The Limit of Information Propagation Speed in Large-Scale Multihop Wireless Networks

Yi Xu, *Member, IEEE*, and Wenye Wang, *Member, IEEE, Member, ACM*

***Abstract*—This paper investigates the speed limit of information propagation in large-scale multihop wireless networks, which provides fundamental understanding of the fastest information transportation and delivery that a wireless network is able to accommodate. We show that there exists a unified speed upper bound for broadcast and unicast communications in large-scale wireless networks. When network connectivity is considered, this speed bound is a function of node density. If the network noise is constant, the bound is a constant when node density exceeds a threshold; if the network noise is an increasing function of node density, the bound decreases to zero when node density approaches infinity. As achieving the speed bound places strict requirements on node locations, we also quantify the gap between the actual achieved speed and the desired bound in random networks in which the relay nodes are not located as desired. We find that the gap converges to zero exponentially as node density increases to infinity.**

***Index Terms*—Information propagation, multihop communication, network connectivity, packet delay, wireless network.**

I. INTRODUCTION

THE wireless networks provide alternative networking services in places where fixed wireline networks are unnecessary or impossible to be deployed. However, the performance of wireless networks is not optimistic because the wireless medium is subject to various communication constraints, such as limited spectrum bandwidth, high environmental noise, intense wireless interference, dynamic channel condition, and fast signal attenuation. As such, understanding and improving the *achievable* network performance have been under intensive study in the wireless circumstances.

Initiated by the seminal work of Gupta and Kumar [1], researchers have investigated the capacity of wireless networks thoroughly. It is found that the throughput per node decreases on the order of $\Theta(1/\sqrt{n})$ as the node population n increases [1]. This finding is pessimistic since it implies the fact that none of the nodes can communicate in the end if the network becomes overcrowded. In the succeeding efforts to improve capacity, it is discovered that higher throughput is indeed obtainable if extra network conditions are considered, for example node mobility [2], infrastructure support [3], transmission cooperation [4], and ultrawide bandwidth [5]. Besides the theoretical bounds, algorithm design [6] has also been proposed to maximize the network capacity.

Different from the previous works, we study in this paper *packet delay* instead of throughput of wireless networks. Packet delay is a performance metric equally important as throughput, especially in the quality-of-service sensitive real-time communications, in which the delay perceived by each packet is more pertinent to the communication quality than the throughput of the entire network. Specifically, we are interested in determining the lower bound on the packet delay in wireless networks. The delay bound provides the understanding of how fast a packet can be transported and paves the way for the further investigation on the feasibility of supporting delay-sensitive traffic in wireless networks.

The delay perceived by a packet is the combined result of various factors including path length, link bandwidth, traffic load, and channel access overhead. Fortunately, these factors can be decoupled as they represent different delay components. For instance, the path length determines the number of hops traversed by the packet; the link bandwidth determines the transmission time on each hop; and the traffic load comes into effect in the form of packet queuing, processing, and wireless medium access delay at each intermediate node. In order to obtain generic results on packet delay, we do not assume any particular traffic pattern in this paper. Instead, we study a lightly loaded network in which the packet queuing, processing, and medium access delays are negligible as compared to the transmission delay. Decoupling the bandwidth-incurred delay and the load-incurred delay allows us to treat them separately and to reach conclusions that are not limited to specific traffic distributions. The additive property of packet delay guarantees that the lower bound discovered in the lightly loaded networks also applies to the arbitrarily loaded networks, though the tightness of the bound needs further investigation in the latter scenario.

To facilitate our study, we define the *information propagation speed* as an equivalent metric of the packet delay. As packet transportation can be viewed as moving a packet from its source node to its destination node over a physical distance, we use the speed metric to quantify the movement progression, namely, the distance traveled through by the packet toward its destination in a given amount of time. Under this definition, we translate the original problem of finding the lower bound of packet delay into the equivalent problem of determining the upper bound of packet propagation speed. The fundamental tradeoff is the selection of few links with long distance per link versus the selection of many links with short distance per link. Obviously, short link distance improves the capacity of the link, and therefore transmission finishes fast on the link, but a packet needs to go through

Manuscript received September 24, 2008; revised October 24, 2009; accepted June 21, 2010; approved by IEEE/ACM TRANSACTIONS ON NETWORKING Editor J. Walrand. Date of publication August 12, 2010; date of current version February 18, 2011. This work was supported by the National Science Foundation (NSF) CAREER Award under Grant CNS-0546289 and the Defense Threat Reduction Agency (DTRA) Project HDTRA 1-08-01-0024.

The authors are with the Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC 27606 USA (e-mail: yxu2@ncsu.edu; wwang@ncsu.edu).

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Digital Object Identifier 10.1109/TNET.2010.2057444

many hops before reaching its destination. Zheng shows in [7] that there exists an upper bound W on the information propagation speed that is attainable under three conditions: 1) every relay node uses an optimal transmission radius R; 2) the transportation distance between the source node and the destination node is a multiple of R; and 3) the relay nodes are aligned with equal separation distance R. We study in this paper the generalized speed limit problem in which the above three conditions may not hold and, in addition, the network connectivity and the node location randomness are taken into account.

To be specific, we are interested in the following three questions. First, if the packet transportation distance between the source and the destination is known not to be a multiple of R, what is the upper bound on the information propagation speed? Obviously, it should be tighter than W since W is unreachable in this case. Second, does network connectivity place any constraint on the speed upper bound and, if yes, how? As achieving the maximum speed W requires using a specified transmission radius R, it is possible that the network is not fully connected by using R. Should this case occur, a packet may not be able to reach all the intended recipients and the speed upper bound W may not be feasible. Third, even if the optimal transmission radius R is used, it may not be possible to find the relay nodes at the desired locations due to the randomness in node distribution. When the relay nodes are not located as desired, information propagates at a lower speed and a gap exists between the actually achieved speed and the upper bound W. How can this gap be quantified? We attempt to answer these questions, which provide the foundation for optimal network planning and protocol design to expedite packet delivery in multihop wireless networks.

As the first contribution, we show that the optimal transmission radius depends on the end-to-end transportation distance. In order to transport a packet at the fastest speed, the relay nodes must be equally spaced and use the same transmission radius. The value of the transmission radius is a divisor of the straight-line distance between the source and the destination nodes. Interestingly, the optimal transmission radius converges to a constant in large-scale wireless networks.

As the second contribution, we determine the speed upper bound under the constraint of network connectivity. We introduce a probabilistic measurement on network connectivity and examine the feasible speed bound in two noise models: the constant-interference model and the increasing-interference model. We show that in the constant-interference model the speed bound is a constant when node density exceeds a threshold, and in the increasing-interference model the speed bound decreases to zero when node density increases to infinity.

As the third contribution, we quantify the gap between the actual achieved speed and the desired upper bound. The gap exists due to the randomness of node locations. We prove that a packet propagates omnidirectionally in large-scale wireless networks and the gap reduces as node density increases. We also show that in both noise models, there exists a threshold node density below which the gap is bounded by constants and above which the gap converges to zero exponentially.

The rest of this paper is structured as follows. We discuss the related work in Section II and formulate the problem of information propagation speed in Section III. The optimal transmission radius and the speed upper bound in large wireless networks are obtained in Section IV. The speed bound under the constraint of network connectivity is determined in Section V. The quantification of speed gap is provided in Section VI. Finally, Section VII concludes this paper.

II. Related Work

A. Throughput Capacity

Since the work by Gupta and Kumar [1], many efforts have been made to understand the fundamental performance limits of wireless networks, most of which have focused on the network throughput. Gupta and Kumar demonstrated that the throughput per node is $\Theta(1/\sqrt{n})$ that decreases to zero as the number of nodes n goes to infinity. In the follow-up research, it was found that higher throughput can be achieved under various conditions and by using different techniques.

Grossglauser and Tse [2] discovered that mobility increases throughput. Node mobility increases the chance of transporting a packet using a short path, which reduces the number of relay transmissions and alleviates the intensity of interference. Garetto *et al.* [8] further proved that the asymptotic capacity of ad hoc networks varies from $\Theta(1/\sqrt{n})$ to $\Theta(1)$ under anisotropic mobility patterns.

Liu *et al.* [3] studied the wireless networks with infrastructure support. They found that throughput increases linearly with the number of base stations if there are sufficient base stations in the network. Kozat and Tassiulas [9] showed similar improvement that the throughput per node is $\Theta(1/\log n)$ if the number of access points is large enough.

Gastpar and Vetterli [10] investigated the network coding techniques and found that throughput can be improved by cooperation in transmissions. It was shown that the throughput per node pair scales as $\log n$ asymptotically when the nodes collaborate in transmissions. Ozgur *et al.* [4] extended the network scenario to multiple source–destination pairs. They showed that an almost linearly scaling capacity $\Theta(n^{1-\epsilon})$ can be achieved by intelligent node cooperations.

B. Delay Bound

All the works discussed above enhance the network utilization by increasing the total volume of transported packets within a given time period. It was meanwhile discovered that by allowing some packet delay, the network throughput can be further improved [11][12]. However, because packet delay cannot be arbitrarily relaxed, for example the real-time communications (e.g., voice conversations) and the time-sensitive messages (e.g., urgent event reports in sensor networks) require timely delivery, mechanisms are needed to guarantee satisfactory packet delay in the delay-constrained communication scenarios.

Krunz and Kim [13] analyzed the packet delay distribution and discard rate in wireless networks, from which they determined the wireless effective bandwidth to enforce the connection admission control. Wang *et al.* [14] proposed two admission control schemes to guarantee the packet delay based on the statistical delay analysis.

Liebeherr *et al.* [15] and Verloop [16] studied packet scheduling policies to minimize the flow-level delays. In bandwidth-sharing networks, multiple flows compete for the limited transmission bandwidth. The competition is coordinated by a scheduling policy. Optimally configuring the scheduling policy minimizes the delay of each flow.

Yang and Kravets [17] considered the wireless medium access delay. They proposed a distributed delay allocation scheme that adjusts the contention window sizes of competing nodes to ensure satisfactory delay performance. Similarly, Bader and Ekici [18] optimized the network throughput and delay by implementing interference-aware packet injection mechanisms.

Link scheduling was also studied to minimize the end-to-end packet delay by coordinating the wireless link transmission orders. Djukic and Valaee [19] designed heuristics to select the optimal transmission order that minimizes the packet delay at each node. Chafekar *et al.* [20] considered the concurrent packet transmissions in multiple flows and presented a scheduling algorithm that minimizes the delivery delays of all the packets.

Though the existing research results on packet delay have provided profound insights into the delay composition in wireless networks and valuable suggestions on the delay control methodologies, they all address the problem from a statistical perspective that minimizes the *average* delay of packet flows. In all these studies, traffic load is the essential factor contributing to packet delay. Quite differently, we consider in this paper the delay of a *single* packet in lightly loaded networks. As there are few other packets competing for the resources, the queuing and medium access delays are negligibly small. Instead, the end-to-end packet delay is dominated by the bandwidth of the links along the packet transportation path.

Till now, only Zheng [7] has studied this bandwidth-limited delay in wireless networks. In the pioneering paper [7], Zheng defined the concept of *information diffusion rate* that measures the time required for disseminating a packet to every node in the network. It was shown that a packet cannot be disseminated faster than a constant upper bound. The fastest rate is achieved only if the packet travels through a straight-line path consisting of R-distance equally spaced relay nodes, and the farthest node in the network is a multiple of R-distance away from the source, where R is a constant.

III. Problem Formulation

We study the generalized packet propagation speed problem in this paper. Particularly, we attempt to discover the speed bound for transporting a packet between arbitrarily located source–destination pairs and take into consideration the network connectivity requirement as well as the node location randomness. As mentioned earlier, we assume a lightly loaded network where the packet delay and hence the packet propagation speed are solely determined by the link bandwidth. The speed upper bound is, nonetheless, still valid for networks with any load distributions. Before starting the investigation, we first describe the network model used in this paper and define the metric *information propagation speed*.

A. Network Model

We study a square-shaped wireless network of n nodes in a very large area $\mathcal{B} = [-(l/2), (l/2)]^2$ ($l \to \infty$) with the following assumptions regarding the node locations and communications.

