wireless networks because of the limited energy supplies from batteries. We investigate in this paper the minimum energy required to fulfill various information delivery goals that correspond to the major communication paradigms in large wireless networks. We characterize the minimum energy requirement in two steps. We first derive the lower bounds on the energy consumption for all the possible solutions that deliver the information as required. We then design routing schemes that accomplish the information delivery tasks by using an amount of energy comparable to the lower bounds. Our work provides the fundamental understandings of energy needs and the efficient solutions for energy usages in major communication scenarios, which contribute to the rational dimensioning and wise utilization of the energy resources in large wireless networks.**

I. Introduction

Large-scale multihop wireless networks offer a flexible communication paradigm complementary to the infrastructure-based networks in places where the deployment cost and convenience are the major concerns in network construction. For example, to provide field surveillance in a wildlife habitat, many sensor nodes (such as cameras, thermometers, or sound detectors) can be installed in the area with internodal wireless communication capabilities to capture the events of interest and transport them cooperatively to the remote control center. As nodes in multihop wireless networks are usually powered by batteries, efficient energy usage is a critical research problem related to the network lifetime and usability. For economic reasons, we expect a wireless network to stay functional for the longest time before its nodes deplete their energy.

Many existing works have investigated the techniques and strategies that reduce energy consumption in wireless networks to extend network lifetimes. Examples of these energy saving efforts include scheduling nodes into the sleep mode periodically [1], minimizing node transmission powers while maintaining the network performance such as connectivity [2], [3], fault tolerance [4], transportation throughput [5], [6] and quality of service [7], and discovering the most energy efficient packet transmission paths [8], [9]. By spending the energy resources in a wireless network wisely, the existing research works have been proved very effective in extending the lifetime of wireless networks.

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However, a fundamental research question remains unanswered yet. Since energy is consumed each time a node transmits a packet to other nodes, every piece of information delivered in the network must incur some amount of energy cost. Hence, our question is: what is the *minimum* amount of energy required in order to fulfill a communication objective? The answer to this question is important in two perspectives. First, it establishes a clear target to aim for when we design energy economic communication protocols in wireless networks. Second, it helps us evaluate the energy saving results obtained from any particular communication schemes by providing a point of reference. Given the limitation of available energy and the criticality of prudent usage in wireless networks, we attempt to determine in this paper the minimum energy requirement for a large wireless network to achieve its communication goals.

We note that a node consumes energy in every functional state, such as exchanging beacons to discover network topology, detecting transmissions in the wireless medium, receiving packets from neighbor nodes, processing packets to determine routing actions, and forwarding packets to the next-hop nodes. As it is quite difficult to characterize the energy consumption in all the functional states, we focus on the energy consumed directly for packet transmissions in this paper. In fact, it has been shown that the energy used to transmit packets takes the dominant share among the total energy consumption and the other energy expenditures (e.g., packet reception and processing) continue to decrease as the circuit technology advances [10]. Speaking accurately, therefore, our results in this paper apply to the energy consumption scenarios where the node transmission energy dominates.

The minimum energy requirement for information delivery in large wireless networks has been studied in a few prior works. In [10], Rodoplu and Meng derived the bits-per-joule capacity of large wireless networks in which every node communicates with a randomly chosen destination. In [11], Wang *et al.* analyzed the total transmission powers needed to support the maximum network throughput in unicast communications. In [12], Jain *et al.* determined a linear program solution to bound the minimum energy required for multicast communications. We present in this paper a comprehensive study on the minimum energy required to achieve various communication goals, which differs from the existing works in a few aspects. First, we investigate the energy consumption for a given packet delivery rather than the throughput of a traffic flow, as compared to [11]. By removing the time ingredient, our study gives the minimum energy for all the possible scheduling and transmission strategies. Second, we derive closed-form bounds rather than a linear program formulation, as compared to [12]. Closed-form results demonstrate the relations between energy consumption and network parameters in a straightforward fashion. Third, we consider all the major communication paradigms, including broadcast, multicast and unicast, as compared to [10]–[12]. Our work hence provides a comprehensive understanding of the minimum energy for communications in large wireless networks.

The contributions of this paper are summarized in two folds. First, we determine the theoretical lower bounds on the minimum energy needed for communications in general and representative network settings. Understanding the fundamental limits in energy saving is critical for network planning such that we will not underestimate the energy demand in large wireless networks. Second, we provide the benchmark energy utilization solutions that achieve the communication goals efficiently by using an amount of energy within constant multiples of the lower bounds. The benchmark solutions demonstrate the feasibility of near-minimum energy consumption for information delivery in large wireless networks.

The rest of this paper is structured as follows. In Section II, we describe our network model and formulate the minimum energy requirement problem. In Section III, we present a preliminary result on the non-zero energy requirement in any one-hop wireless communications. Based on the understanding in Section III, we derive the lower and upper bounds on the minimum energy requirement to fulfill various communication goals in large wireless networks in Sections IV, V and VI. Finally, we summarize our findings in Section VII.

II. Model and Formulation

We investigate in this paper the minimum energy consumption for a large wireless network to achieve its information delivery goals. Specifically, our study is established on the assumptions described below.

A. Network Model

1) Network Size and Node Location: To represent a large wireless network, we assume that a total of n nodes are randomly located in a network occupying a square area $\frac{n}{\lambda}$, where $n \to \infty$ and λ is the finite node density. The locations of different nodes are independent from one another. With $n \to \infty$, the number of nodes appearing in any given region is a Poisson random variable parameterized by λ and the region size. The nodes are assumed to be static. Note that when n is finite, deterministic result is not available for any network measurement due to the randomness of node locations. Therefore, though our results in this paper are asymptotic in nature, they provide an evaluation of the expected network performance when the network consists of finite number of nodes.

2) Noise and Interference: We assume that a total of B Hz bandwidth is shared by all the nodes in their communications. The wireless link between any two nodes is an Additive White Gaussian Noise (AWGN) channel. The spectrum density of the ambient noise is N_0 and we denote the total noise observed by a wireless link as $N = N_0 B$, which is a constant everywhere in the network. Besides the ambient noise, the simultaneously scheduled transmissions impose wireless interference to one another. We model the aggregated interference at an arbitrary location in the network as a random variable uniformly distributed in $[0, I]$, where I is a constant. Our interference model is based on the assumption that any reasonably-designed link scheduling scheme does not allow the interference to be excessively high and such a scheme is implementable by separating the simultaneous transmitters sufficiently far away from one another [13].

