The Latency of Gaining α-Reliability for Message Dissemination in Vehicle-to-Vehicle Networks

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***Abstract*—In many Vehicular Ad-hoc Network applications, such as hazard warning and traffic coordination, the message dissemination in unreliable and highly mobile network environment is a key challenge. In order to understand the relationship between dissemination latency and reliability, we analyze the latency of gaining α-*reliability* that a node correctly receives a message with probability larger than α $(0 < \alpha < 1)$. Under a 1-Dimensional (1-D) network scenario with unreliable channel and constrained vehicle mobility, we derive the minimum latency of gaining almost sure α-reliability, denoted as $t_{min}(\alpha)$. Besides dissemination reliability requirement α, $t_{min}(\alpha)$ also depends on node's original distance from the source, node mobility, channel reliability, and traffic flow. Numerical analysis discloses several interesting insights that 1) $t_{min}(\alpha)$ is dominated by the first attempt to send the message to a destination, 2) node mobility has little impact on $t_{min}(\alpha)$ in emergency information dissemination, and 3) transmission range and node density greatly affect dissemination latency and reliability.**

I. Introduction

Research on Vehicular Ad-hoc Networks (VANETs) aims to improve road safety, traffic efficiency, and driver convenience. Well-known examples are in the area of *safety* applications, such as the hazard warning and collision avoidance system, which enable drivers to response in advance and avoid accidents or traffic jams. In safety applications of VANETs, the continuous information dissemination is a key challenge. A small reduction in driver's available response time or a single vehicle missing the message may result in injury, property damage, and even death. Thus, emergency message dissemination requires fast and reliable information delivery. Message dissemination through VANETs is further complicated by the fact that vehicular networks use *unreliable wireless communication medium* and are highly mobile and frequently disconnected due to *high node mobility*.

Although many dissemination algorithms have been proposed in order to achieve *reliable*, *fast* and *low-cost* information distribution in VANETs (e.g., [1–5]), there exist few analytical studies so far about the fundamental limitations and requirements of information dissemination. Paper [6] derived information propagation speed in a 1-D network. Paper [7] analyzed the performance of message dissemination (such as average per-hop message forwarding distance) in VANETs with two priority levels of messages. Paper [8] developed upper and lower bounds for the time of information propagation between two nodes in a 1-D static network. To our best knowledge, [9] is the first attempt to analyze time-constrained, multi-hop dissemination reliability in VANETs, which derived a lower bound for the probability that a vehicle receives a safety message through multi-hop communication from a source at a distance d away within t seconds. However, results in paper [9] are derived under a 1-D unrealistic network model in which cars are *static* and *equally spaced*. It's still unclear about the dissemination latency with delivery reliability requirement in unreliable and highly mobile vehicle-to-vehicle network.

Meant as a step toward a deeper understanding of the latency and reliability issues, we analyze the *minimum* latency of disseminating a message to a node with probability larger than α $(0 < \alpha < 1)$, which is referred to as α-*reliability*. Analysis of theoretical results aims to provide insights into how network parameters (e.g., vehicle speed, transmission range and node density) affect the latency of gaining α-reliability.

In particular, we consider a 1-D network with probabilistic channel and constrained vehicle mobility—a setting which describes many VANET dissemination applications well. For instance, after detecting a dangerous situation, a warning message is propagated backward to all cars on that road. Based on an *optimal* dissemination strategy, which can spread out message as quickly and reliably as possible, we *analytically* derive the minimum latency $t_{min}(\alpha)$ of gaining almost sure α-reliability. Besides reliability requirement α, $t_{min}(\alpha)$ also depends on destination's *original distance* from source, *node mobility*, *channel reliability*, *transmission range* and *node density*. By setting network parameters according to realistic VANET scenarios [10], *numerical* analysis of $t_{min}(\alpha)$ yields several interesting insights. Specifically, we observe that 1) $t_{min}(\alpha)$ is dominated by the time of the first attempt to send the message to a destination, 2) node mobility has little impact on $t_{min}(\alpha)$ for emergency message dissemination, and 3) transmission range and node density greatly affect $t_{min}(\alpha)$.

The reminder of this paper is structured as follows. In Section II, we introduce our network model, channel model, mobility model and dissemination strategy. In Section III, we derive the minimum latency of gaining α-reliability and discuss this analytical result using numerical analysis with insights into how network parameters affect dissemination performance. Section IV concludes this paper.