- The nodes are static and randomly distributed obeying a Poisson point process [21] with density λ.
- All the nodes share a B-Hz available frequency band.
- Any two nodes can communicate over the direct link between them. The link is characterized by an additive white Gaussian noise (AWGN) channel with path loss exponent $\alpha \geq 2$ [22] and its bandwidth is subject to the *Shannon limit* [23], [24]: $C = B\log_2(1+\mathrm{SNR})$, where SNR is the signal-to-noise ratio. Advanced coding techniques are used such that the link bandwidth approximates the Shannon limit [25].
- A uniform transmission power P is used by every node.
- The noise N present at each node is the sum of the ambient noise N_{A} and the interference noise $N_{\mathrm{I}} : N = N_{\mathrm{A}} + N_{\mathrm{I}}$. For tractable modeling and analysis, we assume that N_{A} is a constant and N_{I} is either a constant or a variable, depending on the density of concurrently transmitting nodes. We consider two cases in this paper: 1) the *constant-interference* noise model in which N_{I} is a constant when λ increases; and 2) the *increasing-interference* noise model in which N_{I} increases as λ increases. In the second model, since N_{I} dominates in the total noise when λ increases, we ignore N_{A} and assume $N = N_{\mathrm{I}}$. In both models, the total noise N is assumed to be homogeneous in the network.
- No directional antenna is used, and no large signal-blocking obstacle exists in the network.
- A packet has fixed length of L bits during transportation.

For the purpose of clearer presentation, we make the following comments and clarifications on the assumptions stated above.

- Throughout this paper, the area of the wireless network $|\mathcal{B}|$ is fixed, though it can be arbitrarily large. When we study the limit case of infinite node density ($\lambda \to \infty$), the density scales by increasing the node population n while keeping the area $|\mathcal{B}|$ constant.
- All the distances in this paper are the Euclidean distance.
- The path-loss attenuation exponent α is 2 in free space. In all other environments, α is bigger than 2 due to multipath fading effects.
- Given two nodes v_i and v_j separated by a distance $d_{v_iv_j}$, the signal strength of v_i as received by v_j is $Pd_{v_iv_j}^{-\alpha}$. Hence, the capacity of the direct link between v_i and v_j is $C_{v_iv_j} = B\log_2(1+(P/N)d_{v_iv_j}^{-\alpha})$.
- If v_i sends a packet to v_j at the full bit rate $C_{v_iv_j}$, a node v_k also receives the same packet if $d_{v_iv_k} \leq d_{v_iv_j}$, since $C_{v_iv_k} \geq C_{v_iv_j}$. On the other hand, if $d_{v_iv_k} > d_{v_iv_j}$, v_k cannot receive the packet correctly, as $C_{v_iv_k} < C_{v_iv_j}$. We define $r_{v_i} = \max_s\{d_{v_is} | C_{v_is} \geq C_{v_iv_j}\}$ as the *transmission radius* of v_i, and $\mathcal{A}_{v_i} = \{s | d_{v_is} \leq r_{v_i}, s \in \mathcal{B}\}$ as the *coverage area* of v_i in this transmission.

B. Information Propagation Speed

In multihop wireless networks, the transportation of a packet is via rebroadcasting. As illustrated in Fig. 1, when node v_0 initiates a packet transportation, it broadcasts the packet to all the neighbors inside its coverage area $\mathcal{A}_{v_0}$, and these neighbors continue to rebroadcast the packet to a farther distance until the packet is received by the destination node. Depending on the routing protocol used, not every intermediate node that hears the packet is required to rebroadcast. Besides, node scheduling is often implemented in large wireless networks to separate the simultaneous transmissions such that their packets do not collide. From the perspective of information theory, the interference from simultaneous transmissions only degrades the

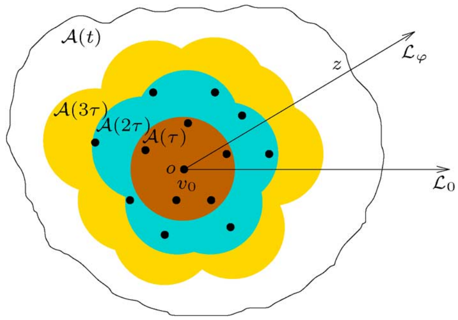

Fig. 1. Information propagation in multihop wireless networks. The packet broadcasting duration is τ for each node.

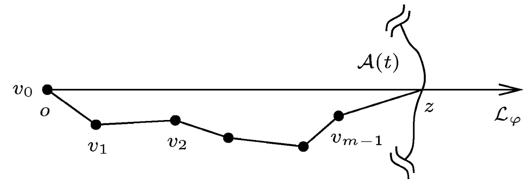

Fig. 2. Packet relay path in direction φ.

quality of wireless channels, but does not necessarily preclude communications. Therefore, theoretically speaking, communications are still possible without node scheduling. However, in order to be consistent with the *de facto* practice, we assume that a random η percentage of nodes are scheduled for transmission at any time. Thus, considering packet routing and node scheduling, only a subset of the nodes that have received the packet from v_0 rebroadcast to transport the packet.

Denote $\mathcal{V}(t)$ as the set of nodes that have received the packet by time t, and $\widetilde{\mathcal{V}}(t) \subset \mathcal{V}(t)$ as the subset that has forwarded the packet by time t. The total area that the packet has reached by time t is then expressed as $\mathcal{A}(t) = \cup_{v_i \in \widetilde{\mathcal{V}}(t)} \mathcal{A}_{v_i}$. In addition, denote $\mathcal{L}_\varphi$ as the line starting from v_0 in the direction $\varphi \in [0, 2\pi)$, and define $\mathcal{L}_\varphi(t) = \mathcal{L}_\varphi \cap \mathcal{A}(t)$. In Fig. 1, $\mathcal{L}_\varphi(t)$ is the line segment $\overline{oz}$. The *information propagation speed* in the direction φ is then defined to be

$$w_\varphi(t) = \frac{|\mathcal{L}_\varphi(t)|}{t} = \frac{\max_s \{d_{v_0 s} | s \in \mathcal{L}_\varphi(t)\}}{t}. \tag{1}$$

By definition, $w_\varphi(t)$ is the distance from v_0 to the farthest location reached by the packet in direction φ divided by the time elapsed since the packet departure from v_0. As we will show in the rest of this paper, maximizing $w_\varphi(t)$ requires all the packet relay nodes use a specified transmission radius. We name such a radius as *optimal transmission radius* in the sense of $w_\varphi(t)$ maximization.

In the rest of this paper, we will formally derive the upper bound on $w_\varphi(t)$, examine the feasibility of the upper bound under the network connectivity constraint, and determine the gap between $w_\varphi(t)$ and its upper bound.

IV. Upper Bound on Speed $w_\varphi(t)$

Zheng shows in [7] that there exists a constant upper bound on $w_\varphi(t)$ that is attainable when several conditions are satisfied simultaneously. One of the conditions requires that the source and the destination be separated by a distance that is multiple of the optimal transmission radius R, where R is a constant independent of the source–destination distance. We note that, in broadcast communications, R might be the best transmission strategy for fast packet dissemination since the number of destinations may be large and their locations may not be known. However, in unicast communications there is only one destination, the location of which is possibly known to the source and relay nodes. If the known source–destination distance is not a multiple of R, we show that there exists a tighter speed upper bound that is achieved at a different transmission radius. For completeness of presentation, we first reproduce the result of [7], as shown next in the case of broadcast communications.

A. Broadcast Communications

Suppose by time t, a packet originated from v_0 has reached the location z in direction φ, as shown in Fig. 2. Let $\mathcal{P} = \{v_0, v_1, \cdots, v_{m-1}\}$ denote the relay path from o to z and $\tau_{v_i} = L/B \log_2(1 + (P/N) r_{v_i}^{-\alpha})$ denote the transmission duration of node v_i. By definition

$$\begin{aligned} w_\varphi(t) &\le \frac{\sum_{i=0}^{m-1} r_{v_i}}{\sum_{i=0}^{m-1} \tau_{v_i}} \\ &\le \max_i \frac{r_{v_i}}{\tau_{v_i}} \\ &\le \frac{B}{L} \max_r r \log_2\left(1 + \frac{P}{N} r^{-\alpha}\right). \end{aligned} \tag{2}$$

The maximum of $r \log_2(1 + (P/N) r^{-\alpha})$ occurs when $r = R_b = (P/N y(\alpha))^{1/\alpha}$, where $y(\alpha)$ is the nonzero root of the equation

$$(1 + y) \log_2(1 + y) = \frac{\alpha}{\ln 2} y. \tag{3}$$

Thus

$$w_\varphi(t) \le W_b = \frac{B}{L} R_b \log_2\left(1 + \frac{P}{N} R_b^{-\alpha}\right). \tag{4}$$

If B, L, P, N, and α are constant, R_b and W_b are constant. This is the same result as Zheng has obtained in [7]. Inequality (4) demonstrates that $w_\varphi(t)$ is upper-bounded by the constant W_b, which is achieved when each step in (2) takes equality, i.e., the following conditions are satisfied: 1) every relay node uses the optimal transmission radius R_b; 2) relay nodes are lined up and equispaced by R_b; and 3) the distance from v_0 to the destination node v_d (or the farthest recipient node in broadcast communications) is a multiple of R_b.

B. Unicast Communications

In unicast communications, though we can always require every node transmit in the radius R_b, the distance $d_{v_0 v_d}$ between the source and the destination nodes may be known and not equal to a multiple of R_b. Should this case occur, we show next that $w_\varphi(t)$ is upper-bounded more tightly by another constant W_u that is achievable when a different transmission radius R_u is used, as specified in the following theorem.

Theorem 1: $w_\varphi(t)$ is maximized when the optimal transmission radius R_u is determined by

$$R_u = \begin{cases} d_{v_0 v_d}, & \text{if } d_{v_0 v_d} < d^* \\ d_{v_0 v_d}/\mathrm{G}\left(d_{v_0 v_d}/R_b\right), & \text{if } d_{v_0 v_d} > d^* \end{cases} \quad (5)$$

in which $d^* = (P/N(2^\alpha - 2))^{1/\alpha}$ and function $\mathrm{G}(\cdot)$ rounds $d_{v_0 v_d}/R_b$ to the nearest integer. In both cases, $w_\varphi(t) \le W_u = (B/L)R_u \log_2(1 + (P/N)R_u^{-\alpha})$.

Remark 1: Theorem 1 states the fact that there exists a threshold distance d^* such that: 1) if the source–destination distance is shorter than d^*, direct transmission from v_0 to v_d achieves the fastest speed; and 2) if the source–destination distance is longer than d^*, the fastest speed is achieved when the optimal transmission radius takes the value closest to R_b and dividing the source–destination distance.

In order to prove Theorem 1, we introduce a few notations and lemmas. Denote $\mathcal{P}^m(v_0, v_d) = \{v_0, v_1, \cdots, v_{m-1}, v_d\}$ ($m \ge 1$) as an m-hop straight-line relay path from v_0 to v_d, and $T(\mathcal{P}^m(v_0, v_d))$ as the packet transmission time along $\mathcal{P}^m(v_0, v_d)$. We have the following lemmas.

Lemma 2: Consider $\mathcal{P}^1(v_0, v_d)$ and $\mathcal{P}^2(v_0, v_d)$. By defining $\min t_{v_0 v_d} = \min\{T(\mathcal{P}^1(v_0, v_d)), T(\mathcal{P}^2(v_0, v_d))\}$, we have

$$\min t_{v_0 v_d} = \begin{cases} \dfrac{L}{B \log_2\left(1 + \frac{P}{N} d_{v_0 v_d}^{-\alpha}\right)}, & \text{if } d_{v_0 v_d} < d^* \\ \dfrac{2L}{B \log_2\left(1 + \frac{P}{N}\left(\frac{d_{v_0 v_d}}{2}\right)^{-\alpha}\right)}, & \text{if } d_{v_0 v_d} > d^*. \end{cases} \quad (6)$$

Remark 2: Lemma 2 states the fact that: 1) if $d_{v_0 v_d} < d^*$, 1-hop direct transmission is faster than any 2-hop relay transmission; 2) if $d_{v_0 v_d} > d^*$, choosing a relay node equidistant from the source and the destination results in the fastest transmission among all the 2-hop relay paths, and it is also faster than the 1-hop direct transmission; 3) if $d_{v_0 v_d} = d^*$, 1-hop direct transmission is as fast as the 2-hop relay transmission with the relay node placed exactly at the middle, and both are faster than any other 2-hop transmissions.