3) Wireless Link Capacity: The strength of a transmitted wireless signal decays over distance with a path loss exponent $\alpha > 2$. However, in order to be consistent with the reality that the received signal cannot be stronger than the transmitted one, we define a unit distance D such that the exponential decay only applies to the distances larger than D. Given two nodes v_i and v_j, if v_i sends a packet to v_j by using a transmitting power P_i, the received signal strength at v_j is $P_{ij} = g_{ij} P_i$ and the channel gain g_{ij} is given by

$$g_{ij} = \begin{cases} 1 & \text{if } d_{ij} \le D \\ d_{ij}^{-\alpha} & \text{if } d_{ij} > D, \end{cases} \tag{1}$$

where d_{ij} is the Euclidean distance between v_i and v_j. To simplify the notations, we normalize all the distances in this paper against D such that we have $D = 1$. Additionally, we assume that the wireless link achieves a throughput arbitrarily close to its capacity limit

$$C_{ij} = B \log_2 \Big(1 + \frac{P_{ij}}{N + I_j}\Big), \tag{2}$$

by using advanced coding techniques [14], where I_j is the aggregated interference at v_j. Note that this information theoretic link model allows any two nodes to communicate directly regardless of their distance, though the link capacity is a decreasing function of distance.

4) Information Delivery Scenarios: The amount of energy consumed in a wireless network to fulfill the information delivery goals depends on what the tasks exactly are. It is straightforward to see that transporting a packet to a near destination consumes much less energy than delivering it to multiple remote recipients. Therefore, we define three information delivery scenarios to stardardize our study.

- Broadcast: the communication goal is to disseminate a piece of information from the source node to all the other nodes in the network.
- Multicast: the objective is to deliver the information from the source node to γ ($0 < \gamma < 1$) percentage of the other nodes, rather than all of them.
- Bordercast: the task is to transport the information from the source node to a recipient located on the network

boundary, which is in fact a special case of unicast.

These three scenarios correspond to the major communication paradigms in large wireless networks. For example, in the broadcast one we need to distribute a message to all the nodes in the network, in the multicast one our message only needs to reach a well-dispersed portion of nodes that will serve as the local redistributors in their respective vicinities, and in the bordercast one a sensor node needs to send an event report to a sink or control center located on the network boundary.

B. Problem Formulation

Let us assume that a piece of information is generated in the form of a L-bit packet by its source node v_0 and the packet needs to be delivered to its destination(s) in one of the three delivery scenarios. We denote a routing scheme that successfully fulfills the communication objective as $\mathcal{S} = \{l_{ij}\}$, where l_{ij} (l_{ij} is directional) is a wireless link used by the routing scheme. Furthermore, we denote the set of transmitter nodes in $\mathcal{S}$ as $\mathcal{T}(\mathcal{S})$ and the energy consumed by a transmitter node $v_i \in \mathcal{T}(\mathcal{S})$ as e_i. Then, the total energy consumption of $\mathcal{S}$ is defined as

$$\Psi(\mathcal{S}) = \sum_{\mathcal{T}(\mathcal{S})} e_i. \tag{3}$$

Our research objective is to determine the minimum energy consumption for all the possible routing schemes that achieve the information delivery goal, i.e.,

$$\widetilde{\Psi} = \min_{\{\mathcal{S}\}} \Psi(\mathcal{S}), \tag{4}$$

where $\{\mathcal{S}\}$ denotes the complete set of routing schemes that deliver the information as required in a given delivery scenario.

As discussed in the introduction section, we only consider the energy used for packet transmissions in this paper. Hence, if node v_i sends the packet to node v_j with transmitting power P_i and at the full link capacity rate C_{ij}, the amount of energy spent by v_i is $e_i = \frac{P_i L}{C_{ij}}$. In general, if v_i transmits the packet simultaneously to multiple receiver nodes, the energy consumption is $e_i = \frac{P_i L}{C_i}$, where C_i is the transmission rate of v_i as determined by the slowest one among the links to the multiple receivers.

Note that we do not consider any specific link scheduling scheme in this work, as long as it guarantees that the aggregated wireless interference is bounded at any location in the network. Different link scheduling schemes may result in different information delivery delays. We intend to determine the fundamental energy requirements for correct information delivery, while the communication delay is not our concern in this paper. Therefore, our results on the minimum energy requirements apply to all the link scheduling schemes.

III. A Single-Hop Examination of Energy Expense

Before we present our results on the minimum energy requirement $\widetilde{\Psi}$, we first explain why there exists a non-zero energy cost associated with each transmission in a wireless network. The minimum energy to transmit a packet over a wireless link was discovered by Uysal-Biyikoglu *et al.* in [15]. To facilitate our following analysis, we present this minimum energy finding in a different way that incorporates the path loss of wireless signals.

A. The Characteristic Curve of Single-Hop Transmission

From our previous discussion, we know that the transmission energy consumed by a node v_i is determined by the transmitting power P_i and the transmitting rate C_i. With P_i and C_i given, however, an intended receiver node v_j may or may not receive the packet, which depends on the link capacity C_{ij} between v_i and v_j. If $C_i \leq C_{ij}$, v_j receives the packet. If $C_i > C_{ij}$, v_j does not receive the packet and the transmission energy of v_i is wasted without revenue. Therefore, from the energy efficiency perspective, we expect every transmission to be successful and investigate the minimum energy required for correct packet reception at the intended receiver nodes.

The ambient noise and wireless interference create a combined barrier $N + I_j$ that prevents the signal of v_i from being received correctly by v_j, which we call a *hurdle* of noise and interference. To transmit successfully, the task of v_i is to select appropriate P_i and C_i to overcome this hurdle $N + I_j$ at v_j. We next define a function called *characteristic curve* (c-curve) to describe mathematically the likelihood of the transmitted signal being received correctly over different transmission distances. The c-curve of a transmission from v_i is defined as

$$\phi_i(x) = H_i g_i(x), \tag{5}$$

where $H_i = \frac{P_i}{2^{C_i/B}-1}$, x is the distance from v_i, and $g_i(x)$ is the generalized channel gain without specifying the particular receiver node, given by $g_i(x) = 1$ if $x \leq 1$ and $g_i(x) = x^{-\alpha}$ if $x > 1$ (recall that all the distances are normalized against D). From the definition of H_i, we have the following equation

$$C_i = B \log_2 \left(1 + \frac{P_i}{H_i}\right). \tag{6}$$

It shows that H_i denotes the maximum height of the hurdle that a transmission can surmount in order to be received correctly by a node within unit distance from v_i. By multiplying $g_i(x)$, the c-curve $\phi_i(x)$ depicts the contour of the surmountable hurdle heights over different transmission distances. As $\phi_i(x) \leq H_i$ for any x, we name H_i the *top height* of $\phi_i(x)$.

We illustrate an example of the c-curve and hurdles in Fig. 1. At location x_1, the c-curve is above the height of the hurdle. Therefore, if a receiver node is located at x_1, it can receive the packet from v_i correctly. At location x_2, the c-curve is below the height of the hurdle. A receiver located at x_2 hence cannot receive the packet. At each location, the hurdle height is a random variable uniformly distributed in $[N, N+I]$ according to our assumption. Note that the heights are independent from one another at different locations.

