Understanding the Tempo-spatial Limits of Information Dissemination in Multi-channel Cognitive Radio Networks

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***Abstract*—Cognitive Radio Networks (CRNs) have emerged to become promising network components for exploiting spectrum opportunistically in order that information can be delivered in circumstances otherwise impossible. Challenging yet open questions are *how fast* and *how far* a packet can be delivered in such networks, in temporal and spatial domains, respectively. The answers to these questions offer a straightforward interpretation of the potentials of CRNs for time-sensitive applications. To tackle these questions, we define two metrics, *dissemination radius* $\|\mathcal{L}(t)\|$ and *propagation speed* $\mathcal{S}(d)$. The former is the maximum Euclidean distance that a packet can reach in time t, and the latter is the speed that a packet transmits between a source and destination at Euclidean distance d apart, which can be used to measure the transmission delay. Further, we determine the *sufficient* and *necessary* conditions under which there exist spatial and temporal limits of information dissemination in CRNs. We find that when information cannot be disseminated to the entire network, the limiting dissemination radius is statistically dominated by an exponential distribution, while the limiting information propagation speed approaches to zero. Otherwise, the dissemination radius approaches to infinity and the propagation speed $\mathcal{S}(d)$ is no lower than some constant κ for large d. The results are validated through simulations.**

I. INTRODUCTION

As a promising technique to more efficiently utilize the valuable yet limited resource, *spectrum*, Cognitive Radio Networks (CRNs) have become an important component of current communication infrastructure for a variety of application scenarios, such as military networks, emergency networks, cognitive mesh networks, and leased networks [1]. In CRNs, there are two types of spectrum users: (i) primary users who have the license to use the spectrum, and (ii) secondary uses who can use the spectrum when it is not used by the primary users. Of late, there has been intensive research on understanding and optimizing performance limits, such as capacity, connectivity, spectrum sensing, spectrum mobility, and spectrum sharing of CRNs [1]–[4], which provides insights on improving spectrum efficiency and traffic capacity in CRNs.

However, the understanding of spectrum efficiency and system capacity are not able to reveal *how fast* and *how far* a packet can be disseminated in a CRN, in temporal and spatial domains, respectively. The answers to these questions offer a straightforward interpretation of the potentials of CRNs for time-sensitive applications. For example, when a CRN is used for emergency rescue in the aftermath of disasters or traffic accidents (e.g., vehicular networks), we need to ensure that *help* or *warning* messages can be disseminated to a sufficiently large area, and estimate how long it takes for such information to reach a chosen destination, which becomes more important than other performance metrics, such as the total network capacity in these circumstances.

Similar problems have been studied for homogeneous wireless networks. Particularly, the conditions for connectivity or percolation in order to ensure that information can be disseminated to the entire network have been explored in [5]–[8]. In addition, information propagation speed or delay has been discussed in recent works [6], [8]. Xu and Wang [6] showed that the information propagation speed in a *fully-connected* network is upper bounded by some constant; and Kong and Yeh [8] found that the transmission latency in a *percolated* network scales at least linearly with the transmission distance. However, these results on homogeneous networks may be not applicable to CRNs because of the unique features of the latter [4]. To be specific, there are two types of users in CRNs and since secondary users exploit channels temporarily unused by primary users, information dissemination among secondary users depends on not only the network topology of secondary users themselves, but also the spatial density of primary users. Furthermore, instead of fixed transmitting parameters assumed in homogeneous networks, secondary users adapt their transmitting parameters, such as transmission power and operating frequency, to avoid or limit interference with the primary users. Dynamic transmitting parameters obviously impact information dissemination among secondary users in both spatial and temporal domains. In addition, cognitive radio is a software-defined radio and can access much more channels than the traditional hardware-defined radio in homogeneous networks [3]. Multiple channels make the information dissemination in CRNs even more complicated. Due to these important differences, how information is disseminated, or the spatial and temporal limits of information dissemination, in multi-channel CRNs, is an unknown problem to be resolved.

Particularly, we focus on the following questions in this paper: (i) *for a large multi-channel CRN, how far can a packet originated from an arbitrary node be disseminated?* (ii) *When a packet can be disseminated to a sufficiently large area, how long does it take this packet to reach a chosen destination?* To tackle these problems, we define two new metrics, the *dissemination radius* $\|\mathcal{L}(t)\|$ and the *propagation speed* $\mathcal{S}(d)$ to study the spatial and temporal limits, respectively. The former is the maximum Euclidean distance that a packet propagates in time t and can be used to characterize the

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dissemination area; and the latter one is the speed that a packet transmits between a source and destination at distance d apart, which can be used to interpret the end-to-end delay. Our main contributions are as follows: (1) We determine the *sufficient* and *necessary* conditions under which there exist theoretical limits of information dissemination among secondary users. Because secondary users can only opportunistically access the channels, these conditions apparently depend on the spatial density λ_p of the coexisting primary users and the number of channels m accessible by secondary users. (2) We find that when the packet cannot percolate to the whole network, the limiting dissemination radius $\|\mathcal{L}(\infty)\|$ is statistically dominated by an exponential distribution and the limiting information propagation speed approaches to zero. Otherwise, the dissemination radius approaches to infinity and the propagation speed $\mathcal{S}(d)$ is no lower than some constant κ.

This paper is organized as follows. We define the problem of information dissemination in CRNs in Section II. We present the analysis and results, focusing on the proof of necessary and sufficient conditions for infinite dissemination radius and limits of information propagation speed in Section III and Section IV, respectively. In Section V, we present simulation results and finally, we conclude in Section VI.

II. Preliminaries and Problem Definition

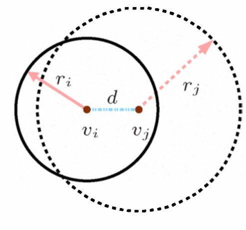
In this section, we first describe assumptions and network models used in the paper and then define the *dissemination radius* and *propagation speed* in order to study the information dissemination in spatial and temporal domains.