Proof: See the Appendix. ■

Next, we generalize the result of Lemma 2 to the cases of any-hop path length and introduce Lemma 3.

Lemma 3: Consider $\mathcal{P}^m(v_0, v_d)$ ($m \ge 1$) and define $\min t_{v_0 v_d} = \min_m\{T(\mathcal{P}^m(v_0, v_d))\}$

$$\min t_{v_0 v_d} = \begin{cases} T\left(\mathcal{P}^1(v_0, v_d)\right), & \text{if } d_{v_0 v_d} < d^* \\ \min_m \left\{T\left(\mathcal{P}_e^m(v_0, v_d)\right)\right\}, & \text{if } d_{v_0 v_d} > d^* \end{cases} \quad (7)$$

in which $\mathcal{P}_e^m(v_0, v_d)$ is an m-hop equidistant relay path connecting v_0 and v_d, i.e., $\mathcal{P}_e^m(v_0, v_d) = \{v_0, v_1, \cdots, v_{m-1}, v_d\}$ and $d_{v_0 v_1} = \cdots = d_{v_{m-1} v_d}$.

Remark 3: Lemma 3 states the fact that: 1) if $d_{v_0 v_d} < d^*$, 1-hop direct transmission is faster than any multihop transmissions; 2) if $d_{v_0 v_d} > d^*$, the fastest transmission must be achieved along a relay path in which the relay nodes are separated equally.

Proof: By Lemma 2, if $d_{v_0 v_d} < d^*$, $T(\mathcal{P}^1(v_0, v_d)) < T(\mathcal{P}^2(v_0, v_d))$. $\forall m \ge 2$, as $d_{v_{m-1} v_d} < d_{v_{m-2} v_d} < \cdots < d_{v_0 v_d} < d^*$, apply the result of Lemma 2 recursively

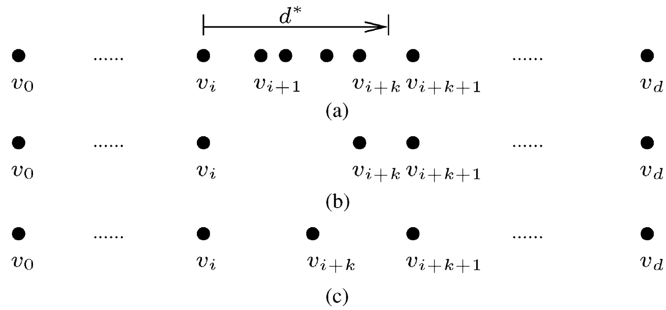

Fig. 3. Node removal and relocation process. (a) Finding the node set $\{v_j | d_{v_i v_j} \le d^*, j = i+1, \cdots, i+k\}$. (b) Removing the nodes $\{v_j, j = i+1, \cdots, i+k-1\}$. (c) Relocating the node v_{i+k} such that $d_{v_i v_{i+k}} = d_{v_{i+k} v_{i+k+1}}$ (if $k > 0$) or the node v_{i+1} such that $d_{v_i v_{i+1}} = d_{v_{i+1} v_{i+2}}$ (if $k = 0$).

$$\begin{aligned} T\left(\mathcal{P}^1(v_0, v_d)\right) &< \sum_{i=0}^{0} T\left(\mathcal{P}^1(v_i, v_{i+1})\right) + T\left(\mathcal{P}^1(v_1, v_d)\right) \\ &< \sum_{i=0}^{1} T\left(\mathcal{P}^1(v_i, v_{i+1})\right) + T\left(\mathcal{P}^1(v_2, v_d)\right) \\ &\vdots \\ &< \sum_{i=0}^{m-2} T\left(\mathcal{P}^1(v_i, v_{i+1})\right) + T\left(\mathcal{P}^1(v_{m-1}, v_d)\right) \\ &= T\left(\mathcal{P}^m(v_0, v_d)\right). \end{aligned}$$

Therefore, $\min t_{v_0 v_d} = T(\mathcal{P}^1(v_0, v_d))$ when $d_{v_0 v_d} < d^*$.

In order to prove $\min t_{v_0 v_d} = \min_m\{T(\mathcal{P}_e^m(v_0, v_d))\}$ when $d_{v_0 v_d} > d^*$, it is equivalent to show that for any path $\mathcal{P}^m(v_0, v_d)$, there is another path $\mathcal{P}_e^{m'}(v_0, v_d)$ that satisfies $T(\mathcal{P}_e^{m'}(v_0, v_d)) \le T(\mathcal{P}^m(v_0, v_d))$. We consider the following node removal and relocation process to prove the existence of $\mathcal{P}_e^{m'}(v_0, v_d)$. For each node $v_i \in \mathcal{P}^m(v_0, v_d)$, we make the two changes below in sequence.

1) **Node removal**. Find the set of nodes $\{v_j | d_{v_i v_j} \le d^*, j = i+1, \cdots, i+k\}$. If $k > 1$, remove the nodes $\{v_j, j = i+1, \cdots, i+k-1\}$ from $\mathcal{P}^m(v_0, v_d)$.
2) **Node relocation**. If v_i is the last relay node or $v_i = v_d$, skip this step. Otherwise, if $k = 0$ (k is the number of nodes found in Step 1), relocate v_{i+1} such that $d_{v_i v_{i+1}} = d_{v_{i+1} v_{i+2}}$; if $k > 0$, relocate v_{i+k} such that $d_{v_i v_{i+k}} = d_{v_{i+k} v_{i+k+1}}$.

This process initiates at v_0, proceeds node by node toward v_d and iterates until there is no more node removal and no more node relocation in the resulting relay path $\mathcal{P}_e^{m'}(v_0, v_d)$. Fig. 3 depicts an example of these two procedures.

First, we show that the resulting relay path has shorter transmission time than the original path, i.e., $T(\mathcal{P}_e^{m'}(v_0, v_d)) \le T(\mathcal{P}^m(v_0, v_d))$. In the node removal step, since $d_{v_i v_{i+k}} \le d^*$, by the first part of Lemma 3 (already proven), 1-hop direct transmission from v_i to v_{i+k} is faster than the k-hop transmission via $v_{i+1}, \cdots, v_{i+k-1}$. Hence, removing $\{v_j, j = i+1, \cdots, i+k-1\}$ results in faster transmission. In the node relocation step, because $d_{v_i v_{i+2}}$ (if $k = 0$) or $d_{v_i v_{i+k+1}}$ (if $k > 0$) is larger than d^*, by Lemma 2, relocation of v_{i+1} (if $k = 0$) or v_{i+k} (if $k > 0$) results in faster transmission. Therefore, $\mathcal{P}_e^{m'}(v_0, v_d)$ has shorter transmission time than $\mathcal{P}^m(v_0, v_d)$.

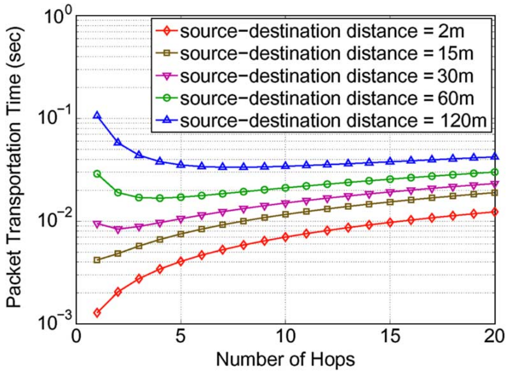

Fig. 4. End-to-end packet transportation time, $B = 100$ KHz, $L = 1024$ bits, $P/N = 10^3$, $\alpha = 2$. When $d_{v_0v_d} = 2$ m, $R_u = 2$ m; when $d_{v_0v_d} = 15, 30, 60$ m, $R_u = 15$ m; when $d_{v_0v_d} = 120$ m, $R_u = 17.14$ m.

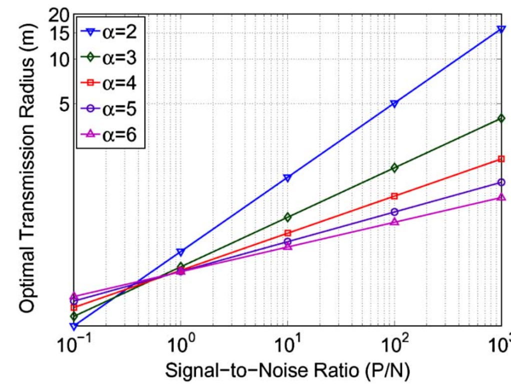

Fig. 5. Optimal transmission radius R.

Second, we prove that the resulting relay path $\mathcal{P}_e^{m'}(v_0, v_d)$ is an equidistant relay path, i.e., $d_{v_0v_1} = \cdots = d_{v_{m'-1}v_d}$. Because the node removal step takes relay nodes away and the number of remaining relay nodes must be nonnegative, it is obvious that the number of relay nodes converges to a value m' $(0 \le m' \le m)$. After that, there are no more removals, but relocations may continue. As the transmission time from v_0 to v_d decreases during the relocations (already proven) and it is nonnegative, it must converge to certain value, after which there are no more relocations. If $d_{v_0v_1}, d_{v_1v_2}, \cdots, d_{v_{m'-1}v_d}$ are not all equal, relocation will continue. Thus, they must be all equal by the end of the relocation process.

Finally, it is safe to substitute m for m'. After the substitution, we have $\min t_{v_0v_d} = \min_m\{T(\mathcal{P}_e^m(v_0, v_d))\}$. ■

We are now ready to prove Theorem 1 based on Lemma 3.

Proof: First, if $d_{v_0v_d} < d^*$, Lemma 3 states that 1-hop direct transmission is the fastest among all the possible relay transmissions. It is obvious to conclude that in order to maximize $w_\varphi(t)$, $R_u = d_{v_0v_d}$.

Second, if $d_{v_0v_d} > d^*$, Lemma 3 shows that $\min t_{v_0v_d} = \min_m\{T(\mathcal{P}_e^m(v_0, v_d))\}$. As

$$T\left(\mathcal{P}_e^m(v_0, v_d)\right) = \frac{mL}{B\log_2\left(1 + \frac{P}{N}\left(\frac{d_{v_0v_d}}{m}\right)^{-\alpha}\right)}$$

solving $\mathrm{d}T(\mathcal{P}_e^m(v_0, v_d))/\mathrm{d}m = 0$, we obtain the optimal number of relay hops $m = \mathrm{G}(d_{v_0v_d}/R_b)$, the optimal transmission radius $R_u = d_{v_0v_d}/\mathrm{G}(d_{v_0v_d}/R_b)$, and the upper bound on the information propagation speed

$$w_\varphi(t) \le W_u = \frac{B}{L}R_u\log_2\left(1 + \frac{P}{N}R_u^{-\alpha}\right). \tag{8}$$

Because R_b is the unique maximizer for $r\log_2(1+(P/N)r^{-\alpha})$, $W_u \le W_b$. When $d_{v_0v_d}$ is a multiple of R_b, $R_u = R_b$ and $W_u = W_b$. Otherwise, $R_u \ne R_b$ and $W_u < W_b$. ■

We plot the packet transportation time for different source–destination distances in Fig. 4 as a visual aid to understand Theorem 1. Unlike R_b and W_b, R_u and W_u are determined not only by B, L, P, N, and α, but also by the source–destination distance $d_{v_0v_d}$. The conditions for $w_\varphi(t) = W_u$ are: 1) every relay node uses the optimal transmission radius R_u; and 2) the relay nodes are aligned and separated from each other by distance R_u. Note $\lim_{d_{v_0v_d}\to\infty} R_u = R_b$, indicating in large-scale networks the generalized speed bound for arbitrarily located source and destination nodes converges to the same constant bound for broadcast communications. As we study large-scale networks in this paper, we will denote $R_b = R_u = R$ and $W_b = W_u = W = (B/L)R\log_2(1 + (P/N)R^{-\alpha})$ in the rest of this paper, where $R = (P/Ny(\alpha))^{1/\alpha}$.