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II. Network and Mobility Models

In this section, we introduce the network, channel and mobility models used in our analysis of the latency with reliability requirement for VANETs message dissemination. Although being simplified, the network, channel and mobility models, capture relevant VANET features. We further present an optimal dissemination strategy in order to derive the minimum latency of gaining α-reliability.

A. Network and Channel Models

We consider a vehicular ad-hoc network consisting of n nodes on a line $\mathcal{B} = [-\frac{L}{2}, \frac{L}{2}]$. Assume at time 0, n nodes $\mathcal{X}_n(0) = \{X_1(0), \ldots, X_n(0)\}$ are uniformly distributed in the network, where $X_i(0)(1 \leq i \leq n)$ is independent of n and any $X_j(0)(i \neq j)$. By definition in [11], $\{\mathcal{X}_n(0)\}$ is a homogeneous Poisson point process. n nodes are Poisson distributed in the network with density $\lambda = \frac{n}{L}$ everywhere.

Although this 1-D network model is simple, it describes many network scenarios well. For example, when there is a traffic collision on a road, alert information must be propagated backward to cars approaching the accident site. Only drivers approaching the accident need this warning message such that they can response in advance to avoid follow-up collisions. The dissemination is restricted to the 1-D road rather than flooding the whole network.

Due to the inherent difficulties of analysis, we use a simplified channel model. When a node transmits a message, all nodes within transmission range R have probability p $(0 < p < 1)$ of correctly receiving this message. This channel model seizes the most relevant feature of wireless transmission, i.e., uncertainty about correct message reception due to shadow fading or hidden terminal.

B. Mobility Model

Rather than moving at random, vehicles tend to move in an organized fashion. Vehicle mobility is restricted in both speed and direction. For instance, car movements are constrained to follow a paved highway within speed limits for that road. In this paper, we focus on an *constrained vehicle mobility* model which catches above mentioned characteristics.

Definition 1. (*Constrained Vehicle Mobility* $M(t)$) Assume time is slotted, each node initially chooses its position $X(0)$ uniformly from network area and selects a movement direction: moving to the right with probability q $(0 \leq q \leq 1)$ or moving to the left with probability $1-q$. Boundary condition is wrapping around. Node moving direction is unvarying over time, while at each time slot, node speed $\mathcal{V}$ follows uniform distribution on $[v_{min}, v_{max}]$ $(0 < v_{min} \leq v_{max} < \infty)$.

Since network border is wrapped, when a node moves out the left/right border, it will reappear at the right/left border. When $q = \frac{1}{2}$, the traffic flow of each direction is symmetric, otherwise, traffic is asymmetric. Regarding to node velocity, average vehicle speed $v = E(\mathcal{V}) = \frac{v_{min}+v_{max}}{2}$ and variance $Var(\mathcal{V}) = \sigma_v^2 = \frac{(v_{max}-v_{min})^2}{12}$. Furthermore, constrained vehicle mobility process $M(t)$ results in *Poisson node distribution all the time*. (The proof is straightforward and omitted due to page limit.)

C. Dissemination Strategy

Assume that message dissemination proceeds in rounds, and transmissions are carefully scheduled in each round to achieve *fast* and *reliable* message dissemination as much as possible. Assume source s initiates a message dissemination at time 0. Without loss of generality (WLOG), let s initially locate at the origin, i.e., $X_s(0) = 0$. The optimal dissemination strategy is in the following, which is similar to the one presented in [9].

Definition 2. (Optimal Dissemination Strategy) Source node s transmits a message at the first round, and each node within its transmission range R has probability p of correctly receiving the message. In the following rounds, the recipient that locates farthest from source site $X_s(0)$ is selected to transmit in order to *speed up* information propagation as much as possible. At the same time, a number of other nodes that have received the message are scheduled for concurrent transmissions according to a greedy rule in order to *maximize the reliability*.

Denote the area between the origin and the farthest node that successfully receives the message as *"covered area"*, and its radius at time t as $D(t)$. To obtain the best possible reliability, assume the optimal dissemination strategy in Definition 2 can make sure that a node has a probability p of correctly receiving the message during each time slot when it's in the covered area. Fig. 1 illustrates this optimal dissemination strategy. The message initiated by source s is forwarded by the farthest receivers in each round. At time t, node a is in the covered area, thus has probability p of receiving the message.

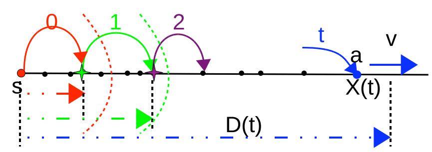

Fig. 1: Optimal Dissemination Strategy: the farthest recipients (star nodes) are selected to spread out the message quickly.