A. Assumptions and Models

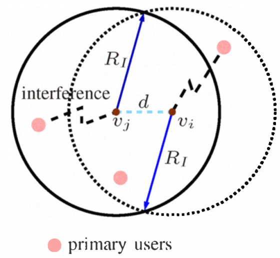
We consider a large CRN consisting of n secondary users $\{v_1, \ldots, v_n\}$, which are distributed independently and uniformly in a region $\Omega = [0, \sqrt{\frac{n}{\lambda}}]^2$ for some constant λ. Let $\mathcal{H}_\lambda = \{X_1, \ldots, X_n\}$ denote the random locations of secondary users and $\mathcal{H}_\lambda$ is a Poisson Point process with density λ as $n \to \infty$ [9]. Instead of *fixed transmission range*, each v_i is assumed to have an independently adaptive transmission range r_i with maximum transmission range $\gamma > 0$, i.e., $\mathbb{P}(r_i < \gamma) = 1$. We focus on $\gamma = 1$ in this paper and due to the *scaling property* [9], [10], the results can be extended to any γ. We further assume that r_i follows a common distribution F_r with probability density f_r for simplicity.

Further, we consider a set of m channels $\{ch_1, \ldots, ch_m\}$. For any $1 \leq k \leq m$, an *overlay* network of primary users, which are distributed as a Poisson Point process with density $\frac{\lambda_p}{\eta}$, are assumed to use the channel ch_k. To model the dynamic traffic of primary users, time is divided into unit slots and at each time slot, primary users on ch_k use this channel with probability η independently. By *thinning theorem* (*Theorem 9.15* [9]), the active (transmitting) primary users on any ch_k are distributed as a Poisson Point process with density λ_p.

Note that there exist two types of interference to secondary users in CRNs: *secondary-to-secondary* and *primary-to-secondary* interference. To characterize the former interference, we make some adaptation to the well-known *protocol model* [6]. Particularly, a successful transmission from secondary user v_i to v_j is feasible if $d = \|X_i - X_j\| < \min(r_i, r_j)$ (see Fig. 1(a)) and $\|X_g - X_j\| \geq (1 + \Delta) \max_{l=1}^{n}(r_l)$ for any other simultaneously transmitting secondary user v_g on the same channel, where $\|\cdot\|$ denotes the Euclidean distance and Δ models the guard zone around v_j in which any simultaneous transmission on the same channel causes collision at v_j. In terms of the latter one, let R_I be the *interference range* of primary users and as shown in Fig. 1(b), two secondary users v_i and v_j are allowed to use the channel ch_k when there are no *active* primary users in $\mathcal{S}(d, R_I)$, which denotes the region covered by the two circles with radius R_I centered at v_i and v_j. Therefore, the probability that a channel ch_k is allowed to be used by the secondary users v_i and v_j without interfering with the coexisting primary users is given by $\mathbb{P}_{sk}(d) = e^{-\lambda_p \|\mathcal{S}(d, R_I)\|}$, where $\|\mathcal{S}(d, R_I)\|$ is the area of $\mathcal{S}(d, R_I)$. And the probability that *at least one* channel can be used by secondary users v_i and v_j is given by $\mathbb{P}_s(d) = 1 - (1 - \mathbb{P}_{sk})^m$. Denote $\mathcal{M} = (\lambda_p, m)$ and the CRN is represented as $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ in the rest of this paper.



(a) $d < \min(r_i, r_j)$



(b) $\mathcal{S}(d, R_I)$

Fig. 1. Two types of interference in CRNs.

B. Percolated CRNs and Challenges

To understand the information dissemination in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$, we first present information dissemination in homogeneous networks, which has been extensively studied [5]–[8], for comparison. Particularly, we consider a homogeneous network where wireless nodes are distributed as a Poisson Point process $\mathcal{H}_\lambda$ with fixed transmission range $r = 1$, which is denoted by a *random geometric graph* $G(\mathcal{H}_\lambda, 1)$ [1] [9], [11], [12]. To ensure that information can be disseminated to the entire network, *full connectivity*, i.e., any pair of nodes are connected by a path, has been assumed as a prerequisite for applications of the network $G(\mathcal{H}_\lambda, 1)$. Later on, it has been shown that the *full connectivity* requirement is over-restrictive, and therefore, another feasible connectivity based on *percolation theory* [10], has been extensively investigated [7], [8], [11].

The main result of percolation theory about $G(\mathcal{H}_\lambda, 1)$ is that there exists a *critical density* λ_c of the node spatial density. The network $G(\mathcal{H}_\lambda, 1)$ is percolated when $\lambda > \lambda_c$ and it is not percolated when $\lambda < \lambda_c$. When the network is percolated, there exists a large connected component of nodes spanning

[1] Note that when the spatial density of primary users $\lambda_p = 0$ and secondary users use the maximum power to transmit ($r = 1$), $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is equivalent to $G(\mathcal{H}_\lambda, 1)$. Thus $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is a subgraph of $G(\mathcal{H}_\lambda, 1)$ and the latter is called the homogeneous network counterpart of the former.

almost the entire network (called *giant component* in [10]), and when the network is not percolated, the network consists only of small isolated connected components [10] [2]. Therefore, in a percolated network, information can still be disseminated to the entire network through the *giant component*.

In this paper, we are interested in how fast information can be disseminated in the *giant component* when the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is percolated and how far information can reach when $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is not percolated. Percolation in CRNs has been studied in the seminal work [4], [13], which greatly improve our understanding on *topology* and *connectivity* of CRNs. However, *multiple channels* and *dynamic transmitting parameters*, which practically characterize the cognitive radio, and make this new technology feasible and fascinating, have never been studied in [4], [13]. In addition, even for the homogeneous network $G(\mathcal{H}_\lambda, 1)$, the exact value of the critical density λ_c is unknown, although the tightest range has been derived as $0.769 < \lambda_c < 3.372$ in [14]. In [4], [13], Ren *et al.* derive a range on the critical density for percolation in a *single-channel* CRN with fixed transmission range, by mapping the CRN on a *discrete square lattice*. Even we cannot obtain the exact value on critical density, we should decrease the interval of the range as much as possible. Motivated by the results that in the homogeneous network, mapping $G(\mathcal{H}_\lambda, 1)$ on a *triangular lattice* provides the tightest upper bound [10] and *clustering coefficient* method provides the tightest lower bound [14] on critical density λ_c, we will extend these methods to derive conditions for percolation in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$. Our methods are different from those used in [4], [13] and to the best of our knowledge, provides the tightest bounds on critical density for percolation of multi-channel CRNs with dynamic transmitting parameters.

It is worthy of noting that the communication links are assumed to be independent (or 1-dependent) of each other in the homogeneous network $G(\mathcal{H}_\lambda, 1)$, which is a prerequisite for the derived results [7], [8], [10], [11]. However, in CRNs, all the communication links depend on the coexisting primary users. That is, the links are not independent, which imposes a big challenge to prevent us from using the existing methods in homogeneous networks. In the next section, we will show how to incorporate the link dependency into the *triangular lattice mapping* and *clustering coefficient* methods to derive the percolation conditions for multi-channel CRNs.