TABLE I
COMPARISON BETWEEN $y(\alpha)$ AND $e^\alpha - 1$

α	$y(\alpha)$	$e^\alpha - 1$	$(\frac{1}{y(\alpha)})^{\frac{1}{\alpha}}$	$(\frac{1}{e^\alpha-1})^{\frac{1}{\alpha}}$
2	3.9216	6.3891	0.5050	0.3956
3	15.8010	19.0855	0.3985	0.3742
4	49.4353	53.5982	0.3771	0.3696
5	142.3249	147.4132	0.3710	0.3684
6	396.3833	402.4288	0.3690	0.3680

C. Solution of R

Till now, we have shown the existence of a unified optimal transmission radius R in large-scale networks. Next, we provide a solution for R. Equation (3) can be rewritten as

$$y = e^{\frac{\alpha y}{1+y}} - 1 \tag{9}$$

which allows us to compute $y(\alpha)$ in a recursive way

$$\begin{cases} y_0(\alpha) = e^\alpha - 1 \\ y_i(\alpha) = e^{\frac{\alpha y_{i-1}(\alpha)}{1+y_{i-1}(\alpha)}} - 1, \quad i = 1, 2, \cdots. \end{cases} \tag{10}$$

Given α, we compute $y_0(\alpha), y_1(\alpha), \cdots, y_i(\alpha)$ until $y_{i-1}(\alpha) = y_i(\alpha)$. Computation with various α values shows that this sequence always converges. The final value of $y_i(\alpha)$ after convergence is the nonzero root of (3). Interestingly, we find $e^\alpha - 1$ is a good approximation to $y(\alpha)$. Table I compares the values of $y(\alpha)$ and $e^\alpha - 1$ for sample α values. We observe that $y(\alpha)$ is well approximated by $e^\alpha - 1$, especially for large α. Hence, $y(\alpha)$ may be obtained numerically from (10) or approximated by (11)

$$y(\alpha) \approx e^\alpha - 1. \tag{11}$$

The optimal transmission radius R is then determined by $R = (P/Ny(\alpha))^{1/\alpha}$ after $y(\alpha)$ is solved. Fig. 5 plots R for sample α values and signal-to-noise ratios. It is observed that R increases as the signal-to-noise ratio increases or α decreases.

V. Upper Bound on Speed $w_\varphi(t)$ Under Network Connectivity Constraint

We have shown that there is a unified upper bound W on $w_\varphi(t)$ for broadcast and unicast communications in large-scale networks. In this section, we study the *feasibility* of this upper bound W. Achieving W requires using the optimal transmission radius R by all the relay nodes, but R may not guarantee the network being connected. If the destination node cannot be reached by the transmission radius R, the maximum speed W is infeasible. As such, we need to understand the maximum information propagation speed constrained by the network connectivity.

A. γ-Feasible Packet Delivery

We define the term γ-*feasible delivery* to provide a probabilistic measurement on the degree of network connectivity as well as the successfulness of packet delivery. The delivery of a packet is γ-feasible if the packet can reach all the intended recipients with a probability no less than γ $(0 \leq \gamma \leq 1)$. Subsequently, we define a transmission radius r to be γ-feasible (denoted as r_γ) if this r provides γ-feasible packet delivery. Note that r_γ implies that the entire network is connected with probability of at least γ by using the transmission radius r. Obviously, any transmission radius larger than r_γ is also γ-feasible. However, because R is the unique maximizer for $r\log_2(1+(P/N)r^{-\alpha})$, $w_\varphi(t)$ is maximized at the r_γ that is closest to R, when γ-feasible packet delivery is required. We define $R_\gamma = \arg\min_{r_\gamma}\{|r_\gamma - R|\}$ to be the γ-feasible optimal transmission radius and $W_\gamma = (B/L)R_\gamma \log_2(1+(P/N)R_\gamma^{-\alpha})$ to be the γ-feasible speed upper bound.

The network connectivity is determined by the node density and the node transmission radius. Given node density, the network is fully connected if the node transmission radius is sufficiently large. Obviously, we are able to construct a minimum spanning tree to connect every node in the network, and the longest edge in this minimum spanning tree serves as a sufficiently large transmission radius. Penrose shows in [26] that the longest edge M_n in the minimum spanning tree over n Poisson distributed random nodes in a unit square satisfies

$$\lim_{n\to\infty} \Pr\left[n\pi M_n^2 - \log(n) \leq \beta\right] = \exp(-e^{-\beta}).$$

Zheng [7] further proves that in an extended network with unit node density, if $\lim_{n\to\infty} c(n) = \infty$

$$\lim_{n\to\infty} \Pr\left[-c(n) \leq \pi M_n^2 - \log(n) \leq c(n)\right] = 1.$$

Scaling the extended network to the dense network, we have

$$\lim_{n\to\infty} \Pr\left[-c(n) \leq n\pi M_n^2 - \log(n) \leq c(n)\right] = 1.$$

Choosing $c(n) = \epsilon\log(n)$ $(\epsilon > 0)$ and replacing n with λ ($n=\lambda$ in a unit square), we have

$$\lim_{\lambda\to\infty} \Pr\left[\sqrt{\frac{(1-\epsilon)\log(\lambda)}{\lambda\pi}} \leq M_\lambda \leq \sqrt{\frac{(1+\epsilon)\log(\lambda)}{\lambda\pi}}\right] = 1. \tag{12}$$

Our network consists of the tiles of unit squares with node density λ over an area $\mathcal{B}$. Thus, (12) applies to our network model as well. Next, we discuss R_γ and W_γ with variable node densities in two different noise models.

B. Noise Models

We assume two noise models in this paper to determine the connectivity-constrained speed upper bound in two representative network environments. The noise models determine the scaling properties of R_γ and W_γ as node density increases. In the first one, the constant-interference model, the density of nodes scheduled for concurrent transmissions is kept constant as λ increases, i.e., $\eta\lambda$ is constant. In this case, $N = N_{\mathrm{A}} + N_{\mathrm{I}}$ is also a constant independent of node density. In the second one, the increasing-interference model, the percentage η of scheduled nodes is constant as λ increases such that $N = N_{\mathrm{I}}$ and N is a linear function of the node density, as proven in the following lemma.

Lemma 4: In a network with uniform transmission power P and randomly distributed nodes in Poisson point process with density λ, the interference at any location is $N_{\mathrm{I}}(\lambda) = \lambda N_{\mathrm{I}}(1)$, where $N_{\mathrm{I}}(1)$ is the interfence at this location when $\lambda = 1$, if the percentage η of simultaneously scheduled nodes is a constant independent of λ.

Remark 4: Lemma 4 states the fact that the interference noise increases linearly as the node density increases.

Proof: Since the nodes are distributed in a Poisson point process and η is independent of λ, the concurrently transmitting nodes are also in Poisson distribution with density $\eta\lambda$. By choosing two arbitrary locations $z_1, z_2 \in \mathcal{B}$ and defining N_{I,z_1,z_2} as the interference at z_1 caused by the transmissions at z_2, we have

$$\begin{aligned} N_{\mathrm{I},z_1,z_2}(\lambda) &= \lim_{\delta\to 0} \frac{N_{\mathrm{I},z_1,z_2(\delta)}(\lambda)}{\delta} \\ &= \lim_{\delta\to 0} \frac{\sum_{k=0}^{\infty} e^{-\eta\lambda\delta}\frac{(\eta\lambda\delta)^k}{k!} kPd_{z_1z_2}^{-\alpha}}{\delta} \\ &= \eta\lambda P d_{z_1z_2}^{-\alpha} \\ &= \lambda N_{\mathrm{I},z_1,z_2}(1) \end{aligned}$$

where δ is a small area around z_2 and $N_{\mathrm{I},z_1,z_2(\delta)}$ is the interference from δ. The total interference at z_1 from all the transmissions in the network is

$$N_{\mathrm{I},z_1}(\lambda) = \int_{z_2\in\mathcal{B}} N_{\mathrm{I},z_1,z_2}(\lambda)\mathrm{d}z_2 = \lambda N_{\mathrm{I},z_1}(1).$$

As z_1 is arbitrary, $N_{\mathrm{I}}(\lambda) = \lambda N_{\mathrm{I}}(1)$. ∎

In the increasing-interference model, since $N = N_{\mathrm{I}}$, we write $N(\lambda) = \lambda N(1)$. The optimal transmission radius R in this model has the form $R = (P/\lambda N(1) y(\alpha))^{1/\alpha}$.

C. γ-Feasible Upper Bound on Speed $w_\varphi(t)$

We discover that the γ-feasible optimal transmission radius R_γ and speed upper bound W_γ can be described by the following two theorems.

Theorem 5: In the constant-interference noise model, there exists a threshold node density λ_C such that

$$R_\gamma(\lambda) = \begin{cases} R\sqrt{\frac{\lambda_C}{\lambda}}, & \lambda < \lambda_C \\ R, & \lambda > \lambda_C \end{cases} \tag{13}$$

$$W_\gamma(\lambda)=\begin{cases}\frac{B}{L}\left(R\sqrt{\frac{\lambda_C}{\lambda}}\right)\\ \quad\times\log_2\left(1+\frac{P}{N}\left(R\sqrt{\frac{\lambda_C}{\lambda}}\right)^{-\alpha}\right), & \lambda<\lambda_C\\ \frac{B}{L}R\log_2\left(1+y(\alpha)\right), & \lambda>\lambda_C\end{cases} \tag{14}$$

where $R=(P/Ny(\alpha))^{1/\alpha}$.

Remark 5: Theorem 5 states the fact that, in the constant-interference model, there is a threshold node density above which $R_\gamma(\lambda)$ and $W_\gamma(\lambda)$ are constants.

Proof: By (12), $\exists\lambda_C^{(1)}$ s.t. $\forall\lambda\geq\lambda_C^{(1)}$

$$\Pr\left[M_\lambda\leq\sqrt{\frac{(1+\epsilon)\log(\lambda)}{\lambda\pi}}\right]\geq\gamma.$$

Let $\lambda_C^{(2)}$ denote the biggest root of the equation

$$\sqrt{\frac{(1+\epsilon)\log(\lambda)}{\lambda\pi}}=\left(\frac{P}{Ny(\alpha)}\right)^{\frac{1}{\alpha}}.$$

If this equation has no real root, define $\lambda_C^{(2)}=0$. Since $\lim_{\lambda\to\infty}\sqrt{(1+\epsilon)\log(\lambda)/\lambda\pi}=0$, $\forall\lambda\geq\lambda_C^{(2)}$

$$\sqrt{\frac{(1+\epsilon)\log(\lambda)}{\lambda\pi}}\leq\left(\frac{P}{Ny(\alpha)}\right)^{\frac{1}{\alpha}}.$$

By denoting $\lambda_C=\max\{\lambda_C^{(1)},\lambda_C^{(2)}\}$, we have $\forall\lambda>\lambda_C$

$$\Pr\left[M_\lambda\leq\left(\frac{P}{Ny(\alpha)}\right)^{\frac{1}{\alpha}}\right]\geq\gamma. \tag{15}$$

This result indicates that when $\lambda>\lambda_C$, $R=(P/Ny(\alpha))^{1/\alpha}$ is γ-feasible, i.e., $R_\gamma(\lambda)=R$. In this case, $W_\gamma(\lambda)=(B/L)R\log_2(1+(P/N)R^{-\alpha})=(B/L)R\log_2(1+y(\alpha))$.

When $\lambda<\lambda_C$, $R=(P/Ny(\alpha))^{1/\alpha}$ is not γ-feasible because Inequality (15) does not hold. Since the network connectivity is determined by the average node degree, we can increase the node transmission radius to keep the network connected when node density is low. The analysis above shows that R is the smallest γ-feasible transmission radius with node density λ_C. By solving $\lambda\pi R'^2=\lambda_C\pi R^2$, we obtain the smallest (also the closest to R) γ-feasible transmission radius with node density λ as $R'=R\sqrt{\lambda_C/\lambda}$. Hence, when $\lambda<\lambda_C$, $R_\gamma(\lambda)=R\sqrt{\lambda_C/\lambda}$ and $W_\gamma(\lambda)=(B/L)(R\sqrt{\lambda_C/\lambda})\log_2(1+(P/N)(R\sqrt{\lambda_C/\lambda})^{-\alpha})$. ■

Theorem 6: In the increasing-interference noise model, there exists a threshold node density λ_I such that

$$R_\gamma(\lambda)=\begin{cases}R_{\mathrm{I}}\sqrt{\frac{\lambda_{\mathrm{I}}}{\lambda}}, & \lambda<\lambda_{\mathrm{I}}\\ R, & \lambda>\lambda_{\mathrm{I}}\end{cases} \tag{16}$$

$$W_\gamma(\lambda)=\begin{cases}\frac{B}{L}\left(R_{\mathrm{I}}\sqrt{\frac{\lambda_I}{\lambda}}\right)\\ \quad\times\log_2\left(1+\frac{P}{\lambda N(1)}\left(R_{\mathrm{I}}\sqrt{\frac{\lambda_{\mathrm{I}}}{\lambda}}\right)^{-\alpha}\right), & \lambda<\lambda_{\mathrm{I}}\\ \frac{B}{L}R\log_2\left(1+y(\alpha)\right), & \lambda>\lambda_{\mathrm{I}}\end{cases} \tag{17}$$

where $R_{\mathrm{I}}=(P/\lambda_{\mathrm{I}}N(1)y(\alpha))^{1/\alpha}$ and $R=(P/\lambda N(1)y(\alpha))^{1/\alpha}$.