Remark 1. Definition 2 ensures that a message will be spread out as *fast* as possible by choosing the farthest node as relay, and as *reliable* as possible by keeping nodes in the covered area with probability p of receiving the message during *every* time slot. Therefore, Definition 2 is an optimal strategy for fast and reliable message dissemination, which will yield the minimum delay of achieving α-reliability. In addition, the analysis in this paper can be easily generalized to *non-optimal* dissemination strategy that can schedule retransmissions for nodes in the covered area every m $(m > 1)$ time slots.

III. Minimum latency of gaining α-reliability

Let $X_i(t)$ be the location of node i at time t. For simplicity, we will omit the index in our following analysis. Due to symmetry of our network model, we consider $X(0) > 0$ and

$X(t) > 0$ WLOG. According to Definition 2, the probability that a node receives the message correctly follows a geometric process during the time slots the node being in the covered area (i.e., $X(t) < D(t)$). Therefore, the dissemination reliability $\rho(t)$ satisfies

$$\rho(t) = 1 - (1-p)^{\sum_{k=0}^{t-1} \mathbf{1}_{D(k)>X(k)}}. \quad (1)$$

Note that under *non-optimal* dissemination strategy, which can schedule retransmissions for nodes in the covered area every m ($m > 1$) time slots, $\rho(t) = 1 - (1-p)^{\frac{1}{m}\sum_{i=0}^{t-1} \mathbf{1}_{\{D(i)\geq X(i)\}}}$. Therefore, the results derived under optimal dissemination strategy can be easily generalized to *non-optimal* dissemination mechanisms by following the same methodology.

Define $\rho(t) > \alpha$ as *α-reliability*, $t_{min}(\alpha) \triangleq \min\{t : P(\rho(t) > \alpha) = 1\}$ as the *minimum latency* to guarantee α-reliability almost surely (a.s.). In this section, we aim to find $t_{min}(\alpha)$ based on the optimal dissemination strategy under our 1-D network model and constrained vehicle mobility.

A. Minimum Latency of gaining α-reliability

In order to find $t_{min}(\alpha) = \{t : \rho(t) \geq \alpha \ a.s.\}$, we need obtain $P(D(t) > X(t))$ first. We start by investigating the radius of covered area $D(t)$.

1) Radius of the covered area: To begin with, we analyze the radius of the covered area, i.e., the distance between the origin and the farthest recipient at time t. Before our analysis, we first present a useful lemma.

Lemma 1. *When a node transmits a message, the probability that k nodes in $[x, y] \subseteq [0, R]$ receive the massage successfully is $P(k) = e^{-p\lambda(y-x)}\frac{[p\lambda(y-x)]^k}{k!}$, where p is the transmission success probability and λ is the node density.*

Proof: Let A_m denote the event that there exist m nodes in $[x, y]$ and B denote the event that k nodes in $[x, y]$ receive the message correctly. Then $P(k) = \sum_{m\geq k} P(B \cap A_m) = \sum_{m\geq k} P(B|A_m) \cdot P(A_m)$. Since mobility process $M(t)$ produces Poisson node distribution all time, we have

$$\begin{aligned} P(k) &= \sum_{m\geq k} \tbinom{m}{k} p^k (1-p)^{m-k} \cdot e^{-\lambda(y-x)} \frac{[\lambda(y-x)]^m}{m!} \\ &= e^{-\lambda(y-x)} \frac{[p\lambda(y-x)]^k}{k!} \sum_{m\geq k} \frac{[(1-p)\lambda(y-x)]^{m-k}}{(m-k)!} \\ &= e^{-p\lambda(y-x)} \frac{[p\lambda(y-x)]^k}{k!}. \end{aligned} \quad (2)$$

Therefore, number of nodes that receive a message correctly follows Poisson distribution with density $p\lambda$. ■

Denote the distance between a sender and the farthest recipient of its message as *1-hop propagation distance S_P*. We have the following theorem regarding S_P.

Theorem 1. $E(S_P) = R - \frac{1}{p\lambda}(1 - e^{-p\lambda R})$,
$E(S_P^2) = R^2 - \frac{2R}{p\lambda} + \frac{2}{(p\lambda)^2}(1 - e^{-p\lambda R})$.