B. The Minimum Energy for Single-Hop Transmission

Based on the c-curve representation, we are now able to derive the minimum energy required for a single-hop trans-

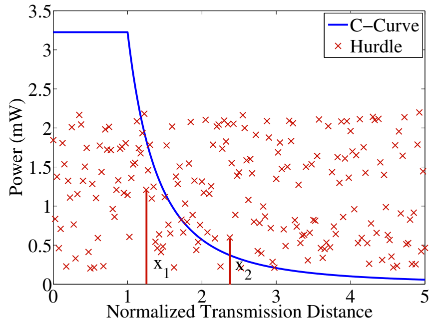


Fig. 1. An example of the characteristic curve $\phi_i(x)$ and the hurdles of noise and interference at various locations, $P_i = 100$ mW, $C_i = 1$ Mbps, $B = 200$ kHz, $\alpha = 2.5$, $N = 0.2$ mW, $I = 2$ mW.

mission of v_i. Plugging Eq. (6) into $e_i = \frac{P_i L}{C_i}$, we have

$$e_i = \frac{P_i L}{B \log_2(1 + \frac{P_i}{H_i})}, \tag{7}$$

which is an increasing function of P_i and reaches the minimum when $P_i = 0$. We denote the minimum of e_i as $\tilde{e}_i$ and obtain

$$\tilde{e}_i = \lim_{P_i \to 0} \frac{P_i L}{B \log_2(1 + \frac{P_i}{H_i})} = \frac{(\ln 2) L H_i}{B}. \tag{8}$$

Eq. (8) shows that the minimum energy consumed in a transmission is determined by the top height H_i of the c-curve. In order to transmit the packet correctly to a node located at a certain distance away, H_i should be sufficiently high such that the c-curve is above the hurdle of noise and interference at the location of the receiver node. For a short transmission distance less than the unit distance, $H_i \geq N$ is the minimum requirement. For longer transmission distances, larger values of H_i are required. In all cases, $\tilde{e}_i \geq \frac{(\ln 2) L N}{B}$ is observed.

The analysis on $\tilde{e}_i$ shows that the transmitter node should use a slightly positive transmitting power close to zero to minimize the energy cost, which results in a low link capacity. In the limiting case of zero transmitting power, the transmission will take infinite time to finish. However, as we do not consider the delay of information delivery in this paper, we consider the arbitrarily small transmitting power as a viable strategy and the minimum energy cost $\tilde{e}_i$ achievable.

C. The Energy Efficiency of Single-Hop Transmission

Our analysis above shows that the minimum energy for a single-hop transmission is determined by H_i of the c-curve, which is in turn related to the transmission radius of v_i. It is straightforward to see that a transmitted packet is received by increased number of nodes or by nodes located at large distances if H_i increases. As our goal is to transport the packet to an amount of nodes (in broadcast and multicast) or over a distance (in bordercast), we next discuss the energy efficiency in a single-hop transmission that contributes to the fulfillment of the ultimate delivery goal. The single-hop energy efficiency provides the necessary mathematical preparation for our subsequent derivations of $\widetilde{\Psi}$.

To measure the single-hop energy efficiency, we define two metrics called the *dissemination utility factor* $\eta_i^{(1)}$ and the *distance utility factor* $\eta_i^{(2)}$ respectively as

$$\eta_i^{(1)} = \frac{\varrho_i^{(1)}}{\tilde{e}_i}, \tag{9}$$

$$\eta_i^{(2)} = \frac{\varrho_i^{(2)}}{\tilde{e}_i}, \tag{10}$$

where $\varrho_i^{(1)}$ is the expected number of nodes that receive the packet from v_i and $\varrho_i^{(2)}$ is the expected farthest distance over which the packet from v_i can be received correctly. We have the following two lemmas regarding $\eta_i^{(1)}$ and $\eta_i^{(2)}$.

Lemma 1: In a transmission of v_i, $\eta_i^{(1)}$ is determined by

$$\eta_i^{(1)} = \begin{cases} \frac{\pi\alpha B\lambda}{(\ln 2)(\alpha-2)LI}\Big(1 - \frac{N^{1-\frac{2}{\alpha}}}{H_i^{1-\frac{2}{\alpha}}}\Big) & \text{if } H_i \leq N + I \\ \frac{\pi\alpha B\lambda\big((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}}\big)}{(\ln 2)(\alpha-2)LIH_i^{1-\frac{2}{\alpha}}} & \text{if } H_i \geq N + I. \end{cases} \tag{11}$$

Proof: When $H_i \leq N + I$, a packet transmitted by v_i is received probabilistically by a node v_j located at distance x away from v_i. The packet is received if $H_i g_i(x) \geq N + I_j$ and not received if $H_i g_i(x) < N + I_j$. Given Poisson distribution with density λ of the nodes around v_i, we have

$$\varrho_i^{(1)} = \lambda \int_0^{(\frac{H_i}{N})^{\frac{1}{\alpha}}} \frac{H_i g_i(x) - N}{I} \cdot 2\pi x \, \mathrm{d}x. \tag{12}$$

Plugging Eq. (8) and (12) into Eq. (9), we obtain

$$\eta_i^{(1)} = \frac{\pi\alpha B\lambda}{(\ln 2)(\alpha - 2)LI}\Big(1 - \frac{N^{1-\frac{2}{\alpha}}}{H_i^{1-\frac{2}{\alpha}}}\Big). \tag{13}$$

When $H_i \geq N + I$, the region around v_i in which other nodes may receive the packet can be divided into two sub-regions: one with radius $(\frac{H_i}{N+I})^{\frac{1}{\alpha}}$ and the other between radius $(\frac{H_i}{N+I})^{\frac{1}{\alpha}}$ and radius $(\frac{H_i}{N})^{\frac{1}{\alpha}}$. All the nodes in the first sub-region can receive the packet from v_i correctly because $H_i g_i(x) \geq N + I$, while the nodes in the second sub-region only receive the packet probabilistically. Therefore, we have

$$\varrho_i^{(1)} = \lambda\pi\Big(\frac{H_i}{N + I}\Big)^{\frac{2}{\alpha}} + \lambda \int_{(\frac{H_i}{N+I})^{\frac{1}{\alpha}}}^{(\frac{H_i}{N})^{\frac{1}{\alpha}}} \frac{H_i g_i(x) - N}{I} \cdot 2\pi x \, \mathrm{d}x. \tag{14}$$

Plugging Eq. (8) and (14) into Eq. (9), we have

$$\eta_i^{(1)} = \frac{\pi\alpha B\lambda\big((N + I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}}\big)}{(\ln 2)(\alpha - 2)LIH_i^{1-\frac{2}{\alpha}}}. \tag{15}$$

The combination of Eq. (13) and (15) finishes the proof. ∎

Lemma 2: In a transmission of v_i, $\eta_i^{(2)} \leq \frac{B}{(\ln 2)LN^{\frac{1}{\alpha}}H_i^{1-\frac{1}{\alpha}}}$.