Remark 6: Theorem 6 states the fact that, in the increasing-interference noise model, $R_\gamma(\lambda)$ and $W_\gamma(\lambda)$ decrease to zero as node density increases to infinity.

Proof: Similar to Theorem 5, $\exists\lambda_{\mathrm{I}}^{(1)}$, s.t. $\forall\lambda\geq\lambda_{\mathrm{I}}^{(1)}$

$$\Pr\left[M_\lambda\leq\sqrt{\frac{(1+\epsilon)\log(\lambda)}{\lambda\pi}}\right]\geq\gamma.$$

Let $\lambda_{\mathrm{I}}^{(2)}$ denote the biggest root of the equation

$$\sqrt{\frac{(1+\epsilon)\log(\lambda)}{\lambda\pi}}=\left(\frac{P}{\lambda N(1)y(\alpha)}\right)^{\frac{1}{\alpha}}.$$

If this equation has no real root, define $\lambda_{\mathrm{I}}^{(2)}=0$. Since $\lim_{\lambda\to\infty}\sqrt{(1+\epsilon)\log(\lambda)/\lambda\pi}/(P/\lambda N(1)y(\alpha))^{1/\alpha}=0$, $\forall\lambda\geq\lambda_{\mathrm{I}}^{(2)}$

$$\sqrt{\frac{(1+\epsilon)\log(\lambda)}{\lambda\pi}}\leq\left(\frac{P}{\lambda N(1)y(\alpha)}\right)^{\frac{1}{\alpha}}.$$

By denoting $\lambda_{\mathrm{I}}=\max\{\lambda_{\mathrm{I}}^{(1)},\lambda_{\mathrm{I}}^{(2)}\}$, we have $\forall\lambda>\lambda_{\mathrm{I}}$

$$\Pr\left[M_\lambda\leq\left(\frac{P}{\lambda N(1)y(\alpha)}\right)^{\frac{1}{\alpha}}\right]\geq\gamma. \tag{18}$$

This shows that when $\lambda>\lambda_{\mathrm{I}}$, $R=(P/\lambda N(1)y(\alpha))^{1/\alpha}$ is γ-feasible. Hence, $R_\gamma(\lambda)=R$ and $W_\gamma(\lambda)=(B/L)R\log_2(1+(P/\lambda N(1))R^{-\alpha})=(B/L)R\log_2(1+y(\alpha))$.

When $\lambda<\lambda_{\mathrm{I}}$, similar to the discussion in Theorem 5, $R_{\mathrm{I}}=(P/\lambda_{\mathrm{I}}N(1)y(\alpha))^{1/\alpha}$ is the smallest γ-feasible transmission radius with node density λ_{I} and $R_{\mathrm{I}}\sqrt{\lambda_{\mathrm{I}}/\lambda}$ is the smallest (thus the closest to R) γ-feasible transmission radius with node density λ. Therefore, when $\lambda<\lambda_{\mathrm{I}}$, $R_\gamma(\lambda)=R_{\mathrm{I}}\sqrt{\lambda_{\mathrm{I}}/\lambda}$ and $W_\gamma(\lambda)=(B/L)(R_{\mathrm{I}}\sqrt{\lambda_{\mathrm{I}}/\lambda})\log_2(1+(P/\lambda N(1))(R_{\mathrm{I}}\sqrt{\lambda_{\mathrm{I}}/\lambda})^{-\alpha})$. ■

D. Comparison of the Noise Models

Interestingly, Theorems 5 and 6 show that the γ-feasible speed upper bound $W_\gamma(\lambda)$ behaves quite differently in the two noise models.

In the constant-interference model, when $\lambda<\lambda_C$, $W_\gamma(\lambda)$ is an increasing function of λ and reaches its maximum at $\lambda=\lambda_C$. When $\lambda>\lambda_C$, $W_\gamma(\lambda)$ is a constant, implying that given sufficiently large node density the information propagation speed is upper-bounded by a constant. An example of $W_\gamma(\lambda)$ in this model is shown in Fig. 6.

In the increasing-interference noise model, when $\lambda<\lambda_{\mathrm{I}}$, the maximizer of $W_\gamma(\lambda)$ is obtained as $\lambda=\lambda_{\mathrm{I}}(y(\alpha-2)/y(\alpha))^{1/((\alpha/2)-1)}$ by solving $\mathrm{d}W_\gamma(\lambda)/\mathrm{d}\lambda=0$, where $y(\alpha-2)$ is the nonzero root of the equation

$$(1+y)\log_2(1+y)=\frac{\alpha-2}{\ln 2}y. \tag{19}$$

If $2\leq\alpha\leq3$, $y(\alpha-2)$ does not exist. In this case $W_\gamma(\lambda)$ is a decreasing function of λ when $0<\lambda<\lambda_{\mathrm{I}}$. If $\alpha>3$, $y(\alpha-2)$ exists and $y(\alpha-2)<y(\alpha)$, indicating $\lambda_{\mathrm{I}}(y(\alpha-2)/y(\alpha))^{1/((\alpha/2)-1)}<\lambda_{\mathrm{I}}$. Therefore, $W_\gamma(\lambda)$ increases until

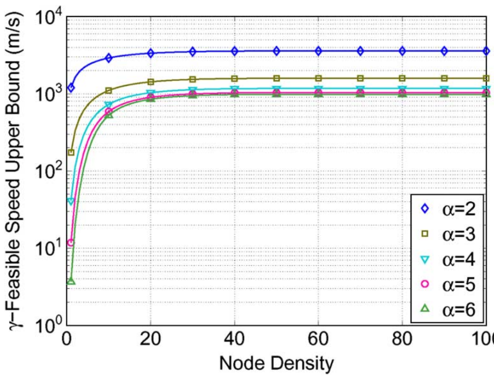

Fig. 6. $W_\gamma(\lambda)$ in the constant-interference model, $B = 100$ KHz, $L = 1024$ bits, $P/N = 10^3$, $\lambda_C = 50$.

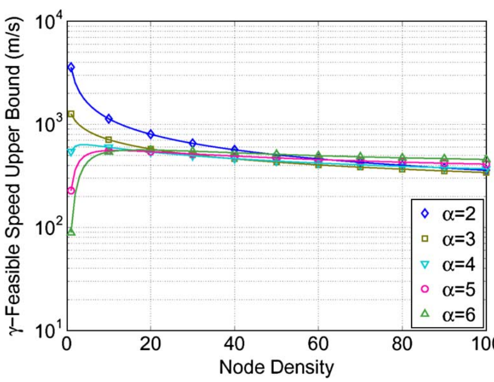

Fig. 7. $W_\gamma(\lambda)$ in the increasing-interference model, $B = 100$ KHz, $L = 1024$ bits, $P/N(1) = 10^3$, $\lambda_\mathrm{I} = 50$.

$\lambda = \lambda_\mathrm{I}(y(\alpha-2)/y(\alpha))^{1/((\alpha/2)-1)}$, and then decreases thereafter. When $\lambda > \lambda_\mathrm{I}$, $W_\gamma(\lambda)$ is always a decreasing function of λ and converges to zero as λ approaches infinity. Thus, in the increasing-interference noise model, information propagation becomes impossible when node density becomes extremely large. The strong interference prevents the transmission of any packet. An example of $W_\gamma(\lambda)$ in this model is shown in Fig. 7, in which it is observed that $W_\gamma(\lambda)$ takes similar values and converges in similar trend when λ is large, regardless of α.

VI. Gap Between $w_\varphi(t)$ and $W_\gamma(\lambda)$

We have shown that, given the parameter γ, there exists an optimal transmission radius $R_\gamma(\lambda)$ that may achieve the maximum information propagation speed $W_\gamma(\lambda)$ in a network with node density λ. However, as we have discussed earlier, actually achieving this maximum speed requires an additional condition that all the relay nodes are aligned and separated from each other by the distance $R_\gamma(\lambda)$. Since the nodes are distributed randomly, it may not be possible to find these perfectly located relay nodes when $\lambda < \infty$. There is always a gap between the actual achievable speed $w_\varphi(t)$ and the bound $W_\gamma(\lambda)$. We quantify this gap in this section.

Note that the speed gap exists due to the location offset of the relay nodes from their desired locations. In addition, when node scheduling is used, it may happen that the nodes closest to the desired locations are not scheduled for transmissions immediately after they receive the data packet, which increases the speed gap further. However, since we study the fastest achievable speed, we assume in this section a smart scheduling scheme

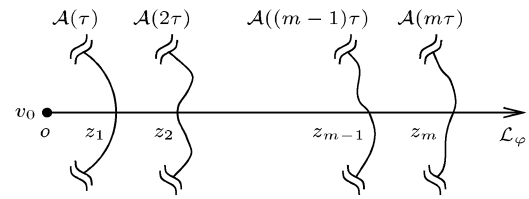

Fig. 8. Information propagation in multihops in direction φ.

that always schedules the nodes in the best locations to relay a data packet immediately after their packet reception. By using such a smart scheduling scheme, the fastest achievable speed is determined solely by the node density λ and independent of the percentage η of scheduled nodes as long as the best relay nodes are included into the schedule.

By definition, the actual information propagation speed is measured by $w_\varphi(t) = |\mathcal{L}_\varphi(t)|/t$. Due to the randomness of node locations, this speed may be faster or slower when the packet travels through different subareas in the network. To evaluate $w_\varphi(t)$ without introducing the subarea bias, we define the *long-term speed* in the direction φ to be

$$w_\varphi = \lim_{t\to\infty} w_\varphi(t) = \lim_{t\to\infty} \frac{|\mathcal{L}_\varphi(t)|}{t}. \tag{20}$$

Since every node uses the same optimal transmission radius $R_\gamma(\lambda)$, the 1-hop transmission time $\tau = L/B\log_2(1+(P/N)R_\gamma^{-\alpha}(\lambda))$ is the same for every node. We rewrite (20) as

$$w_\varphi = \lim_{m\to\infty} \frac{Z_m}{m\tau} = \lim_{m\to\infty} \frac{\sum_{i=1}^m \rho_i}{m\tau} = \frac{\overline{\rho}}{\tau} \tag{21}$$

where $Z_i = d_{oz_i}$, $\rho_i = Z_i - Z_{i-1}$ and $\overline{\rho} = \lim_{m\to\infty}(\sum_{i=1}^m \rho_i/m) = E[\rho_i]$, as shown in Fig. 8.

First, we show that the actual information propagation speed is omnidirectional in large-scale networks. In the long term, a packet is disseminated from the source to the same distance away in any direction. The frontier of propagation is in a circular shape, as specified in the following theorem.

Theorem 7: In a network with homogeneous node distributions, $\forall\varphi_1, \varphi_2 \in [0, 2\pi)$, $w_{\varphi_1} = w_{\varphi_2} = w$.

Remark 7: Theorem 7 states the fact that a packet reaches the same distance away in any direction after sufficiently long propagation time, though it can be faster or slower temporarily in one direction than another.

Proof: By definition, $w_\varphi = \overline{\rho}/\tau$. All we need to show is $\overline{\rho}_{\varphi_1} = \overline{\rho}_{\varphi_2}$. As the nodes are distributed homogeneously, the propagation distances in φ_1 and φ_2 after i hops, Z_{i,φ_1} and Z_{i,φ_2}, are two random variables with the same probability distribution. For the same reason, Z_{i-1,φ_1} and Z_{i-1,φ_2} also have the same probability distribution. Since $\rho_{i,\varphi_1} = Z_{i,\varphi_1} - Z_{i-1,\varphi_1}$ and $\rho_{i,\varphi_2} = Z_{i,\varphi_2} - Z_{i-1,\varphi_2}$, ρ_{i,φ_1} and ρ_{i,φ_2} must have the same probability distribution. Therefore, $\overline{\rho}_{\varphi_1} = E[\rho_{i,\varphi_1}] = E[\rho_{i,\varphi_2}] = \overline{\rho}_{\varphi_2}$. ■

Fig. 9 depicts an example of the speed comparison in different directions. As the packet propagates farther away, the speeds in all directions converge to the same value.