Proof: Based on Lemma 1, the probability of $\{x < S_P \leq y\}$ $(0 \leq x < y \leq R)$ is

$$P(x < S_P \leq y) = (1 - e^{-p\lambda(y-x)})e^{-p\lambda(R-y)}, \quad (3)$$

i.e., no node whose distance from the sender is larger than y receives the message successfully and at least one node in area $[x, y]$ receives the message correctly. In addition, when $x = 0$, $P(S_P = 0) = e^{-p\lambda R}$; when $x < 0$ or $x > R$, $P(S_P = x) = 0$.

Then, *CDF* of S_P follows, when $0 \leq x \leq R$,

$$\begin{aligned} \mathcal{F}_{S_P}(x) &= P(S_P = 0) + \lim_{\Delta x\to 0} \sum_{i=0}^{x/\Delta x} P(i\Delta x < S_P \leq (i+1)\Delta x) \\ &= e^{-p\lambda R} + \lim_{\Delta x\to 0} \sum_{i=0}^{x/\Delta x} (1 - e^{-p\lambda\Delta x})e^{-p\lambda(R-i\Delta x)} \\ &= e^{-p\lambda(R-x)}; \end{aligned} \quad (4)$$

when $x > R$, $\mathcal{F}_{S_P}(x) = 1$; when $x < 0$, $\mathcal{F}_{S_P}(x) = 0$.

As $\mathcal{F}_{S_P}(x)$ is derivative almost everywhere (except at $x = 0$), there exists probability density function (PDF) for S_P, which can be easily obtained by differentiating $\mathcal{F}_{S_P}(x)$.

$$f_{S_P}(x) = \begin{cases} p\lambda e^{-p\lambda(R-x)}; & 0 \leq x \leq R \\ 0; & \text{otherwise} \end{cases} \quad (5)$$

For non-negative random variable S_P,

$$E(S_P) = \int_0^\infty P(S_P > x)\mathrm{d}x = R - \frac{1}{p\lambda}(1 - e^{-p\lambda R}). \quad (6)$$

Further, we can get

$$E(S_P^2) = \int_0^R x^2 f_{S_P}(x)\mathrm{d}x = R^2 - \frac{2R}{p\lambda} + \frac{2(1 - e^{-p\lambda R})}{(p\lambda)^2}. \quad (7)$$

■

Theorem 1 shows the expected propagation distance per transmission. Under medium to high traffic scenarios (i.e., λ is much larger than $\frac{1}{pR}$), $E(S_P) \approx R - \frac{1}{p\lambda}$, which indicates that the expected 1-hop propagation distance scales approximately linearly with transmission range R.

Further taking node mobility into account, the actual dissemination distance per time slot, denoted as *1-hop dissemination distance S_D*, is adding or subtracting $\mathcal{V}$ to S_P depending on the movement direction of the relay node:

$$S_D = \begin{cases} S_P + \mathcal{V} & w.p.\ q \\ S_P - \mathcal{V} & w.p.\ 1-q \end{cases} \quad (8)$$

Consequently, $E(S_D) = E(S_P) + (2q - 1)v$. Let $\{S_D(0), S_D(1), \ldots, S_D(t-1)\}$ be the dissemination distance during each around till time t, which are independent and identically distributed (i.i.d.) with same distribution as S_D. Therefore, the radius of covered area is $D(t) = \sum_{i=0}^{t-1} S_D(i)$.

2) Probability of a node in the covered area: Given a node initially located at $X(0) > 0$, if it is moving to the right (moving away from the source), its position at time t is $X(t) = X(0) + \sum_{i=0}^{t-1} \mathcal{V}(i)$; if it is moving to the left (moving toward the source), $X(t) = X(0) - \sum_{i=0}^{t-1} \mathcal{V}(i)$, where $\mathcal{V}(i)$ is the node speed during time slot i. $\{\mathcal{V}(i), i \geq 1\}$ are i.i.d. with the same distribution as $\mathcal{V}$.

In the following analysis, we assume $X(t) > 0$, as for $X(t) < 0$ the results also hold. Then, a node is in the covered area if $D(t) > X(t)$; otherwise, it's outside the covered area. We give a lower bound of $P(D(t) > X(t))$ based on the following lemma in [12].

Lemma 2. *Suppose $X_1, X_2, \ldots, X_k$ are independent random variables satisfying $X_i \geq -M$ $(M > 0)$, for $1 \leq i \leq k$. Let $X = \sum_{i=1}^{k} X_i$ and $||X|| = \sqrt{\sum_{i=1}^{k} E(X_i^2)}$. Then we have the following bound for random variable X, given any $\eta > 0$:*

$$P(X \leq E(X) - \eta) \leq e^{-\frac{\eta^2}{2(||X||^2 + M\eta/3)}}. \quad (9)$$

The lower bound of $P(D(t) > X(t))$ is presented in the following theorem.