Proof: It is obvious to see that $\varrho_i^{(2)} \leq (\frac{H_i}{N})^{\frac{1}{\alpha}}$ because any node located more than $(\frac{H_i}{N})^{\frac{1}{\alpha}}$ distance away from v_i cannot receive the packet. Then, plugging $\varrho_i^{(2)} \leq (\frac{H_i}{N})^{\frac{1}{\alpha}}$ and Eq. (8) into Eq. (10), we have

$$\eta_i^{(2)} \leq \frac{B}{(\ln 2) L N^{\frac{1}{\alpha}} H_i^{1-\frac{1}{\alpha}}}, \tag{16}$$

which finishes the proof. ■

Based on our understanding of the energy efficiency per hop of transmission, we next present our findings on the minimum energy required to achieve various information delivery goals.

IV. The Minimum Energy for Broadcast Information Delivery

In the broadcast information delivery scenario, a packet needs to be disseminated from the source node v_0 to all the other nodes in the network. For clarity, we denote $\widetilde{\Psi}$ as $\widetilde{\Psi}^{(b)}$ in this section. We derive both a lower bound and an upper bound on $\widetilde{\Psi}^{(b)}$ to characterize the minimum energy requirement. The lower bound is obtained by evaluating the energy efficiency of each transmission that satisfies the network connectivity requirement for information broadcast in large wireless networks. The upper bound is demonstrated by constructing a routing scheme that delivers the packet as required with an amount of energy within a multiple of the lower bound.

A. Lower Bound

For a packet to be disseminated to all the nodes in the network, each relay node must use a sufficiently large transmission radius such that there exists a path between the source node and each other node in the network. Gupta and Kumar shows in [2] that the critical transmission radius is $r(n) = \sqrt{\frac{\ln n + c(n)}{\pi n}}$ in a dense network with $\lambda = n$, where $c(n) \to \infty$ as $n \to \infty$. To connect every node in a large wireless network with probability one, a transmission radius not smaller than $r(n)$ is required. If we scale the network density from n to λ, we have

$$r(n) = \sqrt{\frac{\ln n + c(n)}{\pi \lambda}} \tag{17}$$

as the minimum requirement on the node transmission radius in our network model. Combining the transmission radius and our result on the energy efficiency, we have the following theorem on $\widetilde{\Psi}^{(b)}$.

Theorem 1: In a large wireless network with random node locations, the minimum energy required to disseminate a packet to all the nodes in the network is lower bounded by

$$\widetilde{\Psi}^{(b)} \geq \frac{\xi^{(b)} (\ln n)^{\frac{\alpha}{2}-1} n}{\lambda^{\frac{\alpha}{2}}}, \tag{18}$$

where $\xi^{(b)} = \frac{(\ln 2)(\alpha-2) L I N^{1-\frac{2}{\alpha}}}{\pi^{\frac{\alpha}{2}} \alpha B ((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}})}$ is a constant, with probability one as $n \to \infty$.

Proof: For an arbitrary transmitter node v_i in the network, with probability one we have $r_i \geq \sqrt{\frac{\ln n}{\pi \lambda}}$ in order to keep the network connected, as known from Eq. (17). Transmitting a packet successfully to a node located at r_i distance away requires at least

$$H_i \geq N r_i^{\alpha} \geq N \Big(\frac{\ln n}{\pi \lambda}\Big)^{\frac{\alpha}{2}}. \tag{19}$$

We hence have $H_i \geq N + I$ as $n \to \infty$. By Lemma 1, $\eta_i^{(1)}$ is a decreasing function of H_i in the regime $H_i \geq N + I$ and

$$\begin{aligned} \eta_i^{(1)} &\leq \frac{\pi \alpha B \lambda \big((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}}\big)}{(\ln 2)(\alpha-2) L I N^{1-\frac{2}{\alpha}} \big(\frac{\ln n}{\pi \lambda}\big)^{\frac{\alpha}{2}-1}} \\ &= \frac{\alpha B \pi^{\frac{\alpha}{2}} \lambda^{\frac{\alpha}{2}} \big((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}}\big)}{(\ln 2)(\alpha-2) L I N^{1-\frac{2}{\alpha}} (\ln n)^{\frac{\alpha}{2}-1}}. \end{aligned} \tag{20}$$

The total energy required to disseminate the packet to all the nodes in the network is then bounded by

$$\widetilde{\Psi}^{(b)} \geq \frac{n}{\eta_i^{(1)}} \geq \frac{(\ln 2)(\alpha-2) L I N^{1-\frac{2}{\alpha}} (\ln n)^{\frac{\alpha}{2}-1} n}{\alpha B \pi^{\frac{\alpha}{2}} \lambda^{\frac{\alpha}{2}} \big((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}}\big)}, \tag{21}$$

which is equivalent to Eq. (18) by substituting $\xi^{(b)}$. ■

B. Upper Bound

We next design a packet routing scheme and prove that the proposed scheme delivers the packet to all the nodes in the network by using an amount of energy that is a multiple of the lower bound. The energy consumption of this example scheme hence provides an upper bound on the minimum energy requirement because other schemes may perform even better than this proposed one. We summarize the result of our broadcast routing scheme in the theorem below.

Theorem 2: In a large wireless network with random node locations, the minimum energy required to disseminate a packet to all the nodes in the network is upper bounded by

$$\widetilde{\Psi}^{(b)} \leq \frac{k^{(b)} \xi^{(b)} (\ln n)^{\frac{\alpha}{2}-1} n}{\lambda^{\frac{\alpha}{2}}}, \tag{22}$$

where $k^{(b)} = \frac{2^{\frac{3\alpha}{2}} \pi \alpha (N+I) \big((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}}\big)}{(\alpha-2) I N^{1-\frac{2}{\alpha}}}$ is a constant, with probability one as $n \to \infty$.

Proof: The simpliest broadcast routing scheme is to flood the packet in the network by requiring every node transmit it once using $r(n)$ as the transmission radius. This simple scheme is however not energy efficient. It consumes a total amount of energy that is significantly higher than the lower bound. We present next a slightly modified version of the simple flooding, which reduces the energy expense while guaranteeing the packet to be received by all the nodes in the network with probability one.