C. Limits of Information Dissemination

Assume that information is disseminated through broadcasting in CRNs and consider a packet is sent by a node v_0, which is assumed to be located at the origin $o \in \mathbb{R}^2$, at time $t = 0$. Let us denote $\mathcal{V}(t)$ as the cluster of nodes that have received the packet by time t. The *dissemination area* at t, $\mathcal{A}(t) \in \mathbb{R}^2$, that is, the total area covered by $\mathcal{V}(t)$, can be expressed by $\mathcal{A}(t) \triangleq \bigcup_{v_i \in \mathcal{V}(t)} \mathcal{B}(X_i, 1)$, where $\mathcal{B}(x, r)$ is a circle with a radius r centering at point $x \in \mathbb{R}^2$ and X_i is the location of v_i.

[2]This phase transition in the macroscopic behavior of large wireless networks is defined as *critical phenomenon* in *continuum percolation* [10].

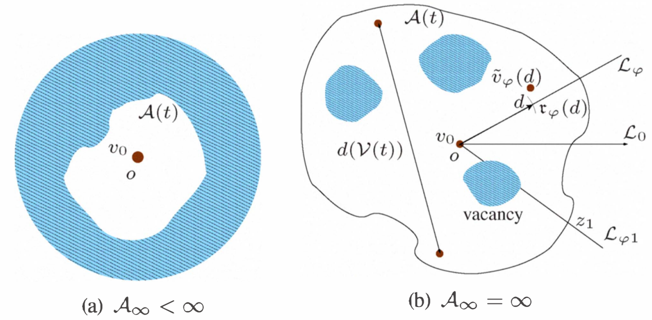


Fig. 2. An illustration of dissemination area after long time t.

Illustrations of dissemination area for sufficiently large t are shown in Fig. 2. Since the network is not fully connected, any sufficiently large area is only partially covered by $\mathcal{A}(t)$. The uncovered area is shown in Fig. 2 as shaded areas. Let $\mathcal{A}_\infty = \lim_{t\to\infty} \mathcal{A}(t)$ and we illustrate $\mathcal{A}_\infty < \infty$ and $\mathcal{A}_\infty = \infty$ in Fig. 2(a) and Fig. 2(b) respectively. Denote $\mathcal{L}_\varphi$ as the line starting from the origin o in the direction $\varphi \in [0, 2\pi)$ and $\mathcal{L}_\varphi(t) = oz$, where $z = \arg\max_{v \in \mathcal{L}_\varphi \cap \mathcal{A}(t)} \|v\|$ is the farthest intersection point between $\mathcal{L}_\varphi$ and $\mathcal{A}(t)$. For example, in Fig. 2(b), $\mathcal{L}_{\varphi 1}(t)$ is the line segment $\overline{oz_1}$. The length of $\mathcal{L}_\varphi(t)$, $\|\mathcal{L}_\varphi(t)\|$, is defined as the *transmitting distance* at t.

Definition 1 (Dissemination radius $\|\mathcal{L}(t)\|$): The *dissemination radius* at time t is defined as $\|\mathcal{L}(t)\| = \max_{\varphi \in [0,2\pi)} \|\mathcal{L}_\varphi(t)\|$; and the *limiting dissemination radius* is defined as $\|\mathcal{L}(\infty)\| = \lim_{t\to\infty} \|\mathcal{L}(t)\|$.

The *dissemination radius* indicates how *far* a packet can reach in the spatial domain in a large network. Next, we move on to the temporal domain.

Definition 2 (Information propagation speed $\mathcal{S}_\varphi(d)$): Let $\mathfrak{r}_\varphi(d)$ be the point on $\mathcal{L}_\varphi$ with $\|\mathfrak{r}_\varphi(d)\| = d$ (see Fig. 2(b)). Define $\mathcal{T}(v_0, v) \triangleq \arg\min_{t \geq 0}\{v \in \mathcal{V}(t)\}$ as the delay of the node v. Denote $\tilde{v}_\varphi(d)$ as the node closest to $\mathfrak{r}_\varphi(d)$ which can receive the packet, i.e., $\tilde{v}_\varphi(d) = \arg\min_{v \in \mathcal{V}_\infty} \|v - \mathfrak{r}_\varphi(d)\|$, where $\mathcal{V}_\infty = \lim_{t\to\infty} \mathcal{V}(t)$. When $\|\mathcal{L}(\infty)\| = \infty$, the *Information Propagation Speed* in direction φ is defined as $\mathcal{S}_\varphi(d) \triangleq \frac{d}{\mathcal{T}(v_0, \tilde{v}_\varphi(d))}$. And when $\|\mathcal{L}(\infty)\| < \infty$, $\mathcal{S}_\varphi(d) \triangleq \frac{d}{\mathcal{T}(v_0, \tilde{v}_\varphi(d))}$ for $d \leq \|\mathcal{L}(\infty)\|$, and $\mathcal{S}_\varphi(d) \triangleq 0$ for $d > \|\mathcal{L}(\infty)\|$. The *limiting propagation speed* $\mathcal{S}_\varphi(\infty)$ is defined as $\lim_{d\to\infty} \mathcal{S}_\varphi(d)$. The definition of $\mathcal{S}(d)$ denotes the propagation speed in an arbitrary direction.

III. How Far Can Information Be Disseminated in a Multi-Channel CRN?

In this section, we identify the sufficient and necessary conditions under which information can be disseminated to the entire network. We first prove the existence of the critical phenomenon in cognitive radio networks by *coupling* [10], then derive the sufficient condition for percolated cognitive radio networks by continuous-to-discrete mapping and the necessary condition by using a *Link Correlation Coefficient (LCC)* approach.

A. Critical Phenomenon in CRNs

Although *critical phenomenon* has been observed and proved in large homogeneous networks [7], [8], [10], [11] and *percolation* for single-channel CRNs has been studied in [4], [13], note that there is no proof to show the *critical phenomenon* in large CRNs. Therefore, before we derive the percolation conditions for the multi-channel cognitive radio networks, we first prove that the *critical phenomenon* [10] also exists in the CRNs.