Theorem 2. *The probability that a node, initially at $X(0) > 0$, is in the covered area at time t satisfies:*

1) *if the node moves away from the source, let $Y \triangleq S_D - \mathcal{V}$, $\eta(t) = tE(Y) - X(0)$, when $t > X(0)/E(Y)$,*

$$P(D(t) > X(t)) \geq 1 - e^{-\frac{\eta(t)^2}{2tE(Y^2) + 4v_{max}\eta(t)/3}}; \quad (10)$$

2) *if the node moves toward the source, let $Y' \triangleq S_D + \mathcal{V}$, $\eta'(t) = tE(Y') - X(0)$, when $t > X(0)/E(Y')$ while $X(t) > 0$,*

$$P(D(t) > X(t)) \geq 1 - e^{-\frac{\eta'(t)^2}{2tE(Y'^2) + 2(v_{max} - v_{min})\eta'(t)/3}}. \quad (11)$$

Proof: For case 1), $X(t) = X(0) + \sum_{i=0}^{t-1} \mathcal{V}(i)$. Then,

$$P(D(t) \leq X(t)) = P\{\sum_{i=0}^{t-1} (S_D(i) - \mathcal{V}(i)) \leq X(0)\} \quad (12)$$

Let $Y_i \triangleq S_D(i) - \mathcal{V}(i)$ and $\eta(t) \triangleq E(\sum_{i=0}^{t} Y_i) - X(0)$,

$$P(D(t) \leq X(t)) = P\left(\sum_{i=0}^{t} Y_i \leq E(\sum_{i=0}^{t} Y_i) - \eta(t)\right) \quad (13)$$

Since $S_D(i)$ are i.i.d. random variables satisfying $-v_{max} \leq S_D(i) \leq R + v_{max}$, and $\mathcal{V}(i)$ are i.i.d. random variables satisfying $v_{min} \leq \mathcal{V}(i) \leq v_{max}$, Y_i are i.i.d. random variables satisfying $Y_i \geq -2v_{max}$. Denote $Y = S_D - \mathcal{V}$ and $\eta(t) = tE(Y) - X(0)$ by omitting the index. Based on Lemma 2, for any $\eta(t) > 0$, i.e., $t > X(0)/E(Y)$,

$$P\{D(t) \leq X(t)\} \leq e^{-\frac{\eta(t)^2}{2tE(Y^2) + 4v_{max}\eta(t)/3}}, \quad (14)$$

where

$$E(Y) = E(S_P) - 2(1-q)v,$$
$$E(Y^2) = E\{(S_P)^2\} - 4(1-q)vE(S_P) + 4(1-q)(\sigma_v^2 + v^2),$$
$$\eta(t) = t\,(E(S_P) - 2(1-q)v) - X(0), \quad (15)$$

in which $E(S_P)$ and $E\{(S_P)^2\}$ are shown in Theorem 1. Therefore, if the node moves away from the source, for $t > X(0)/E(Y)$, we have Eq. (10).

For case 2), $X(t) = X(0) - \sum_{i=0}^{t-1} \mathcal{V}(i)$. When $X(t) < 0$, the node begins moving away from the source which is same as 1). Thus, we only consider $X(t) > 0$.

$$P(D(t) \leq X(t)) = P\{\sum_{i=0}^{t-1} (S_D(i) + \mathcal{V}(i)) \leq X(0)\} \quad (16)$$

Let $Y'_i \triangleq S_D(i) + \mathcal{V}(i)$, and $Y' \triangleq S_D + \mathcal{V}$. $\{Y'_0, Y'_1, \ldots, Y'_{t-1}\}$ are i.i.d. random variables satisfying $Y'_i \geq -(v_{max} - v_{min})$. Denote $\eta'(t) = E(\sum_{i=0}^{t-1} Y'_i) - X(0)$. As $Y'_i = Y'$ in distribution, $\eta'(t) = tE(Y') - X(0)$. When $t > X(0)/E(Y')$, from Lemma 2, we have

$$P\{D(t) \leq X(t)\} \leq e^{-\frac{\eta'(t)^2}{2tE(Y'^2) + 2(v_{max} - v_{min})\eta'(t)/3}}. \quad (17)$$