From Eq. (17), we know that $r_i = \sqrt{\frac{2 \ln n}{\pi \lambda}}$ is a sufficiently large transmission radius to connect every node in the network with probability one. To reduce redundant transmissions, we divide the network into small square cells each with side length $\frac{r_i}{\sqrt{2}}$ and group all the nodes in the same cell as a block, as illustrated in Fig. 2. In each block, we let one and only one node transmit the packet, which is the first node that receives the packet in its block. Each selected transmitter node configures $H_i = (N+I)(2r_i)^{\alpha}$ when sending the packet. The

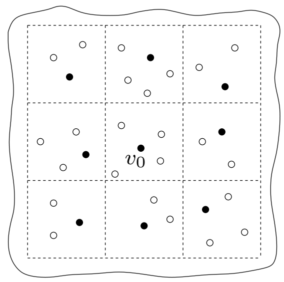


Fig. 2. The broadcast routing scheme. For clarity, we have only shown the region around the source node v_0. The side length of each cell is $\frac{r_i}{\sqrt{2}}$. The dark nodes are selected to transmit while the white nodes are not.

flooding process begins with the source node v_0 and continues until every non-empty block has transmitted once.

We first prove that this routing scheme delivers the packet from v_0 to all the other nodes in the network with probability one. By using $H_i = (N+I)(2r_i)^\alpha$, we observe that each transmitter node v_i is able to send the packet correctly to any node within distance $2r_i$. Since any two nodes in the same cell are separated at most by distance r_i, which is the diagonal of the cell, a node v_j must be able to receive the packet from v_i directly if v_j can receive the packet from some node in v_i's block when the original simple flooding is used with transmission radius r_i. Therefore, same as the simple flooding, our modified scheme delivers the packet throughout the network with probability one.

We next determine the amount of energy consumed in our modified flooding scheme. With $H_i = (N+I)(2r_i)^\alpha$, each transmitter node consumes $\tilde{e}_i = \frac{(\ln 2)L(N+I)(2r_i)^\alpha}{B}$ and the total energy expenditure in the network is bounded by $\Psi^{(b)} \leq \frac{2n}{\lambda r_i^2}\tilde{e}_i$. Plugging $r_i = \sqrt{\frac{2\ln n}{\pi\lambda}}$, we obtain

$$\widetilde{\Psi}^{(b)} \leq \Psi^{(b)} \leq \frac{2^{\frac{3\alpha}{2}}(\ln 2)L(N+I)(\ln n)^{\frac{\alpha}{2}-1}n}{B\pi^{\frac{\alpha}{2}-1}\lambda^{\frac{\alpha}{2}}}. \tag{23}$$

Substituting $k^{(b)}$ and $\xi^{(b)}$, Eq. (23) is equivalent to (22). ■

V. The Minimum Energy for Multicast Information Delivery

In this section, we study the minimum energy requirement for multicast information delivery, in which we do not need to disseminate a piece of information to every node in the network. Instead, the delivery is satisfactory if γ $(0 < \gamma < 1)$ percentage of the nodes receive the information. If other nodes are interested in the information, they may request it from nearby nodes that have the information. Therefore, our primary goal in this scenario is to guarantee the information delivery to γ percentage of nodes. Similar to our approach in the broadcast scenario, we determine the lower and upper bounds on $\widetilde{\Psi}$, which we denote as $\widetilde{\Psi}^{(m)}$ in this section for clarity.

A. Lower Bound

The lower bound on $\widetilde{\Psi}^{(m)}$ can be determined by using the concept of *continuum percolation* [16]. In a large wireless network with infinite number of nodes and random node locations, if every node can transmit a packet successfully to its neighbors within the unit distance, then the continuum percolation theory states that the amount of nodes that eventually receive the packet from the source node v_0 depends on the node density λ. Specifically, there exists a critical density λ_c $(1.43 < \lambda_c < 1.44)$ such that the size of the cluster containing v_0 takes a non-zero percentage of the network size with some positive probability if $\lambda > \lambda_c$ and a zero percentage with probability one if $\lambda < \lambda_c$, where the cluster is defined as the set of nodes that receive the packet. Based on the percolation theory, we have the following theorem on $\widetilde{\Psi}^{(m)}$.

Theorem 3: In a large wireless network with random node locations, the minimum energy required to disseminate a packet to γ $(0 < \gamma < 1)$ percentage of nodes in the network is lower bounded by

$$\widetilde{\Psi}^{(m)} \geq \begin{cases} \frac{\xi_1^{(m)} n}{\lambda^{\frac{\alpha}{2}}} & \text{if } \lambda \leq \lambda_c\left(\frac{N}{N+I}\right)^{\frac{2}{\alpha}} \\ \frac{\xi_2^{(m)} n}{\lambda} & \text{if } \lambda \geq \lambda_c\left(\frac{N}{N+I}\right)^{\frac{2}{\alpha}}, \end{cases} \tag{24}$$

where $\xi_1^{(m)} = \frac{(\ln 2)(\alpha-2)\gamma L I N^{1-\frac{2}{\alpha}}\lambda_c^{\frac{\alpha}{2}-1}}{\pi\alpha B((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}})}$ is a constant and $\xi_2^{(m)} = \frac{(\ln 2)(\alpha-2)\gamma L I (N+I)^{1-\frac{2}{\alpha}}}{\pi\alpha B((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}})}$ is also a constant, with probability one as $n \to \infty$.

Proof: We first examine the case $\lambda \leq \lambda_c(\frac{N}{N+I})^{\frac{2}{\alpha}}$. From the percolation theory and the scaling property [16], we know that $r_i \geq \sqrt{\frac{\lambda_c}{\lambda}}$ is required with probability one in order to disseminate a packet to a non-zero percentage of nodes when λ is given, which further requires $H_i \geq N r_i^\alpha \geq N(\frac{\lambda_c}{\lambda})^{\frac{\alpha}{2}}$. When $\lambda \leq \lambda_c(\frac{N}{N+I})^{\frac{2}{\alpha}}$, we have $H_i \geq N(\frac{\lambda_c}{\lambda})^{\frac{\alpha}{2}} \geq N + I$ and

$$\eta_i^{(1)} \leq \frac{\pi\alpha B\lambda^{\frac{\alpha}{2}}\left((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}}\right)}{(\ln 2)(\alpha-2)LIN^{1-\frac{2}{\alpha}}\lambda_c^{\frac{\alpha}{2}-1}}, \tag{25}$$

as known from Lemma 1. The total energy required to disseminate the packet to γn number of nodes is therefore lower bounded by

$$\widetilde{\Psi}^{(m)} \geq \frac{\gamma n}{\eta_i^{(1)}} \geq \frac{(\ln 2)(\alpha-2)LIN^{1-\frac{2}{\alpha}}\lambda_c^{\frac{\alpha}{2}-1}\gamma n}{\pi\alpha B\lambda^{\frac{\alpha}{2}}\left((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}}\right)}, \tag{26}$$

which is the first part in Eq. (24) by substituting $\xi_1^{(m)}$.