As we mentioned in Section II-B, the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is a subgraph of the homogeneous network $G(\mathcal{H}_\lambda, 1)$, i.e., $G(\mathcal{H}_\lambda, F_r, \mathcal{M}) \subset G(\mathcal{H}_\lambda, 1)$. Therefore, when the node density $\lambda < \lambda_c$, the homogeneous network $G(\mathcal{H}_\lambda, 1)$ is not percolated, and thus the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is not percolated by *coupling* [10]. On the other hand, as shown in Section. III-B, when the node density λ is large enough, the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is percolated. Let $\mathcal{E}$ denote the event that $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is percolated. Denote $\mathbb{P}_\lambda(\mathcal{E})$ as the probability that $\mathcal{E}$ happens. Note that $\mathcal{E}$ is a *tail event* [15] and thus by *Kolmogorov's 0-1* theorem [15], $\mathbb{P}_\lambda(\mathcal{E})$ is either 0 or 1. Since $\mathbb{P}_\lambda(\mathcal{E})$ is nondecreasing with λ, and we have shown that when λ is small, $\mathbb{P}_\lambda(\mathcal{E}) = 0$ and when λ is large, $\mathbb{P}_\lambda(\mathcal{E}) = 1$, therefore, there exists a positive value $\lambda_{c,c}$, below which $\mathbb{P}_\lambda(\mathcal{E}) = 0$ and above which $\mathbb{P}_\lambda(\mathcal{E}) = 1$. This proves the *critical phenomenon* in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ and $\lambda_{c,c}$ is called *critical density* of $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ in this paper.

B. Conditions for Percolated Cognitive Radio Networks

When a network is percolated, there exists a giant component, which consists of the number of nodes in the order of the network size n. In this case, the network is *almost surely connected* such that a packet can be disseminated to most nodes in the network. Therefore, to find out the limits of *dissemination radius*, i.e., how far a packet can be disseminated, we first identify the *sufficient* and *necessary* conditions for a CRN to be percolated. As discussed in Section III-A, critical density $\lambda_{c,c}$ provides the condition for percolation. However, as mentioned in Section II-B, it is infeasible to obtain the exact value of critical density, and thus to determine the conditions for a CRN to be percolated is quite challenging. Particularly, we have the following necessary and sufficient conditions for a cognitive radio network to percolate.

Theorem 1: For a large multi-channel CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$, given the number of channels m and the spatial density of primary users λ_p, there exists a critical density $\lambda_{c,c}$ on secondary users, above which $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is percolated and below which $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is not percolated. Specifically, we have

$$\lambda_{c,c} < \min_{0<\|e\|<0.5} \frac{1.21 \log\left(\frac{1}{1-\sqrt{\frac{\mathbb{P}_c}{\mathbb{P}_a}}}\right)}{\|e\|^2(1 - F_r(2\|e\|))}, \qquad (1)$$

where $\mathbb{P}_c$ is the bond open probability sufficient for percolation on a dependent triangular lattice given in Appendix B and $\mathbb{P}_a = 1 - \left(1 - e^{-\lambda_p \alpha}\right)^m$ with α given in Appendix A. Furthermore, almost surely,

$$\lambda_{c,c} > \frac{1}{\Gamma(1 - \mathcal{C}_{LCC})}, \qquad (2)$$

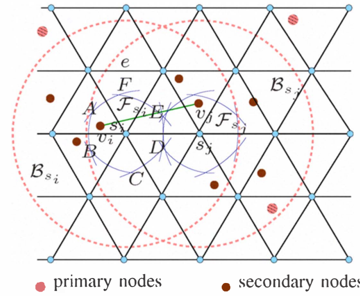


Fig. 3. The triangular lattice $\mathcal{D}$ with the "flower" $\mathcal{F}_{s_i} = ABCDEF$.

where $\Gamma = 2\pi \int_0^1 \int_0^r x(1 - F_r(x))\mathbb{P}_a dx dF_r$, and $\mathcal{C}_{LCC}$ is the *Link Correlation Coefficient* given in Appendix C.

Note that the unique feature of CRNs is the coexistence of primary users and secondary users and the latter can only opportunistically use channels for communication. Therefore, the dynamics of the primary users (i.e., λ_p and R_I) and the number of accessible channels m can affect the information dissemination, and thus the percolation of secondary users (see Eq. (1) and (2), where $\mathbb{P}_c$, $\mathbb{P}_a$ and Γ are functions of λ_p, R_I and m.) We next investigate how λ_p, R_I and m impact percolation of secondary users in details.

We first derive the sufficient condition, i.e., the upper bound on critical density $\lambda_{c,c}$ (i.e., Eq. (1)), for percolation of the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ by using the technique of *continuous-to-discrete* percolation mapping since the percolation on the discrete lattice has been well-understood [16]. Note that a mapping of the homogeneous network $G(\mathcal{H}_\lambda, 1)$ on a square lattice has been used in the existing work [8]. Although this square lattice mapping is easy for analysis, we find that the triangular lattice mapping can provide tighter results and thus is used in our study. Particularly, to obtain the sufficient condition, we begin by constructing a triangular lattice, denoted by $\mathcal{D}$ over the plane, with edge length $\|e\| < \frac{1}{2}$, as shown in Fig. 3. Each site (vertex) s of the lattice is enclosed in an area $\mathcal{F}_s$, which is called "flower" in this paper. $\mathcal{F}_s$ is formed by the six arcs of circles, each of radius $\|e\|$ and centered at the midpoints of the six edges incident on s. This formation ensures that for any points $x \in \mathcal{F}_{s_i}$ and $y \in \mathcal{F}_{s_j}$, $\|x - y\| \le 2\|e\| < 1$. Denote $\mathcal{B}_s$ as the circle centering at s with a radius $R_I + \|e\|$, i.e., $\mathcal{B}_s = \mathcal{B}(s, R_I + \|e\|)$. Note that the distance between any point outside $\mathcal{B}_{s_i} \cap \mathcal{B}_{s_j}$ and any point inside $\mathcal{F}_{s_i} \cap \mathcal{F}_{s_j}$ is larger than R_I, the interference range of primary users. Therefore, if there are no primary users using the channel ch_k within $\mathcal{B}_{s_i} \cap \mathcal{B}_{s_j}$, the channel ch_k can be used by any secondary user within $\mathcal{F}_{s_i} \cap \mathcal{F}_{s_j}$, according to the *primary-to-secondary* interference model given in Section II-A. And for simplicity, the channel ch_k is said to be *available* within the region $\mathcal{F}_{s_i} \cap \mathcal{F}_{s_j}$ in this case. We define a site s_i to be *open* if there exist secondary users with the transmission range $r \ge 2\|e\|$ within $\mathcal{F}_{s_i}$. A bond (i.e., edge or link) $e = s_i s_j$ is declared *open* if there exist at least one channel *available* within $\mathcal{F}_{s_i} \cap \mathcal{F}_{s_j}$ and both s_i and s_j are *open*. By this definition, an *open* bond $s_i s_j$ indicates some secondary nodes v_i inside $\mathcal{F}_{s_i}$ and v_j inside $\mathcal{F}_{s_j}$ with $\|X_i - X_j\| \le 2\|e\| < \min(r_i, r_j)$ and some channels which can be used by secondary users v_i and v_j without interference to the primary users (X_i, X_j are the locations of v_i, v_j). Equivalently speaking, an open bond in $\mathcal{D}$ implies a communication link of the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ around (see Fig. 3 and existence of communication links in Section II-A). Consequently, bond percolation of the triangular lattice $\mathcal{D}$ implies percolation in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$.