It can be obtained that

$$E(Y') = E(S_P) + 2qv,$$
$$E(Y'^2) = E\{(S_P)^2\} + 4qvE(S_P) + 4q(\sigma_v^2 + v^2), \quad (18)$$

$$\eta'(t) = tE(Y') - X(0) = t(E(S_P) + 2qv) - X(0). \quad (19)$$

Therefore, if the node moves toward the source, when $t > X(0)/E(Y')$ while $X(t) > 0$, we have Eq. (11). ∎

Theorem 2 estimates the probability that a node is in the covered area at time t. When this lower bound is close to 1 at time t, a node is in the covered area a.s., which means that it has probability p of receiving a message during t^{th} slot. Thus, Theorem 2 can be used to estimate $\rho(t)$, i.e., the probability of correctly receiving a message within time t.

3) Toward deriving $t_{min}(\alpha)$: In order to find the minimum latency of achieving α-reliability a.s., we give a lower bound of $P(\rho(t) > \alpha)$, then obtain $t_{min}(\alpha)$ by letting the lower bound approach 1.

Theorem 3. *Given a small number $\epsilon > 0$, when the destination moves away from the source, if $C(t) > \log_{1-p}^{1-\alpha}$,*

$$t_{min}(\alpha) = \min\{t : e^{-\frac{\left(C(t) - \log_{1-p}^{1-\alpha}\right)^2}{2C(t)}} < \epsilon\}, \quad (20)$$

where $C(t) = \sum_{\lceil \frac{X(0)}{E(Y)} \rceil}^{t-1} (1 - e^{-\frac{\eta(i)^2}{2iE(Y^2) + 4v_{max}\eta(i)/3}})$;
when the destination moves toward the source, if $C'(t) > \log_{1-p}^{1-\alpha}$,

$$t_{min}(\alpha) = \min\{t : e^{-\frac{\left(C'(t) - \log_{1-p}^{1-\alpha}\right)^2}{2C'(t)}} < \epsilon\}, \quad (21)$$

where $C'(t) = \sum_{\lceil \frac{X(0)}{E(Y')} \rceil}^{t-1} (1 - e^{-\frac{\eta'(i)^2}{2iE(Y'^2) + 2(v_{max} - v_{min})\eta'(i)/3}})$.

Proof: As $\rho(t) = 1 - (1-p)^{\sum_{i=0}^{t-1} \mathbf{1}_{\{D(i) \geq X(i)\}}}$,

$$P(\rho(t) > \alpha) = P((1-p)^{\sum_{i=0}^{t-1} \mathbf{1}_{\{D(i) \geq X(i)\}}} < 1 - \alpha)$$
$$= P(\sum_{i=0}^{t-1} \mathbf{1}_{\{D(i) \geq X(i)\}} > \log_{1-p}^{1-\alpha}) \quad (22)$$

Since $\mathbf{1}_{\{D(i) \geq X(i)\}}, i \geq 0$ are i.i.d. indicator functions,

$$E\left\{\left(1_{\{D(i) \geq X(i)\}}\right)^2\right\} = P(D(i) > X(i)). \quad (23)$$

Using Lemma 2, when $\sum_{i=0}^{t-1} P\{D(i) \geq X(i)\} > \log_{1-p}^{1-\alpha}$, we have the lower bound of $P(\rho(t) > \alpha)$,

$$P(\rho(t) > \alpha) \geq 1 - e^{-\frac{\left(\sum_{i=0}^{t-1} P(D(i)>X(i)) - \log_{1-p}^{1-\alpha}\right)^2}{2\sum_{i=0}^{t-1} P(D(i)>X(i))}}. \qquad (24)$$

From Theorem 2, if a node moves away from the source,

$$\sum_{i=0}^{t-1} P(D(i) > X(i)) \geq C(t), \qquad (25)$$

where $C(t) \triangleq \sum_{\lceil \frac{X(0)}{E(Y)} \rceil}^{t-1} (1 - e^{-\frac{\eta(i)^2}{2iE(Y^2)+4v_{max}\eta(i)/3}})$ and $\lceil \cdot \rceil$ is the ceil function; if a node moves toward the source,

$$\sum_{i=0}^{t-1} P(D(i) > X(i)) \geq C'(t), \qquad (26)$$

where $C'(t) \triangleq \sum_{\lceil \frac{X(0)}{E(Y')} \rceil}^{t-1} (1 - e^{-\frac{\eta'(i)^2}{2iE(Y'^2)+2(v_{max}-v_{min})\eta'(i)/3}})$. Consequently, when the node moves away from the source,

$$P(\rho(t) > \alpha) \geq 1 - e^{-\frac{\left(C(t) - \log_{1-p}^{1-\alpha}\right)^2}{2(C(t))}}; \qquad (27)$$

when the node moves toward the source,

$$P(\rho(t) > \alpha) \geq 1 - e^{-\frac{\left(C'(t) - \log_{1-p}^{1-\alpha}\right)^2}{2(C'(t))}}. \qquad (28)$$