We next prove the case $\lambda \geq \lambda_c(\frac{N}{N+I})^{\frac{2}{\alpha}}$. We know from Lemma 1 that for all $H_i \geq N$ we always have

$$\eta_i^{(1)} \leq \frac{\pi\alpha B\lambda}{(\ln 2)(\alpha-2)LI}\left(1 - \left(\frac{N}{N+I}\right)^{1-\frac{2}{\alpha}}\right) \tag{27}$$

because $\eta_i^{(1)}$ is an increasing function of H_i when $H_i \leq N+I$ and a decreasing function of H_i when $H_i \geq N+I$. Similar to Eq. (26), we have

$$\widetilde{\Psi}^{(m)} \geq \frac{\gamma n}{\eta_i^{(1)}} \geq \frac{(\ln 2)(\alpha-2)LI(N+I)^{1-\frac{2}{\alpha}}\gamma n}{\pi\alpha B\lambda((N+I)^{1-\frac{2}{\alpha}} - N^{1-\frac{2}{\alpha}})}, \tag{28}$$

which is the second part in Eq. (24) by substituting $\xi_2^{(m)}$. ■

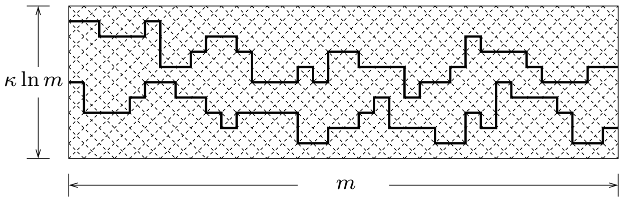
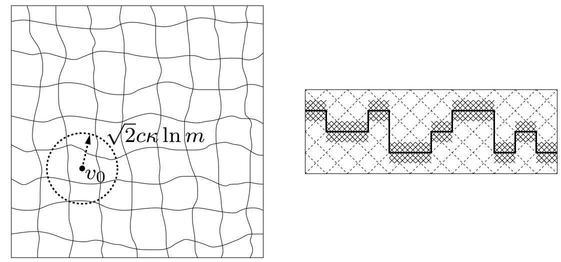


Fig. 3. An example of the disjoint paths in the highway system.



(a) (b)

Fig. 4. The packet transportation in the highway system. (a) The packet traverses all the horizontal and vertical paths. (b) When the packet traverses a path, all the nodes in the shadow area receive the packet correctly.

B. Upper Bound

Similar to our approach in the broadcast scenario, we design a routing scheme to deliver the packet and use the energy consumption of this example scheme as an upper bound on the minimum energy requirement. The challenge in the routing scheme design is that we must guarantee at least γn number of nodes receive the packet correctly. Before we present our solution for the packet delivery, we introduce a discovery by Franceschetti *et al.* [17] that will serve as the theoretical foundation for our routing design.

Franceschetti *et al.* found in [17] that in a large wireless network there exists a communication highway system consisting of many horizontal and vertical paths that span the entire network under certain conditions. To construct this highway system, we partition the network into diamond-shaped cells with constant side length c and require that every node use a sufficiently large transmission radius to communicate successfully with any other node in its neighboring cells. If we take an arbitrary strip area of dimension $\kappa \ln m \times m$ from the network, where $m = \frac{\sqrt{n}}{c\sqrt{2\lambda}}$ is the network dimension measured in the unit of cell diagonal, then with probability one we can find at least $\delta \ln m$ number of disjoint paths that connect the two short edges of the strip area for any $\delta > 0$ that satisfies

$$\delta \ln \frac{p}{1-p} + \kappa \ln(6(1-p)) + 1 < 0, \tag{29}$$

where $\kappa > 0$, $p = 1 - e^{-\lambda c^2}$, and $\frac{5}{6} < p < 1$. An example of the disjoint paths in a strip area is illustrated in Fig. 3. Note that the distance between an arbitrary node and its nearest path in the highway system does not exceed $\sqrt{2}c\kappa \ln m$, which is straightforward to observe in Fig. 3.

We find from Eq. (29) that we can construct the highway system with an arbitrary path density such that $\frac{\delta}{\kappa} = \gamma$ for any $0 < \gamma < 1$. To see this finding, we substitute $\gamma\kappa$ for δ in Eq. (29) and let $\kappa = \ln m$ to obtain

$$\gamma < \frac{-\ln(6(1-p))}{\ln \frac{p}{1-p}} \tag{30}$$

as $n \to \infty$. The right hand side of Eq. (30) is a monotonically increasing function of p in the interval $(0, 1)$ when p changes in $(\frac{5}{6}, 1)$. Therefore, given any $\gamma \in (0, 1)$, we can always find a constant $p(\gamma)$ such that Eq. (30) is satisfied. We next design a routing scheme to deliver the packet from v_0 to at least γn nodes in the network by utilizing the highway system. The result is summarized in the theorem below.

Theorem 4: In a large wireless network with random node locations, the minimum energy required to disseminate a packet to γ $(0 < \gamma < 1)$ percentage of nodes in the network is upper bounded by

$$\widetilde{\Psi}^{(m)} \leq \begin{cases} \frac{k_1^{(m)} \xi_1^{(m)} n}{\lambda^{\frac{\alpha}{2}}} & \text{if } \lambda \leq -8\ln(1-p(\gamma)) \\ \frac{k_2^{(m)} \xi_2^{(m)} n}{\lambda} & \text{if } \lambda \geq -8\ln(1-p(\gamma)), \end{cases} \tag{31}$$

$k_1^{(m)} = \frac{(2\sqrt{2})^\alpha \pi\alpha(-\ln(1-p(\gamma)))^{\frac{\alpha}{2}-1}(N+I)\left((N+I)^{1-\frac{2}{\alpha}}-N^{1-\frac{2}{\alpha}}\right)}{(\alpha-2)\gamma I N^{1-\frac{2}{\alpha}}\lambda_c^{\frac{\alpha}{2}-1}}$ and $k_2^{(m)} = \frac{8\pi\alpha(N+I)^{\frac{2}{\alpha}}\left((N+I)^{1-\frac{2}{\alpha}}-N^{1-\frac{2}{\alpha}}\right)}{(\alpha-2)\gamma I}$ are constants, $p(\gamma)$ satisfies Eq. (30), with probability one as $n \to \infty$.

Proof: We present a packet delivery scheme via the highway system and demonstrate that the energy consumed in this scheme does not exceed the amount given in Eq. (31). Our scheme works as follows. For a given delivery objective γ, we choose a constant $p(\gamma)$ that satisfies Eq. (30) and determine $c = \max\{\sqrt{\frac{-\ln(1-p(\gamma))}{\lambda}}, \frac{1}{2\sqrt{2}}\}$. We then partition the network into diamond-shaped cells with side length c and designate one node in each cell as a relay to forward the packet when the nodes in the cell receive the packet. We set $\kappa = \ln m$, configure $H_0 = (N + I)(\sqrt{2}c\kappa \ln m)^\alpha$ for v_0, and $H_i = (N + I)(2\sqrt{2}c)^\alpha$ for any relay node v_i. The packet dissemination process begins with v_0 and ends when every non-empty cell has forwarded the packet.