As we will show next that w is determined by the node density λ, we write $w = w(\lambda) = \overline{\rho}(\lambda)/\tau$. We define the gap between the actual speed $w(\lambda)$ and its upper bound $W_\gamma(\lambda)$ as

$$\varepsilon(\lambda) = \frac{W_\gamma(\lambda) - w(\lambda)}{W_\gamma(\lambda)} = \frac{R_\gamma(\lambda) - \overline{\rho}(\lambda)}{R_\gamma(\lambda)}. \tag{22}$$

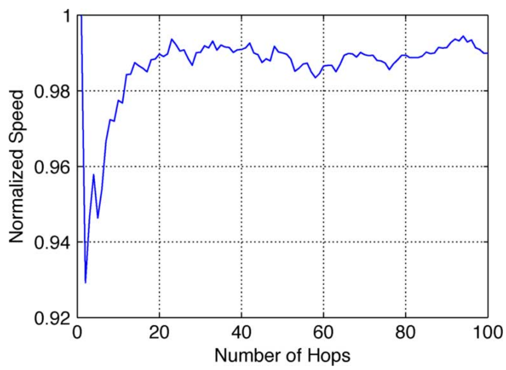

Fig. 9. Comparison of the packet propagation speeds in six randomly chosen directions, in which the normalized speed is defined as the ratio of the minimum speed to the maximum speed in the six directions, $\lambda = 30$.

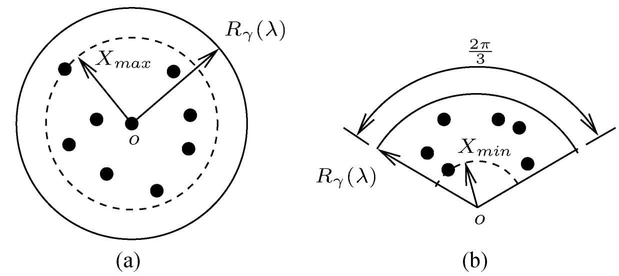

Fig. 10. Definitions of (a) $X_{\max}$ and (b) $X_{\min}$.

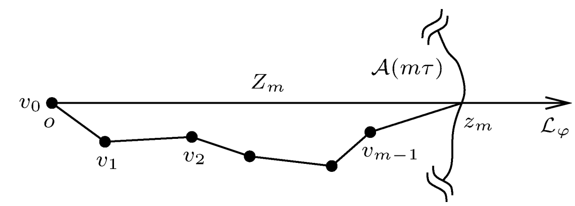

Fig. 11. Propagation distance Z_m in direction φ.

Theorem 8: In a network where the nodes are randomly distributed in a Poisson point process with density λ, $\forall \lambda_1 < \lambda_2$, $\varepsilon(\lambda_1) > \varepsilon(\lambda_2)$ almost surely.

Remark 8: Theorem 8 states the fact that $\varepsilon(\lambda)$ is a strictly decreasing function of λ with probability 1.

Proof: By definition, $\varepsilon(\lambda) = 1 - (\overline{\rho}(\lambda)/R_\gamma(\lambda))$. To prove $\forall \lambda_1 < \lambda_2$, $\varepsilon(\lambda_1) > \varepsilon(\lambda_2)$, it is equivalent to show $(\overline{\rho}(\lambda_1)/R_\gamma(\lambda_1)) < (\overline{\rho}(\lambda_2)/R_\gamma(\lambda_2))$.

First, we show $\overline{\rho}(\lambda_1) < \overline{\rho}(\lambda_2)$ almost surely. We start with a network of node density λ_2. Suppose a packet originated by node v_0 has propagated over a distance of $Z_m(\lambda_2)$ to reach location $z_m(\lambda_2)$ in an arbitrary direction φ after m hops and denote $\mathcal{P} = \{v_0, v_1, \cdots, v_{m-1}\}$ as the m-hop relay path traveled through by the packet to reach $z_m(\lambda_2)$. Now, reduce the node density to λ_1 by randomly removing each node (except v_0) from the network with probability $(\lambda_2 - \lambda_1)/\lambda_2$. From the properties of Poisson process, we know that the nodes in the resulting network are Poisson-distributed with density λ_1. Since removing any $v_i \in \{v_1, v_2, \cdots, v_{m-1}\}$ disrupts $\mathcal{P}$, the survival probability of $\mathcal{P}$ is

$$\Pr[\mathcal{P} \text{ survives}] = \left(\frac{\lambda_1}{\lambda_2}\right)^{m-1}.$$

When $m \to \infty$, $\Pr[\mathcal{P} \text{ survives}] \to 0$, implying $z_m(\lambda_2)$ is almost unreachable in the resulting network. Denoting $Z_m(\lambda_1)$ as the propagation distance of the packet in direction φ after m hops in the resulting network, we have $\lim_{m\to\infty} \Pr[Z_m(\lambda_1) < Z_m(\lambda_2)] = 1$, which gives

$$\Pr\left[\overline{\rho}(\lambda_1) < \overline{\rho}(\lambda_2)\right] = \lim_{m\to\infty} \Pr\left[\frac{Z_m(\lambda_1)}{m} < \frac{Z_m(\lambda_2)}{m}\right] = 1$$

i.e., $\overline{\rho}(\lambda_1) < \overline{\rho}(\lambda_2)$ almost surely.

Next, we show $R_\gamma(\lambda_1) \geq R_\gamma(\lambda_2)$. Theorems 5 and 6 indicate that $R_\gamma(\lambda)$ is a decreasing function (not always strictly though) of λ in both noise models. Hence, $\forall \lambda_1 < \lambda_2$, $R_\gamma(\lambda_1) \geq R_\gamma(\lambda_2)$.

Combining $\overline{\rho}(\lambda_1) < \overline{\rho}(\lambda_2)$ and $R_\gamma(\lambda_1) \geq R_\gamma(\lambda_2)$, we obtain $(\overline{\rho}(\lambda_1)/R_\gamma(\lambda_1)) < (\overline{\rho}(\lambda_2)/R_\gamma(\lambda_2))$ almost surely. ■

Theorem 8 points out that $\varepsilon(\lambda)$ reduces as λ increases. The next theorem provides a quantified measurement of $\varepsilon(\lambda)$.

Theorem 9: In a network where the nodes are randomly distributed in a Poisson point process with density λ and the optimal transmission radius $R_\gamma(\lambda)$ is used, defining $a = \lambda\pi R_\gamma^2(\lambda)$, $g_1(a) = \int_0^1 e^{a(x^2-1)}\mathrm{d}x$ and $g_2(a) = \int_0^1 e^{-(1/3)ax^2}\mathrm{d}x$

$$g_1(a) \leq \varepsilon(\lambda) \leq g_2(a). \tag{23}$$

Remark 9: Theorem 9 provides the bounds on the convergence rate of the speed gap as the node density increases.

Proof: First, we define two relevant random variables that will be used in this proof. As depicted in Fig. 10(a), we define $X_{\max}$ as the distance from a node to its farthest neighbor within the transmission radius $R_\gamma(\lambda)$. In Fig. 10(b), we draw a sector at an arbitrary location o with radius $R_\gamma(\lambda)$ and central angle $2\pi/3$, and define $X_{\min}$ as the distance from o to the nearest node found in this sector.

Next, we prove $\varepsilon(\lambda) \geq g_1(a)$. As shown in Fig. 11, letting $\mathcal{P} = \{v_0, v_1, \cdots, v_{m-1}\}$ denote the relay path traveled by a packet from v_0 to reach z_m in m hops

$$Z_m \leq \sum_{i=0}^{m-2} d_{v_i v_{i+1}} + d_{v_{m-1} z_m} \leq \sum_{i=0}^{m-1} X_{\max,i} + R_\gamma(\lambda)$$

where $X_{\max,i}$ is the $X_{\max}$ occurring at v_i. Then

$$\begin{aligned}\overline{\rho}(\lambda) &= \lim_{m\to\infty} \frac{Z_m}{m}\\ &\leq \lim_{m\to\infty} \frac{\sum_{i=0}^{m-1} X_{\max,i} + R_\gamma(\lambda)}{m}\\ &= \lim_{m\to\infty} \frac{\sum_{i=0}^{m-1} X_{\max,i}}{m}\\ &= E[X_{\max}]\end{aligned}$$

since $X_{\max,i}$ has i.i.d. probability distribution. We obtain $E[X_{\max}]$ as follows. According to the Poisson distribution, with probability e^{-a}, a node v_i has no neighbor (i.e.,

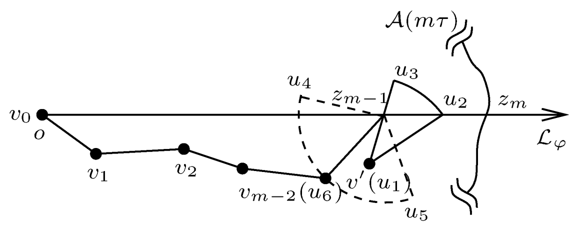

Fig. 12. mth-hop propagation distance $\rho_m = Z_m - Z_{m-1}$ in direction φ, $d_{z_{m-1}u_4} = d_{z_{m-1}u_5} = d_{z_{m-1}u_6} = d_{u_1u_2} = d_{u_1u_3} = R_\gamma(\lambda)$, $\angle u_4 z_{m-1} u_6 = \angle u_5 z_{m-1} u_6 = \pi/3$.

$X_{\max} = 0$). With probability $1 - e^{-a}$, v_i has at least one neighbor (i.e., $X_{\max} > 0$). Given $0 < x \leq R_\gamma(\lambda)$

$$\Pr[X_{\max} \leq x | X_{\max} > 0] = \frac{1}{1-e^{-a}} \sum_{k=1}^{\infty} e^{-a}\frac{a^k}{k!}\left(\frac{\pi x^2}{\pi R_\gamma^2(\lambda)}\right)^k = \frac{e^{-a}}{1-e^{-a}}\left(e^{\frac{ax^2}{R_\gamma^2(\lambda)}} - 1\right).$$

The conditional expectation is

$$E[X_{\max}|X_{\max} > 0] = \int_0^{R_\gamma(\lambda)} x \mathrm{d}\Pr[X_{\max} \leq x | X_{\max} > 0] = \int_0^{R_\gamma(\lambda)} \frac{e^{-a}}{1-e^{-a}}\left(\frac{2ax^2}{R_\gamma^2(\lambda)}\right) e^{\frac{ax^2}{R_\gamma^2(\lambda)}} \mathrm{d}x = \frac{R_\gamma(\lambda)}{1-e^{-a}}\left(1 - \int_0^1 e^{a(x^2-1)}\mathrm{d}x\right).$$

The unconditional expectation is

$$E[X_{\max}] = e^{-a}\cdot 0 + (1-e^{-a})\cdot E[X_{\max}|X_{\max} > 0] = R_\gamma(\lambda)\left(1 - \int_0^1 e^{a(x^2-1)}\mathrm{d}x\right).$$

Thus

$$\varepsilon(\lambda) \geq \frac{R_\gamma(\lambda) - E[X_{\max}]}{R_\gamma(\lambda)} = \int_0^1 e^{a(x^2-1)}\mathrm{d}x.$$

Finally, we prove $\varepsilon(\lambda) \leq g_2(a)$. As Fig. 12 illustrates, denote z_{m-1} as the farthest location that a packet has reached in direction φ after $m-1$ hops, and $\mathcal{P} = \{v_0, v_1, \cdots, v_{m-2}\}$ as the relay path traveled by the packet to reach z_{m-1}. Draw a sector at z_{m-1} with radius $R_\gamma(\lambda)$ and central angle $2\pi/3$, as illustrated by the dashed-line encompassed area in Fig. 12, where $\angle u_4 z_{m-1} u_6 = \angle u_5 z_{m-1} u_6 = \pi/3$. Note that for any node v' in this sector, $d_{v_{m-2}v'} \leq R_\gamma(\lambda)$, implying that v' must have received the packet by time $(m-1)\tau$ and forwarded the packet by time $m\tau$, i.e., $v' \in \widetilde{\mathcal{V}}(m\tau)$. Since z_m is the farthest location from o on $\mathcal{L}_\varphi$ covered by $\widetilde{\mathcal{V}}(m\tau)$, $d_{oz_m} \geq d_{ou_2}$, where u_2 is the farthest location reached by v' on $\mathcal{L}_\varphi$. Thus, $\rho_m = d_{z_{m-1}z_m} \geq d_{z_{m-1}u_2}$. By triangle inequality

$$\rho_m \geq d_{z_{m-1}u_2} \geq d_{u_1u_2} - d_{u_1z_{m-1}} = R_\gamma(\lambda) - d_{u_1z_{m-1}}.$$