$C(t)$ and $C'(t)$ are defined in (25) and (26) respectively. Therefore, $t_{min}(\alpha)$ are as (20) and (21) ∎

Theorem 3 shows that besides α, the minimum latency of gaining α-reliability depends on the following parameters: destination node's initial position $X(0)$, node speed and direction, transmission range R, channel reliability p, and node density λ. Next, we will examine the effects of these parameters on $t_{min}(\alpha)$ using *numerically* analysis.

B. Numerical Analysis

In order to investigate how network parameters affect $t_{min}(\alpha)$, the network settings are set according to *realistic* scenarios [10]. We focus on a two-lane straight road with symmetric traffic ($q = \frac{1}{2}$). The average speed is set to be $10m/s$ ($22mph$), $20m/s$ ($44mph$), and $30m/s$ ($67mph$), which are corresponding to low, medium and high mobility, respectively. Node density λ, depending on traffic flow, is set to be 0.03 (15 vehicles/km/lane), 0.08 (40 vehicles/km/lane), and 0.13 (65 vehicles/km/lane), which are corresponding to light, medium and heavy traffic [13], respectively. The transmission range R of a car can vary from $50m$ to $300m$. The channel reliability p changes from 0.5 (low reliability) to 0.9 (high reliability). Initial node position $X(0)$ varies from $250m$ to $2000m$. We consider a node that moves away from the source unless especially pointed out. Although lower transmission interval could prevent unsafe situation in higher speeds, it results in more saturated channel and it is more likely to cause collisions among simultaneous transmissions. In this paper, we use $100ms$ for transmission interval which seems to be reasonable for most of the scenarios and have been used in other research [14].

1) Dependence on α: We first investigate how long a message dissemination takes to achieve various α-reliability. To this purpose, we consider low traffic condition ($\lambda = 0.03$) with high mobility ($30m/s$) and low channel reliability $p = 0.5$. Vehicle's transmission range $R = 100m$.

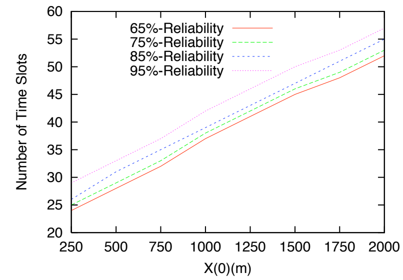

Fig. 2: Values of $t_{min}(\alpha)$ for varying α.

In Fig. 2, $t_{min}(\alpha)$ increases slightly from achieving 65%-reliability to 95%-reliability. This indicates that several retransmissions are needed to increase delivery reliability after a node in the covered area. In other words, $t_{min}(\alpha)$ is dominated by the time of the first attempt to send a message to a node.

Remark 2. In emergency message dissemination that requires high delivery reliability within a short time limit, the primary concerns of a dissemination mechanism should be first to spread a message to target receivers as fast as possible, then increase reliability by several retransmissions once the nodes are in the covered area.

2) Dependence on Channel Reliability: The dependence of $t_{min}(\alpha)$ on channel reliability p, with $\alpha = 99\%$, $v = 30m/s$, $R = 100m$ in medium traffic scenario is reported in Fig. 3.

The figure clearly shows that increasing p can reduce $t_{min}(\alpha)$. Notice that the decrement of $t_{min}(\alpha)$ when p grows from 0.5 to 0.7 is larger than the decrement when p grows from 0.7 to 0.9. In other words, the benefit on $t_{min}(\alpha)$ of increasing channel reliability from low to medium is larger than that from medium to high.

3) Dependence on Mobility: The dependence of $t_{min}(\alpha)$ on node mobility at $\alpha = 99\%$ under light traffic ($\lambda = 0.03$) is shown in Fig. 4, where $R = 100m$ and $p = 0.75$.

As duration of a time slot is $100ms$, average speed is only 3 meter/time slot even for high mobility. This explains why *node speed only slightly affects* $t_{min}(\alpha)$, as shown in Fig. 4. In addition, we find that movement *direction* also has little impact on $t_{min}(\alpha)$. Further, under medium/high node density, the impact of mobility is even smaller. Figures regarding these results are omitted here due to limited space.