We first prove that our proposed routing scheme fulfills the information delivery goal by transporting the packet from v_0 to at least γn number of nodes in the network. By the configurations of $p(\gamma)$, c and H_i, we know that the highway system exists and the path density $\frac{\delta}{\kappa}$ is at least γ. Besides, the configuration of H_0 guarantees that all the nodes within distance $\sqrt{2}c\kappa \ln m$ from v_0 receive the packet correctly and at least one of them serves as an entry point in the highway system. When each cell forwards the packet, the packet is transported along the paths in the highway system. Note that with probability one there are at least γm horizontal paths and at least γm vertical paths, and these paths intersect one another, as illustrated in Fig. 4(a). When the dissemination process stops, the packet must have traversed all the paths in the highway system. Let us examine one of the horizontal paths in detail. As shown in Fig. 4(b), when the packet traverses the path, it is received by at least $2\lambda c^2 m$ nodes. Since the γm horizontal paths are disjoint, the packet must be received by at least $2\gamma\lambda c^2 m^2 = \gamma n$ number of nodes by the time dissemination stops.

We next compute the energy consumption in our proposed routing scheme. In the first hop of packet transportation, v_0 consumes $\tilde{e}_0 = \frac{(\ln 2)L(N+I)(\sqrt{2}c\kappa \ln m)^\alpha}{B}$. In all the other relay hops, the total energy consumed is bounded by

$$\sum_i \tilde{e}_i \le \frac{(\ln 2)L(N+I)(2\sqrt{2})^\alpha c^{\alpha-2} n}{B\lambda}, \tag{32}$$

because there are at most $\frac{n}{\lambda c^2}$ relay nodes and each consumes $\tilde{e}_i = \frac{(\ln 2)L(N+I)(2\sqrt{2}c)^\alpha}{B}$. Note that $\tilde{e}_0 = o(\sum_i \tilde{e}_i)$ as $n \to \infty$ and hence $\tilde{e}_0$ can be ignored. Depending on λ, the cell side length c takes the following values

$$c = \begin{cases} \sqrt{\frac{-\ln(1-p(\gamma))}{\lambda}} & \text{if } \lambda \le -8\ln(1-p(\gamma)) \\ \frac{1}{2\sqrt{2}} & \text{if } \lambda \ge -8\ln(1-p(\gamma)). \end{cases} \tag{33}$$

Plugging Eq. (33) into Eq. (32) and noting $\widetilde{\Psi}^{(m)} \le \sum_i \tilde{e}_i$, we obtain the upper bound on $\widetilde{\Psi}^{(m)}$ as Eq. (31). ■

By comparing Eq. (24) and (31), we note that both the lower and upper bounds on $\widetilde{\Psi}^{(m)}$ have two separate scaling regimes regarding the node density λ and the scaling factors of λ match each other in the corresponding regimes. However, the boundary density that separates the regimes differs from (24) to (31). In fact, either one of the two boundary densities can be changed to exactly match the other. Due to space constraint, we skip this part, which does not change the bound tightness.

VI. The Minimum Energy for Bordercast Information Delivery

In the bordercast information delivery scenario, a node needs to send a piece of information to a destination located on the network boundary. As we study a large network with $n \to \infty$, any arbitrary node can be chosen as the network center. We hence assume that the source node v_0 is located at the center of the network. We next characterize the minimum energy requirement for bordercast, denoted as $\widetilde{\Psi}^{(u)}$, by deriving the lower and upper bounds on $\widetilde{\Psi}^{(u)}$.

A. Lower Bound

Similar to our proof technique used in the multicast study, we determine the lower bound on $\widetilde{\Psi}^{(u)}$ by using the percolation theory again. We have the following theorem on $\widetilde{\Psi}^{(u)}$.

Theorem 5: In a large wireless network with random node locations, the minimum energy required to transport a packet from the network center to the network boundary is lower bounded by

$$\widetilde{\Psi}^{(u)} \ge \begin{cases} \frac{\xi_1^{(u)} n^{\frac{1}{2}}}{\lambda^{\frac{\alpha}{2}}} & \text{if } \lambda \le \lambda_c \\ \frac{\xi_2^{(u)} n^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}} & \text{if } \lambda \ge \lambda_c, \end{cases} \tag{34}$$

where $\xi_1^{(u)} = \frac{(\ln 2)LN\lambda_c^{\frac{\alpha-1}{2}}}{2B}$ and $\xi_2^{(u)} = \frac{(\ln 2)LN}{2B}$ are constants, with probability one as $n \to \infty$.

Proof: We know from our previous discussion that $r_i \ge \sqrt{\frac{\lambda_c}{\lambda}}$ is required with probability one for each node v_i in order to disseminate a packet to a non-zero percentage of nodes. If $r_i < \sqrt{\frac{\lambda_c}{\lambda}}$, with probability one the packet is received only by finite number of nodes as $n \to \infty$, implying that the packet does not reach the network boundary. Hence, $r_i \ge \sqrt{\frac{\lambda_c}{\lambda}}$ is a basic requirement for bordercast, which further requires $H_i \ge N r_i^\alpha \ge N(\frac{\lambda_c}{\lambda})^{\frac{\alpha}{2}}$. Besides, $H_i \ge N$ is also required. Otherwise, the transmission from v_i fails definitely. Combining the two requirements, we have $H_i \ge \max\{N(\frac{\lambda_c}{\lambda})^{\frac{\alpha}{2}}, N\}$.

From Lemma 2, we know that the bound on $\eta_i^{(2)}$ is a decreasing function of H_i. When $\lambda \le \lambda_c$, we have $N(\frac{\lambda_c}{\lambda})^{\frac{\alpha}{2}} \ge N$, $H_i \ge N(\frac{\lambda_c}{\lambda})^{\frac{\alpha}{2}}$, and

$$\eta_i^{(2)} \le \frac{B\lambda^{\frac{\alpha-1}{2}}}{(\ln 2)LN\lambda_c^{\frac{\alpha-1}{2}}}. \tag{35}$$

The minimum energy requirement is therefore bounded by

$$\widetilde{\Psi}^{(u)} \ge \frac{\frac{1}{2}\sqrt{\frac{n}{\lambda}}}{\eta_i^{(2)}} \ge \frac{(\ln 2)LN\lambda_c^{\frac{\alpha-1}{2}} n^{\frac{1}{2}}}{2B\lambda^{\frac{\alpha}{2}}}, \tag{36}$$

which is equivalent to the first part in Eq. (34) by substituting $\xi_1^{(u)}$. When $\lambda \ge \lambda_c$, we have $N \ge N(\frac{\lambda_c}{\lambda})^{\frac{\alpha}{2}}$, $H_i \ge N$, and

$$\eta_i^{(2)} \le \frac{B}{(\ln 2)LN}. \tag{37}$$

The minimum energy is hence bounded by

$$\widetilde{\Psi}^{(u)} \ge \frac{\frac{1}{2}\sqrt{\frac{n}{\lambda}}}{\eta_i^{(2)}} \ge \frac{(\ln 2)LN n^{\frac{1}{2}}}{2B\lambda^{\frac{1}{2}}}, \tag{38}$$

which is equivalent to the second part in Eq. (34) after substituting $\xi_2^{(u)}$. ■