As the previous inequality holds for all the v' in the sector

$$\rho_m \geq \max_{\{v'\}}\left\{R_\gamma(\lambda) - d_{u_1z_{m-1}}\right\} = R_\gamma(\lambda) - X_{\min}$$

where $X_{\min}$ is defined in Fig. 10(b). Replacing m with i

$$\overline{\rho}(\lambda) = \lim_{m\to\infty}\frac{\sum_{i=1}^m \rho_i}{m} \geq R_\gamma(\lambda) - \lim_{m\to\infty}\frac{\sum_{i=1}^m X_{\min,i}}{m} = R_\gamma(\lambda) - E[X_{\min}]$$

where $X_{\min,i}$ is the $X_{\min}$ occurring at z_{i-1} and $X_{\min,i}$ has i.i.d. probability distribution. Next, we compute $E[X_{\min}]$. We know from the Poisson distribution that with probability $e^{-(1/3)a}$, there is no node in the sector (i.e., $X_{\min} = R_\gamma(\lambda)$), and with probability $1 - e^{-(1/3)a}$, there is at least one node in the sector (i.e., $X_{\min} < R_\gamma(\lambda)$). Given $0 \leq x < R_\gamma(\lambda)$

$$\Pr[X_{\min} \leq x | X_{\min} < R_\gamma(\lambda)] = 1 - \Pr[X_{\min} > x | X_{\min} < R_\gamma(\lambda)] = 1 - \frac{e^{-\frac{1}{3}\lambda\pi x^2} - e^{-\frac{1}{3}a}}{1 - e^{-\frac{1}{3}a}}.$$

The conditional expectation is

$$E[X_{\min}|X_{\min} < R_\gamma(\lambda)] = \int_0^{R_\gamma(\lambda)} x\mathrm{d}\Pr[X_{\min} \leq x | X_{\min} < R_\gamma(\lambda)] = \int_0^{R_\gamma(\lambda)} \frac{\frac{2}{3}\lambda\pi x^2 e^{-\frac{1}{3}\lambda\pi x^2}}{1 - e^{-\frac{1}{3}a}} = \frac{R_\gamma(\lambda)}{1 - e^{-\frac{1}{3}a}}\left(\int_0^1 e^{-\frac{1}{3}ax^2}\mathrm{d}x - e^{-\frac{1}{3}a}\right).$$

The unconditional expectation is

$$E[X_{\min}] = e^{-\frac{1}{3}a}R_\gamma(\lambda) + \left(1 - e^{-\frac{1}{3}a}\right) \times E[X_{\min}|X_{\min} < R_\gamma(\lambda)] = R_\gamma(\lambda)\int_0^1 e^{-\frac{1}{3}ax^2}\mathrm{d}x.$$

Thus

$$\varepsilon(\lambda) \leq \frac{R_\gamma(\lambda) - (R_\gamma(\lambda) - E[X_{\min}])}{R_\gamma(\lambda)} = \int_0^1 e^{-\frac{1}{3}ax^2}\mathrm{d}x.$$

■

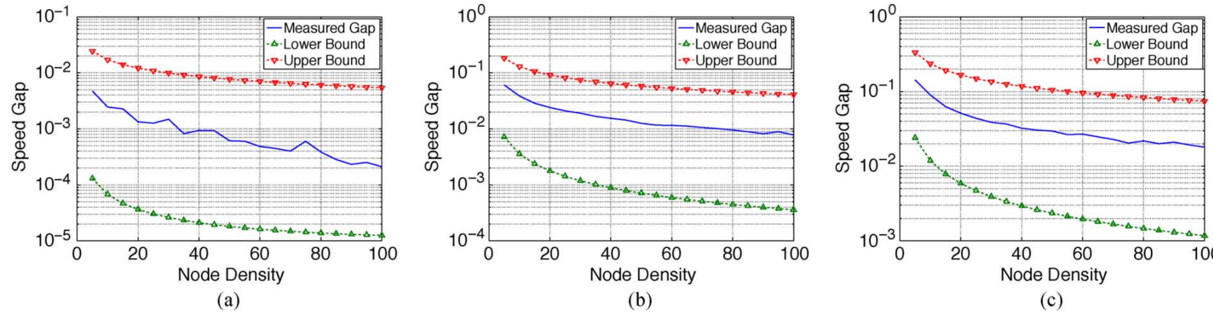

Fig. 13. Speed gap in the constant-interference noise model, $P/N = 10^3$. (a) $\alpha = 2$; (b) $\alpha = 4$; (c) $\alpha = 6$.

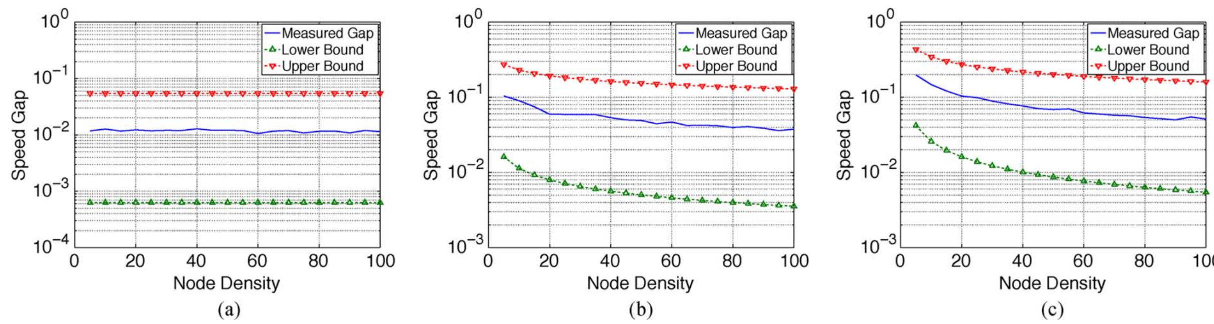

Fig. 14. Speed gap in the increasing-interference noise model, $P/N(1) = 10^3$. (a) $\alpha = 2$; (b) $\alpha = 4$; (c) $\alpha = 6$.

The speed gap $\varepsilon(\lambda)$ and its bounds are shown in Figs. 13 and 14 for different α values. Based on the result of Theorem 9, we are able to determine the asymptotic convergence rate of the speed gap as the node density approaches infinity. In order to present this asymptotic rate, we first introduce Lemma 10.

Lemma 10: Define $h_1(b) = \int_0^1 k^{b(x^2-1)}\mathrm{d}x$ and $h_2(b) = \int_0^1 k^{-(1/3)bx^2}\mathrm{d}x$, where $k > 1$, $b > 0$. $\forall 0 < c < 1$ and $\epsilon > 0$, $h_1(b) > c^b$ and $h_2(b) < c^{b^{1-\epsilon}}$ as $b \to \infty$.

Remark 10: Lemma 10 states the fact that the tails of $h_1(b)$ and $h_2(b)$ are bounded by exponential functions.

Proof: When $0 < c < 1$, $\exists 0 < x_0 < 1$ s.t. $k^{x_0^2-1} > c$. Because $\forall b > 0$

$$\int_0^1 \left(\frac{k^{x^2-1}}{c}\right)^b \mathrm{d}x \geq \int_{x_0}^1 \left(\frac{k^{x^2-1}}{c}\right)^b \mathrm{d}x \geq (1-x_0)\left(\frac{k^{x_0^2-1}}{c}\right)^b$$

as $b \to \infty$

$$\int_0^1 \left(\frac{k^{x^2-1}}{c}\right)^b \mathrm{d}x \geq (1-x_0)\left(\frac{k^{x_0^2-1}}{c}\right)^b \to \infty > 1.$$

Therefore, $h_1(b) = \int_0^1 k^{b(x^2-1)}\mathrm{d}x > c^b$ as $b \to \infty$.

When $0 < c < 1$, $\epsilon > 0$ and $x > 0$, as $b \to \infty$, $c^{b^{-\epsilon}} \to 1$ and $(k^{-(1/3)x^2}/c^{b^{-\epsilon}})^b \to 0$. Hence, $\int_0^1 (k^{-(1/3)x^2}/c^{b^{-\epsilon}})^b\mathrm{d}x \to 0 < 1$, which gives $h_2(b) = \int_0^1 k^{-(1/3)bx^2}\mathrm{d}x < c^{b^{1-\epsilon}}$ as $b \to \infty$. ■

The asymptotic convergence rate of $\varepsilon(\lambda)$ is then summarized in the next two theorems.

Theorem 11: In the constant-interference noise model, $\exists 0 < c_1, c_2 < 1$ and $\forall 0 < c < 1$, $\epsilon > 0$

$$\begin{cases} c_1 \leq \varepsilon(\lambda) \leq c_2, & \lambda < \lambda_C \\ c^\lambda < \varepsilon(\lambda) < c^{\lambda^{1-\epsilon}}, & \lambda > \lambda_C \text{ as } \lambda \to \infty. \end{cases}$$

Remark 11: Theorem 11 states the fact that, in the constant-interference noise model, the speed gap converges to zero exponentially with exponent $\lambda^{1-\epsilon}$, where ϵ is an arbitrarily small positive real number.

Proof: When $\lambda < \lambda_C$, $R_\gamma(\lambda) = R\sqrt{\lambda_C/\lambda}$ and $a = \lambda_C \pi R^2$. By Theorem 9, letting $c_1 = g_1(\lambda_C \pi R^2)$ and $c_2 = g_2(\lambda_C \pi R^2)$, we have $c_1 \leq \varepsilon(\lambda) \leq c_2$.

When $\lambda > \lambda_C$, $R_\gamma(\lambda) = R$ and $a = \lambda\pi R^2$. Choose $k = e^{\pi R^2}$ and $b = \lambda$. By Theorem 9 and Lemma 10, $\forall 0 < c < 1$ and $\epsilon > 0$, as $\lambda \to \infty$

$$\varepsilon(\lambda) \geq g_1(a) = h_1(\lambda) > c^\lambda$$

and

$$\varepsilon(\lambda) \leq g_2(a) = h_2(\lambda) < c^{\lambda^{1-\epsilon}}.$$

■

Theorem 12: In the increasing-interference noise model, $\exists 0 < c_1, c_2 < 1$ and $\forall 0 < c < 1$, $\epsilon > 0$

$$\begin{cases} c_1 \leq \varepsilon(\lambda) \leq c_2, & \lambda < \lambda_I \\ c^{\lambda^{1-\frac{2}{\alpha}}} < \varepsilon(\lambda) < c^{\lambda^{\left(1-\frac{2}{\alpha}\right)(1-\epsilon)}}, & \lambda > \lambda_I \text{ as } \lambda \to \infty. \end{cases}$$

Remark 12: Theorem 12 states that, in the increasing-interference noise model, the speed gap converges to zero exponentially with exponent $\lambda^{(1-(2/\alpha))(1-\epsilon)}$, where ϵ is an arbitrarily small positive real number.

Proof: When $\lambda < \lambda_I$, $R_\gamma(\lambda) = R_I\sqrt{\lambda_I/\lambda}$ and $a = \lambda_I \pi R_I^2$. By Theorem 9, letting $c_1 = g_1(\lambda_I \pi R_I^2)$ and $c_2 = g_2(\lambda_I \pi R_I^2)$, we have $c_1 \leq \varepsilon(\lambda) \leq c_2$.