According to our definition, given neighboring bonds $s_i s_j$ and $s_j s_k$, the regions $\mathcal{B}_{s_i} \cap \mathcal{B}_{s_j}$ and $\mathcal{B}_{s_j} \cap \mathcal{B}_{s_k}$ are overlapping, which indicates that in the constructed bond percolation model on the triangular lattice $\mathcal{D}$, bonds (links or edges) are *not independent*. We then investigate the condition that the discrete bond percolation model on $\mathcal{D}$ percolates and we find that there exists a certain value $\mathbb{P}_c$ (see Appendix B) such that if the bond is *open* with probability $\mathbb{P}_o > \mathbb{P}_c$, the dependent discrete model percolates. A reverse mapping then can be carried out back to the continuous plane, which finally provides the percolation condition for $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$.

Proof of Eq. (1) of Theorem 1: By *Thinning Theorem* [10], the probability that at least one secondary user with transmission range $r > 2\|e\|$ lies in $\mathcal{F}_{s_i}$ is $1 - e^{-\lambda[1-F_r(2\|e\|)]\mathbb{A}}$, where $\mathbb{A} = \|\mathcal{F}_{s_i}\| \approx 0.8277\|e\|^2$ is the area of the flower $\mathcal{F}_{s_i}$, and F_r is the distribution of the transmission range r. The probability that no primary users using the channel ch_k in $\mathcal{B}_{s_i} \cap \mathcal{B}_{s_j}$ is $e^{-\lambda_p \alpha}$, where α denotes the area of $\mathcal{B}_{s_i} \cap \mathcal{B}_{s_j}$ given in Appendix A. Thus the probability that at least one channel *available* within $\mathcal{F}_{s_i} \cap \mathcal{F}_{s_j}$ is $\mathbb{P}_a = 1 - \left(1 - e^{-\lambda_p \alpha}\right)^m$. Therefore, by definition, the probability that a bond $s_i s_j$ is open is given by $\mathbb{P}_o = \left(1 - e^{-\lambda[1-F_r(2\|e\|)]\mathbb{A}}\right)^2 \mathbb{P}_a$. By the percolation requirement $\mathbb{P}_o > \mathbb{P}_c$, we obtain the upper bound (Eq. (1)) on the critical density. ■

Next, we study the necessary condition when $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ percolates, i.e., the lower bound on critical density (Eq. (2)). There exist two methods in the homogeneous network $G(\mathcal{H}_\lambda, 1)$ to derive the necessary condition for percolation: the *multi-type branching process method* [10] and the *clustering coefficient method* [14]. The former employs an argument based on comparison of $G(\mathcal{H}_\lambda, 1)$ with a suitable branching process to provide an upper bound on the expected number of nodes contained in a component(see p45 of [10] for more details). The latter incorporates *clustering effect* in the random geometric graph into the multi-type branching process method and thus can obtain tighter bounds. The *clustering effect* can be measured by *clustering coefficient* [14], which is defined as follows:

Definition 3: Given three nodes v_i, v_j, and v_k in the network $G(\mathcal{H}_\lambda, 1)$, the *clustering coefficient* is the conditional probability that nodes v_i and v_j are adjacent given that v_i and v_j are both adjacent to node v_k. Here two nodes are called *adjacent* when they are within each other's transmission range.

Clustering coefficient captures the spatial dependency among neighboring links in the homogeneous network. By using *clustering coefficient method*, the main result for percolation of $G(\mathcal{H}_\lambda, 1)$ in [14] is the following lemma:

Lemma 2: Given the mean node degree μ and *clustering coefficient* $\mathcal{C}_c$ of $G(\mathcal{H}_\lambda, 1)$. If $\mu < \frac{1}{1-\mathcal{C}_c}$, the network $G(\mathcal{H}_\lambda, 1)$ is not percolated.

However, in CRNs, besides this spatial dependency, neighboring links are interdependent due to the coexisting primary users and the time-varying transmission range of secondary users, as shown in Appendix C. In order to extend the result of the *clustering coefficient method* (Lemma 2) to the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$, we define *Link Correlation Coefficient (LCC)* as follows, which account for all the mentioned dependency among neighboring links in CRNs.

Definition 4: Given a CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ and three secondary users v_i, v_j and v_k, the *Link Correlation Coefficient (LCC)* $\mathcal{C}_{LCC}$ is the conditional probability that the communication link $v_j v_k$ exists given that links $v_i v_j$ and $v_i v_k$ exist.

Note that the proof for Lemma 2 in [14] requires neither homogeneous nodes nor uniform transmission range. The essential of the *clustering coefficient method* is to incorporate the link dependency into the traditional *multi-type branching process method* [10]. And the fundamental difference between the *cluster coefficient* and *Link Correlation Coefficient* is that the latter accounts for more link dependency existing in CRNs and thus provides a tighter bound. Therefore, by the same proof, we can obtain similar result to Lemma 2. Particularly, given mean node degree μ and *Link Correlation Coefficient (LCC)* $\mathcal{C}_{LCC}$, the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is not percolated if $\mu < \frac{1}{1-\mathcal{C}_{LCC}}$.

In $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$, the mean degree of each secondary user is $\mu = \int_0^1 2\pi\lambda \int_0^r x(1 - F_r(x))\mathbb{P}_a dx dF_r$. With the *Link Correlation Coefficient (LCC)* $\mathcal{C}_{LCC}$ in Appendix C, we can obtain the lower bound on $\lambda_{c,c}$ given in Theorem 1 (Eq. (2)). This completes the proof of Theorem 1.