B. Upper Bound

We obtain the upper bound on $\widetilde{\Psi}^{(u)}$ by designing a routing scheme similar to the one used for multicast. As there is only one destination in bordercast, we do not need to transport the packet through all the paths in the highway system. Instead, one horizontal path and one vertical path are sufficient for the delivery. We present this routing scheme and the upper bound on $\widetilde{\Psi}^{(u)}$ in the theorem below.

Theorem 6: In a large wireless network with random node locations, the minimum energy required to transport a packet from the network center to the network boundary is upper bounded by

$$\widetilde{\Psi}^{(u)} \le \begin{cases} \frac{k_1^{(u)}\xi_1^{(u)} n^{\frac{1}{2}}}{\lambda^{\frac{\alpha}{2}}} & \text{if } \lambda \le -8\ln(1-p(\gamma)) \\ \frac{k_2^{(u)}\xi_2^{(u)} n^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}} & \text{if } \lambda \ge -8\ln(1-p(\gamma)), \end{cases} \tag{39}$$

where $k_1^{(u)} = \frac{(2\sqrt{2})^{\alpha+1}(N+I)(-\ln(1-p(\gamma)))^{\frac{\alpha-1}{2}}}{\gamma N\lambda_c^{\frac{\alpha-1}{2}}}$ and $k_2^{(u)} = \frac{8(N+I)}{\gamma N}$ are constants, $\gamma \in (0,1)$ is an arbitrary fractional number, $p(\gamma)$ is an arbitrary probability that satisfies Eq. (30), with probability one as $n \to \infty$.

Proof: Our bordercast routing scheme utilizes the percolation highway system and works as follows. First, we choose γ to be an arbitrary fractional number (e.g., $\gamma = 0.2$). Note

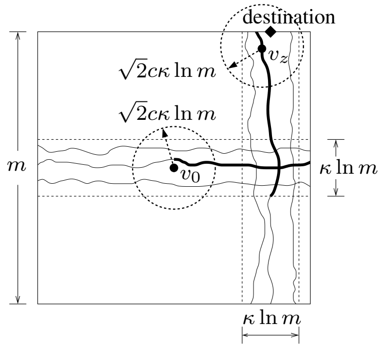


Fig. 5. An example of the bordercast routing scheme. Among the group of paths within the horizontal strip area, we choose the shortest one to transport the packet. Similarly, in the vertical strip area, we also choose the shortest path. The two selected paths are highlighted. The packet is transmitted by v_0 into the highway system, then routed to v_z, and finally forwarded to the destination. The configuration $H_0 = (N+I)(\sqrt{2c\kappa}\ln m)^\alpha$ guarantees that v_0 injects the packet correctly into the horizontal path and $H_z = (N+I)(\sqrt{2c\kappa}\ln m)^\alpha$ ensures that the destination receives the packet correctly, since v_0 and the destination node are within distance $\sqrt{2c\kappa}\ln m$ from the horizontal path and the vertical path respectively.

that this γ is used for highway construction only. It does not denote the percentage of nodes to receive the packet. Second, we construct the highway system exactly as the one used for multicast information delivery by following the same steps and using the same configurations. Third, in the highway system we choose one horizontal path and one vertical path that are close to the source node and the destination node to transport the packet, by requiring the cells along the selected paths forward the packet while all the other cells not forward. Finally, the last relay node v_z delivers the packet to the destination with configuration $H_z = (N+I)(\sqrt{2c\kappa}\ln m)^\alpha$. We illustrate an example of this routing scheme in Fig. 5.

The total energy consumption in the network for the packet delivery consists of three parts. In the first hop, v_0 consumes $\tilde{e}_0 = \frac{(\ln 2)L(N+I)(\sqrt{2c\kappa}\ln m)^\alpha}{B}$. In the last hop, v_z consumes $\tilde{e}_z = \frac{(\ln 2)L(N+I)(\sqrt{2c\kappa}\ln m)^\alpha}{B}$. In the middle, energy is consumed on the horizontal and vertical paths. We know from Fig. 3 that there are $2\kappa m \ln m$ cells and at least $\delta \ln m$ disjoint paths in a strip area. Therefore, the shortest path in a strip area traverses at most $\frac{2\kappa m \ln m}{\delta \ln m} = \frac{2m}{\gamma} = \frac{2\sqrt{n}}{\gamma c\sqrt{2\lambda}}$ number of cells. Since the packet is transported through half length in each of the horizontal and vertical paths and each cell along the paths consumes $\tilde{e}_i = \frac{(\ln 2)L(N+I)(2\sqrt{2}c)^\alpha}{B}$, the energy consumed in the highway system is bounded by

$$\sum_i \tilde{e}_i \le \frac{\sqrt{2}(2\sqrt{2})^\alpha(\ln 2)L(N+I)c^{\alpha-1}n^{\frac{1}{2}}}{\gamma B\lambda^{\frac{1}{2}}}. \quad (40)$$

Note that $\tilde{e}_0 = \tilde{e}_z = o(\sum_i \tilde{e}_i)$ as $n \to \infty$, so we can ignore $\tilde{e}_0$ and $\tilde{e}_z$. Plugging Eq. (33) into Eq. (40) and noting $\widetilde{\Psi}^{(u)} \le \sum_i \tilde{e}_i$, we obtain the upper bound on $\widetilde{\Psi}^{(u)}$ as Eq. (39). ■

Similar to the multicast scenario, we observe that the boundary density demarcating the scaling regimes differs from Eq. (34) to Eq. (39), which however does not change the tightness of our bounds as discussed before.

VII. Conclusion

In this paper, we have characterized the minimum energy requirement in large wireless networks to fulfill various information delivery goals that include broadcast, multicast and bordercast. In each communication scenario, we have derived both the lower and upper bounds on the minimum energy requirement $\widetilde{\Psi}$ as functions of the network size n and the node density λ. Our results show that the minimum energy to deliver a piece of information is in general an increasing function of the network size and a decreasing function of the node density, by proving the quantified relations among $\widetilde{\Psi}$, n and λ. As energy efficient communication is critical in energy-constrained environment, our work provides the fundamental understanding toward the effective energy management in large-scale multihop wireless networks to extend the lifetime and usability of these networks.

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