When $\lambda > \lambda_I$, $R_\gamma(\lambda) = (P/\lambda N(1)y(\alpha))^{1/\alpha}$ and $a = \lambda^{1-(2/\alpha)}\pi(P/N(1)y(\alpha))^{2/\alpha}$. Choose $k = e^{\pi(P/N(1)y(\alpha))^{2/\alpha}}$

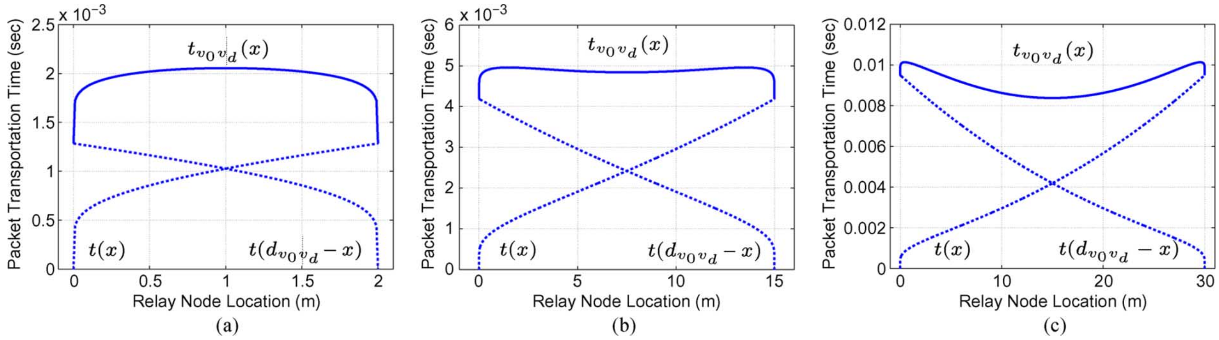

Fig. 15. Packet transportation time via 1-hop and 2-hop relay paths, $B = 100$ KHz, $L = 1024$ bits, $P/N = 10^3$, $\alpha = 2$. (a) $0 < d_{v_0v_d} = 2$ m $\leq 2x_2$. (b) $2x_2 < d_{v_0v_d} = 15$ m $< \infty$. (c) $2x_2 < d_{v_0v_d} = 30$ m $< \infty$.

and $b = \lambda^{1-(2/\alpha)}$. By Theorem 9 and Lemma 10, $\forall 0 < c < 1$ and $\epsilon > 0$, as $\lambda \to \infty$

$$\varepsilon(\lambda) \geq g_1(a) = h_1\left(\lambda^{1-\frac{2}{\alpha}}\right) > c^{\lambda^{1-\frac{2}{\alpha}}}$$

and

$$\varepsilon(\lambda) \leq g_2(a) = h_2\left(\lambda^{1-\frac{2}{\alpha}}\right) < c^{\lambda^{\left(1-\frac{2}{\alpha}\right)(1-\epsilon)}}.$$

■

Theorems 11 and 12 reveal that in both noise models there is a threshold node density, below which $\varepsilon(\lambda)$ is bounded by constants (the constants are determined by the choice of parameter γ) and above which $\varepsilon(\lambda)$ converges to zero exponentially in the rates of $c^{\lambda^{1-\epsilon}}$ and $c^{\lambda^{(1-(2/\alpha))(1-\epsilon)}}$, respectively.

VII. Conclusion

In this paper, we have studied the packet delay problem in lightly loaded large-scale multihop wireless networks in terms of the packet propagation speed. We find that there exists an upper bound, determined by the network parameters, on the information propagation speed. This upper bound is different for broadcast communications and unicast communications, but the two bounds converge in large-scale networks. As a necessary condition for achieving this upper bound, all the relay nodes must use an optimal transmission radius. We also reveal that, when network connectivity is considered, the feasible speed upper bound is a function of node density. If the noise in the network is constant, the speed bound is constant when node density exceeds a threshold. Otherwise, if the noise is an increasing function of node density, the speed bound decreases to zero as node density grows to infinity. Finally, we prove that a packet propagates omnidirectionally in large-scale random networks, and the gap between its actual speed and the upper bound decreases exponentially when node density increases to infinity. The work in this paper provides fundamental understanding of the achievable fastest information delivery in large-scale wireless networks, which is instrumental to the delay-minimization routing protocol design in wireless networks. The speed upper bound found in this paper also applies to the wireless networks with arbitrary traffic loads and routing protocols since heavy load and nonoptimal path selection incur extra packet transportation delay. The tightness of the speed upper bound, however, will need to be reconsidered in such cases.

Appendix
Proof of Lemma 2

Proof: Define $t(x) = L/B\log_2(1+(P/N)x^{-\alpha})$ and $t_{v_0v_d}(x) = t(x) + t(d_{v_0v_d} - x)$, in which $x \in [0, d_{v_0v_d}]$ is the distance between the source node and the relay node. When $x = 0$ or $x = d_{v_0v_d}$, it is a 1-hop transmission. Otherwise, it is a 2-hop transmission. In order to find the minimum of $t_{v_0v_d}(x)$, we determine the convexity of $t(x)$ and $t_{v_0v_d}(x)$ first.

By definition, function $t(x)$ has the following properties:

$$t'(x) = \frac{(\ln 2)\alpha L P x^{-\alpha-1}}{BN\left(1+\frac{P}{N}x^{-\alpha}\right)\ln^2\left(1+\frac{P}{N}x^{-\alpha}\right)} > 0$$

$$t''(x) = \frac{\alpha L P x^{-\alpha-2}}{(\ln 2)^2 BN\left(1+\frac{P}{N}x^{-\alpha}\right)^2\log_2^3\left(1+\frac{P}{N}x^{-\alpha}\right)} \cdot \left[2\alpha\frac{P}{N}x^{-\alpha} - \left(\alpha+1+\frac{P}{N}x^{-\alpha}\right)\ln\left(1+\frac{P}{N}x^{-\alpha}\right)\right].$$

Define $y = (P/N)x^{-\alpha}$ and $f(y) = 2\alpha y - (\alpha+1+y)\ln(1+y)$. We have $f'(y) = 2\alpha - [\ln(1+y) + \alpha/(1+y) + 1]$. It is not difficult to find the following properties of $f'(y)$.

1) $f'(0) = \alpha - 1 > 0$.
2) $f'(y)$ increases monotonically when $y \in [0, \alpha-1)$.
3) $f'(y)$ decreases monotonically when $y \in (\alpha-1, \infty)$.
4) $\lim_{y\to\infty} f'(y) = -\infty$.

These properties indicate $\exists y_1 > 0$ s.t.

$$\begin{cases} f'(y) > 0, & 0 \leq y < y_1 \\ f'(y) < 0, & y_1 < y < \infty. \end{cases}$$

Since $f(0) = 0$, $\exists y_2 > y_1$ s.t.

$$\begin{cases} f(y) > 0, & 0 < y < y_2 \\ f(y) < 0, & y_2 < y < \infty. \end{cases}$$

Define $x_2 = (P/Ny_2)^{1/\alpha}$ and $f(x) = 2\alpha(P/N)x^{-\alpha} - (\alpha + 1 + (P/N)x^{-\alpha})\ln(1+(P/N)x^{-\alpha})$. Then

$$\begin{cases} f(x) < 0, & 0 < x < x_2 \\ f(x) > 0, & x_2 < x < \infty. \end{cases}$$

Since $\forall x > 0$, $(\alpha L P x^{-\alpha-2}/(\ln 2)^2 BN(1+(P/N)x^{-\alpha})^2\log_2^3(1+(P/N)x^{-\alpha})) > 0$, we have

$$\begin{cases} t''(x) < 0 \text{ (i.e., } t(x) \text{ is strictly concave)}, & 0 < x < x_2 \\ t''(x) > 0 \text{ (i.e., } t(x) \text{ is strictly convex)}, & x_2 < x < \infty. \end{cases}$$

Next, we determine the convexity of $t_{v_0v_d}(x)$. Note that $t_{v_0v_d}(x)$ is symmetric with respect to $x = d_{v_0v_d}/2$. In addition, $t'_{v_0v_d}(d_{v_0v_d}/2) = 0$ and $t''_{v_0v_d}(0) = t''_{v_0v_d}(d_{v_0v_d}) < 0$ (because $t''(0) = -\infty$, $t''(d_{v_0v_d}) < \infty$, $t''_{v_0v_d}(0) = t''_{v_0v_d}(d_{v_0v_d}) = t''(0) + t''(d_{v_0v_d}) = -\infty$). The convexity as well as the minimum of $t_{v_0v_d}(x)$ is discussed in three cases.

1) $d_{v_0v_d} \in (0, x_2]$. $t(x)$ and $t(d_{v_0v_d} - x)$ are concave on $[0, d_{v_0v_d}]$, so $t_{v_0v_d}(x)$ is concave with no local minimum, as shown in Fig. 15(a).
2) $d_{v_0v_d} \in (x_2, 2x_2]$. $t(x)$ is concave on $[0, x_2]$ and convex on $[x_2, d_{v_0v_d}]$. $t(d_{v_0v_d} - x)$ is convex on $[0, d_{v_0v_d} - x_2]$ and concave on $[d_{v_0v_d} - x_2, d_{v_0v_d}]$. Therefore, $t_{v_0v_d}(x)$ must be concave on $[d_{v_0v_d} - x_2, x_2]$, while either concave or convex on $[0, d_{v_0v_d} - x_2] \cup [x_2, d_{v_0v_d}]$. However, $t''_{v_0v_d}(0) = t''_{v_0v_d}(d_{v_0v_d}) < 0$ indicates concavity. Thus, $t_{v_0v_d}(x)$ is concave on $[0, d_{v_0v_d}]$ with no local minimum, also as shown in Fig. 15(a).
3) $d_{v_0v_d} \in (2x_2, \infty)$. Similar to the discussion in Case 2, $t_{v_0v_d}(x)$ must be convex on $[x_2, d_{v_0v_d} - x_2]$, while either concave or convex on $[0, x_2] \cup [d_{v_0v_d} - x_2, d_{v_0v_d}]$. Again, $t''_{v_0v_d}(0) = t''_{v_0v_d}(d_{v_0v_d}) < 0$ indicates concavity. Therefore, $t_{v_0v_d}(x)$ has one local minimum at $x = d_{v_0v_d}/2$ and two local maxima in $[0, x_2]$ and $[d_{v_0v_d} - x_2, d_{v_0v_d}]$, as shown in Fig. 15(b) and (c).

Summarizing all the three cases, $t_{v_0v_d}(x)$ has at most one local minimum which occurs at $x = d_{v_0v_d}/2$. Hence

$$\min t_{v_0v_d} = \min\left\{t_{v_0v_d}(0), t_{v_0v_d}\left(\frac{d_{v_0v_d}}{2}\right), t_{v_0v_d}(d_{v_0v_d})\right\}$$
$$= \begin{cases} \frac{L}{B\log_2\left(1+\frac{P}{N}d_{v_0v_d}^{-\alpha}\right)}, & \text{if } d_{v_0v_d} < d^* \\ \frac{2L}{B\log_2\left(1+\frac{P}{N}\left(\frac{d_{v_0v_d}}{2}\right)^{-\alpha}\right)}, & \text{if } d_{v_0v_d} > d^*. \end{cases}$$

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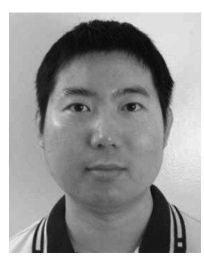

Yi Xu (M'04) received the B.E. degree in electrical engineering from the Huazhong University of Science and Technology, Wuhan, China, in 1998; the M.E. degree in computer engineering from the National University of Singapore, Singapore, in 2002; and the Ph.D. degree in computer engineering from the North Carolina State University, Raleigh, in 2010, respectively.

He worked as a Research Engineer with the Institute for Infocomm Research, Singapore, from 2001 to 2005. He is currently a Post-Doctoral Research Associate with the Department of Electrical and Computer Engineering, North Carolina State University. His research interests include performance modeling and analysis of large-scale wireless networks, network resilience to mobility and failure, and network security.

Wenye Wang (M'98) received the M.S.E.E. degree and Ph.D. degree in computer engineering from the Georgia Institute of Technology, Atlanta, in 1999 and 2002, respectively.

She is now an Associate Professor with the Department of Electrical and Computer Engineering, North Carolina State University, Raleigh. Her research interests include mobile and secure computing, modeling and performance analysis of single- and multihop wireless networks, network topology, and architecture design.

Dr. Wang has been a Member of the Association for Computing Machinery (ACM) since 1998, and a Member of the Eta Kappa Nu and Gamma Beta Phi honorary societies since 2001. She is a recipient of the NSF CAREER Award 2006. She is the co-recipient of the 2006 IEEE GLOBECOM Best Student Paper Award—Communication Networks and the 2004 IEEE Conference on Computer Communications and Networks (ICCCN) Best Student Paper Award.