C. Analysis of Dissemination Radius

One of our main objectives of this study is to find *how far* information can be disseminated in a multi-channel CRN. The problem is formulated to find the limits of *dissemination radius*, which is dependent on whether a CRN is percolated or not. The *critical phenomenon* in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ has been proved in Section III-A and Theorem 1 further provides sufficient and necessary conditions for percolation in the cognitive radio network. Next, we analyze the dissemination radius and have the following results.

Corollary 1: In a multi-channel CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$, given the number of channels m and the spatial density of primary users λ_p, if the spatial denstiy of secondary users $\lambda > \lambda_{c,c}$, a packet b sent by the origin node v_0 can be disseminated to the entire network with some positive probability, i.e., $\mathbb{P}(\|\mathcal{L}(\infty)\| = \infty) > 0$.

Proof: When the density of secondary users $\lambda > \lambda_{c,c}$, the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is percolated, i.e., there exists a giant component $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M}))$ of secondary users. When a packet b sent by $v_0 \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M}))$, all secondary users in $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M}))$ can receive this packet. That is, the packet b can be disseminated to the entire network through this component. Moreover, when the network is percolated, the probability $\mathbb{P}(v_0 \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M})))$ is equal to the fraction of secondary users contained in the *giant component*, which is positive by the definition of *percolation* [10]. This completes the proof. ∎

Corollary 2: When the spatial density of secondary users in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is less than the critical density of the homogeneous network $G(\mathcal{H}_\lambda, 1)$, i.e., $\lambda < \lambda_c$, information can only be disseminated to *finite* area, i.e., $\mathbb{P}(\|\mathcal{L}(t)\| < \infty) = 1$. Particularly, we have

$$\mathbb{P}(\|\mathcal{L}(\infty)\| > x) \leq \xi_1 e^{-\xi_2 x}, \tag{3}$$

for some positive constants ξ_1 and ξ_2.

Proof: As discussed in Section II-B, the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is a subgraph of its homogeneous network counterpart $G(\mathcal{H}_\lambda, 1)$. Therefore, considering the origin node v_0 and the component of secondary users $\mathbb{C}_{v_0} \in G(\mathcal{H}_\lambda, 1)$ consisting of v_0, the secondary users which can receive the packet b sent by v_0 are upper bounded by $\mathbb{C}_{v_0}$. When $\lambda < \lambda_c$, $\mathbb{C}_{v_0} \in G(\mathcal{H}_\lambda, 1)$ is almost surely finite [10]. That is, $\mathbb{P}(dia(\mathbb{C}_{v_0}) < \infty) = 1$, where $dia(\mathcal{C})$ denotes the diameter of the component $\mathbb{C}_{v_0}$. Note that $\|\mathcal{L}(t)\| \leq dia(\mathbb{C}_{v_0})$ for any t and thus, $\mathbb{P}(\|\mathcal{L}(\infty)\| < \infty) = 1$. Moreover, as shown in [10] (see Lemma 3.3, page 68), when the network $G(\mathcal{H}_\lambda, 1)$ is not percolated, for any component $\mathbb{C} \in G(\mathcal{H}_\lambda, 1)$, $\mathbb{P}(dia(\mathbb{C}) > x) < \xi_1 e^{-\xi_2 x}$ for some positive constants ξ_1 and ξ_2. Therefore, $\mathbb{P}(\|\mathcal{L}(\infty)\| > x) < \mathbb{P}(dia(\mathbb{C}) > x) < \xi_1 e^{-\xi_2 x}$. This completes the proof. ∎

An interesting question that remains unanswered is that when $\lambda_c < \lambda < \lambda_{cc}$, how far information can be disseminated. Intuitively, when $\lambda < \lambda_{cc}$, the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is not percolated and thus it seems that information can only be disseminated to a finite number of nodes. However, on the other hand, given $\lambda > \lambda_c$, a *giant component* $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$ exists in its homogeneous network counterpart $G(\mathcal{H}_\lambda, 1)$. The giant component $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$ is disconnected in $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ because many of its links are *inactive* due to the interference to the primary users. In the next section, we will show that any *inactive* link in $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ will be *active* within finite time, which implies that when $\lambda_c < \lambda < \lambda_{cc}$, information can still be disseminated to the entire network through the component $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$ by *store-and-forward* strategy. Putting all together, when the density $\lambda > \lambda_c$, $\mathbb{P}(\|\mathcal{L}(\infty)\| = \infty) > 0$.

IV. How Fast Can Information Be Propagated in a Multi-Channel CRN?

In this section, we investigate how fast information can be disseminated in large multi-channel CRNs. Particularly, we partition the spatial density λ of secondary users into three regions and have the following results.

Theorem 3: (i) When the density $\lambda_c < \lambda < \lambda_{c,c}$ and the origin node $v_0 \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$, a packet b sent by v_0 can be disseminated to the entire network at some constant speed κ; (ii) When $\lambda > \lambda_{c,c}$ and $v_0 \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$, the packet b can be disseminated to the entire network within finite time; (iii) When $\lambda < \lambda_c$, the packet b can only be disseminated to a finite number of nodes and the limiting speed $\mathcal{S}_\varphi(\infty) = 0$.

A. Differences from Earlier Studies

We first study the information propagation speed when the spatial density of secondary users $\lambda_c < \lambda < \lambda_{c,c}$ and $v_0 \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$. This problem is similar to the *first passage percolation* problem in the *random geometric graph* [16] and related problems have been studied in homogeneous networks ([7], [8], [12]). In [7], [8], [12], the authors investigate the delay incurred by the topology dynamics in homogeneous networks, where nodes or links have independent or degree-dependent dynamic behaviors, which can be modeled by a *dynamic site percolation* on *random geometric graph* $G(\mathcal{H}_\lambda, 1)$. Particularly, given a link $v_i v_j \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$ and a packet b sent by some node $u \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$, denote $\mathcal{T}_{ij}$ as the time needed for the node v_j to receive b from v_i, after the latter has received b. And the delay $\mathcal{T}(u, u)$ between any nodes $u, v \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$ can be coupled as the *first passage time*, i.e., $\mathcal{T}(u,u) \triangleq inf_{l(u,v)}\{\sum_{v_i v_j \in l(u,v)} \mathcal{T}_{i,j}\}$, where $l(u, v)$ is an arbitrary path from the node u to v. By using *subadditive ergodic theorem* [17], the main result about the delay $\mathcal{T}(u,u)$ in [7], [8], [12] is given as follows:

Lemma 4: The delay $\mathcal{T}(u,u)$ asymptotically scales with the Euclidean distance $d(u, v)$ between u, v, that is,

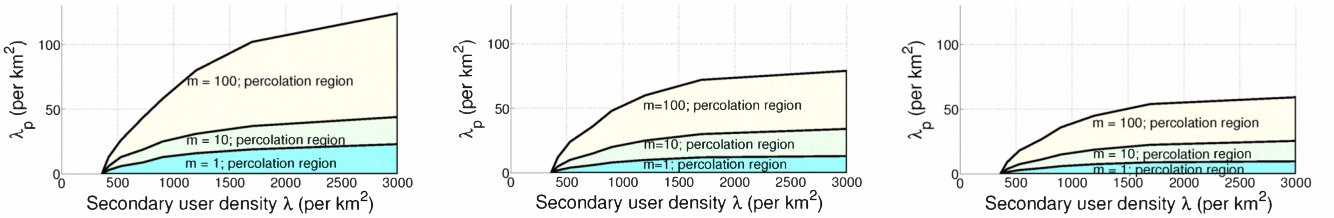
$$\mathbb{P}\Big(\lim_{d(u,v)\to\infty} \frac{d(u,v)}{\mathcal{T}(u,u)} = \delta\Big) = 1,$$

for some constant $0 < \delta < \infty$.

Next, we are interested in whether we can obtain similar results to Lemma 4 in CRNs. We find that the fundamental requirement for proof of Lemma 4 in [7], [8], [12] is that the expected time $E(\mathcal{T}_{i,j}) < \infty$. Note that if we only focus on secondary users, a CRN can be represented as a dynamic homogeneous network, where the impact of primary users are modeled as link or node dynamics. The difference between CRNs and homogeneous networks studied in [7], [8], [12] is that the network dynamics of the latter is dependent on the homogeneous nodes themselves and is assumed by authors in [7], [8], [12] to satisfy the constraint $E(\mathcal{T}_{i,j}) < \infty$. However, the dynamics in the CRNs depends on the traffic and density of coexisting primary users and thus the assumption $E(\mathcal{T}_{i,j}) < \infty$ may not hold, which prevent us from applying Lemma 4 to CRNs directly. If we can show that $E(\mathcal{T}_{i,j}) < \infty$ in CRNs, Lemma 4 also holds in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ by the same proof in [7], [8], [12]. The next lemma prove this.

Lemma 5: Given any secondary users v_i and v_j in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ with the link $v_i v_j \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$, denote $\mathcal{T}_{i,j}$ as the time needed by v_j to receive the packet b from v_i directly after v_i has received b. We have $\mathbb{P}(\mathcal{T}_{i,j} < \infty) = 1$ and $E(\mathcal{T}_{i,j}) < \infty$.

Proof: Assume that the node v_i receives the packet at time 0 for simplicity. Thus, $\mathcal{T}_{i,j} = 0$ when at time $t = 0$, $\|X_i - X_j\| < \min\{r_i, r_j\}$ (recall that X_i, X_j are locations of v_i, v_j) and at least one channel can be used by nodes v_i and v_j (with probability $\mathbb{P}_s(d)$, see Section II-A). Denote $Q = \min\{r_i, r_j\}$ and $Y = \|X_i - X_j\|$. It is easy to show that their distributions $\mathbb{P}(Q < q) = 1 - (1 - F_r(q))^2$ for $q \leq 1$ and $\mathbb{P}(Y \leq y) = \frac{\pi y^2}{\pi} = y^2$ for $y \leq 1$. Thus $\mathbb{P}(Y < Q) = \int_0^1 \int_0^Q d\mathbb{P}(Y <



(a) Percolation regions; $R_I = 100$ $(meters)$ (b) Percolation regions; $R_I = 120$ $(meters)$ (c) Percolation regions; $R_I = 200$ $(meters)$

Fig. 4. Percolation regions under different interference range R_I.

$y)d\mathbb{P}(Q < q)$ and $\varepsilon = \mathbb{P}(\mathcal{T}_{i,j} = 0) = \mathbb{P}(Y < Q)\mathbb{P}_s(d)$. Now consider $\mathbb{P}(\mathcal{T}_{i,j} > t)$. Recall that time has been slotted into units (see Section II-A) and at each time slot, the packet b is successfully delivered with probability ε independently, i.e., $\mathbb{P}(\mathcal{T}_{i,j} > t) \leq (1-\varepsilon)^t < e^{-t\varepsilon}$. Therefore, $\mathbb{P}(\mathcal{T}_{i,j} < \infty) = 1$ and $E(\mathcal{T}_{i,j}) = \int_0^\infty \mathbb{P}(\mathcal{T}_{i,j} > t)dt < \infty$. ■

B. Asymptotic Information Propagation Speed

In Section IV-A, we have shown that when the node density $\lambda_c < \lambda < \lambda_{c,c}$ and $v_0 \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$, Lemma 4 holds in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$, which indicates that the asymptotic propagation speed is a constant. This completes the proof of Part (i) of Theorem 3. We next consider the node density $\lambda > \lambda_{c,c}$. If $v_0 \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M}))$, information can be disseminated to the entire network through the giant component $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M}))$ immediately, considering the propagation delay of each link is negligible (see [7], [8], [12]). If $v_0 \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1)) \backslash \mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M}))$, let ψ be the first time that some node of $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M}))$ receives this packet. Note that at $t = \psi$, the packet can be disseminated to all nodes of $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M}))$. Therefore, to prove part (ii) of Theorem 3, we only need to show that ψ is finite with probability 1.