Remark 3. Fig. 4 reveals that node mobility has little impact on the latency and reliability of emergency message dissemination in which transmission interval is usually very short.

Despite that, we have to aware that node mobility and channel reliability could be correlated since p is probably smaller under high mobility. Thus, we plan to examine the joint effects of node mobility and channel reliability on VANET message dissemination in our future work.

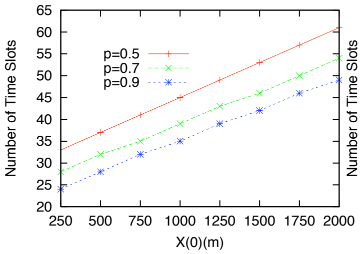

Fig. 3: $t_{min}(99\%)$ for varying channel reliability p.

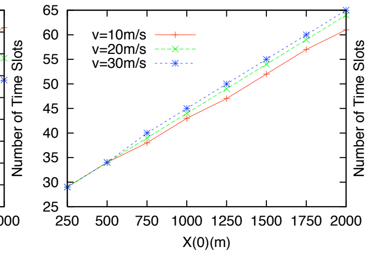

Fig. 4: $t_{min}(99\%)$ under low, medium, and high mobility.

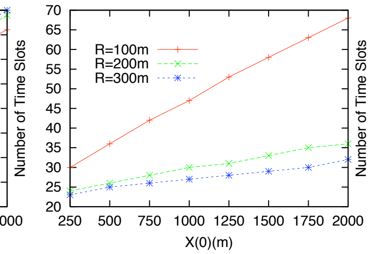

Fig. 5: $t_{min}(99\%)$ for various transmission range R.

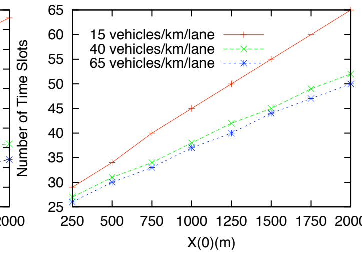

Fig. 6: $t_{min}(99\%)$ under various traffic density λ.

4) Dependence on Transmission Range: Fig. 5 shows the dependence of $t_{min}(\alpha)$ on transmission range with $\alpha = 99\%$, $p = 0.75$, $v = 30m/s$, and $\lambda = 0.08$ (medium traffic).

From the figure, it is clearly to see the beneficial effect on $t_{min}(\alpha)$ of increasing R especially for nodes initially located far away from the source. Interestingly, the benefit of increasing R from $200m$ to $300m$ is small. Since larger R causes more interference for concurrent transmissions, i.e., there is a trade-off between increasing R and decreasing p, the joint effects of R, p, and mobility on message dissemination in VANETs require future work.

5) Dependence on Traffic Density: The dependence of $t_{min}(\alpha)$ on traffic density with $\alpha = 99\%$, $p = 0.75$, $v = 30m/s$, and $R = 100$ is shown in Fig. 6.

From the figure, we can tell that when network density is low, the latency of gaining α-reliability is longer, especially for nodes that initially located far away from the source. Hence, it is harder to achieve high reliability within a given time limit in sparse area. Fig. 6 suggests that density-aware dissemination strategy might be feasible for frequently disconnected network due to time and location related traffic density in VANETs.

Remark 4. In summary, Figs. 2-6 show that transmission range and node density greatly affect dissemination latency and reliability, while mobility has surprisingly little impact.

IV. Conclusion

In this paper, we have analyzed latency of information dissemination in the context of vehicular networks. We have derived the minimum latency of gaining α-reliability using an optimal dissemination strategy. Besides reliability requirement α, $t_{min}(\alpha)$ also depends on node mobility, channel reliability, traffic flow, and transmission range. Numerical analysis of $t_{min}(\alpha)$ reveals several interesting insights, including 1) $t_{min}(\alpha)$ is dominated by the time when a node falls into the covered area; 2) node mobility has little impact on $t_{min}(\alpha)$ when time slot duration is small as in emergency information dissemination for safety applications of VANETs; and 3) transmission range and node density greatly affect dissemination latency and reliability.

As future work, we will consider the latency of achieving α-reliability under more realistic network, mobility and channel models. We will also address issues about the joint effects of channel reliability and node mobility on dissemination latency and reliability. Furthermore, we will design fast, reliable, and sound dissemination strategy according to our analysis as well as taking account of overhead, energy consumption and so on.

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