Proof of Part (ii) of Theorem 3: Let $\{Z_k\}_{k\geq 1}$ denote the i.i.d random variables which have the same distribution with $\mathcal{T}_{i,j}$. Thus along the infinite path in $G(\mathcal{H}_\lambda, 1)$, b can be spread to at least $N = \sup_n\{\sum_{k=1}^n Z_k < t\}$ nodes within time t. At any time point, each node $v \in \mathbb{C}_\infty(G(\mathcal{H}_\lambda, 1))$ has the same probability $\theta > 0$ belonging to $\mathbb{C}_\infty(G(\mathcal{H}_\lambda, F_r, \mathcal{M}))$. Thus we have $\mathbb{P}(\psi > t) < (1-\theta)^N$. Considering Z_k is bounded with probability 1, N approaches to infinity as t increases. Therefore, ψ is finite with probability 1. ■

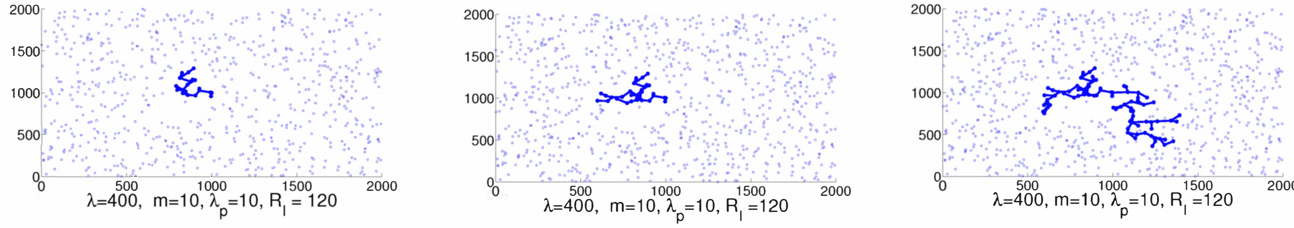
Finally we study the spatial density $\lambda < \lambda_c$.

Proof of Part (iii) of Theorem 3: When $\lambda < \lambda_c$, information cannot be disseminated to the node farther than $\|\mathcal{L}(\infty)\|$ and $\mathcal{S}_\varphi(d)$ is defined as 0 when $d > \|\mathcal{L}(\infty)\|$. Hence $\mathcal{S}_\varphi(\infty) = \lim_{d\to\infty}(\mathcal{S}_\varphi(d)) = 0$ almost surely considering $\mathbb{P}(\|\mathcal{L}(\infty)\| < \infty) = 1$. This completes the proof. ■

V. Simulation Studies

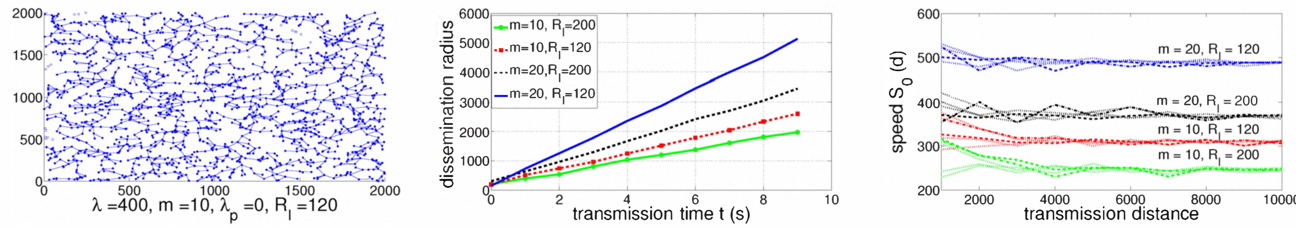
We have performed a series of simulations to validate the theoretical results concerning percolation conditions, dissemination radius and information propagation speed in large multi-channel CRNs. In the simulation, secondary users are distributed independently and uniformly with density λ and on each channel, primary users are distributed uniformly and independently with density $\frac{\lambda_p}{\eta}$. Time is slotted into units and at each time slot, any primary user is *active* with probability η independently. This deployment ensures that *active* primary users are distributed as a Poisson Point process with density λ_p at any time slot by *thinning theorem* [9]. The transmission range of each secondary user is randomly generated according to the same distribution $Uniform(60, 100)$ (meters).

To illustrate the impact of primary users on the information dissemination among secondary users, simulations on the percolation of the CRNs under variant interference ranges R_I and spatial density λ_p of primary users are shown in Fig. 4. Particularly, Fig. 4 illustrates percolation conditions of the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ in terms of spatial density λ and λ_p and the number of channels m. As shown in Fig. 4, there exist some bounds on λ_p for any R_I and m, above which the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ cannot percolate, regardless of how large λ is. This is because when the density of primary users is large, i.e., the primary traffic is high, secondary users cannot gain enough spectrum opportunity for communication and thus percolation. Moreover, Fig. 4 shows that the percolation region will increase, i.e., the critical denstiy λ_{cc} of secondary users for percolation will decrease, as the number of channel m increases or R_I decreases. Note that the probability that some channel *available* for secondary users within neighboring regions $\mathbb{P}_a = 1 - \left(1 - e^{-\lambda_p \alpha}\right)^m$ increases as m increases or R_I decreases. This agrees with our theoretical results in Theorem 1 that both the upper bound (Eq. (1)) and the lower bound (Eq. (2)) on $\lambda_{c,c}$ decreases as $\mathbb{P}_a$ increases. Simulations of how information is disseminated among secondary users, when $\lambda_c < \lambda < \lambda_{c,c}$, are shown in Fig. 5. To be specific, given $m = 10$, $\lambda_p = 10$ (per km^2) and $R_I = 120$ (meters), the node density $\lambda = 400$ (per km^2) is less than the lower bound of critical density $\lambda_{c,c}$ (Eq. (2) in Section III-B), thus the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ is not percolated. On the other hand, $\lambda = 400$ (per km^2) is larger than the critical density λ_c (see Fig. 4), thus as shown in Fig. 5(a), 5(b) and 5(c), information can be disseminated to the entire network step by step, which validates our analysis in Theorem 3. To compare information dissemination in the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$ and its homogeneous network counterpart (i.e., $\lambda_p = 0$), information dissemination in the latter has been shown in Fig. 6(a), which illustrates that information can be disseminated to the entire network instantaneously (with negligible propagation delay on each link). This verifies our theoretical analysis in Section III



(a) t = 0s (b) t = 1s (c) t = 2s

Fig. 5. Snapshots: a packet is disseminated to the entire network step by step when $\lambda_c < \lambda < \lambda_{c,c}$.



(a) t = 0s, homogeneous network (b) Dissemination radius $\mathcal{L}(t)$ (c) Propagation speed $\mathcal{S}_\varphi(d)$

Fig. 6. Dissemination radius and propagation speed analysis.

and IV that although the existence of primary users in CRNs will not influence the *condition* (i.e., $\lambda > \lambda_c$), it will impact the *speed*, of information to percolate to the entire network, when the density of secondary users is not too large (i.e., $\lambda < \lambda_{c,c}$).

The average dissemination radius $\mathcal{L}(t)$ based on 100 independent simulations with $\lambda = 400$ (per km^2) is shown in Fig. 6(b). Note that the dissemination radius $\mathcal{L}_t$ approaches to infinity as time goes on under variant R_I and m, which validates our theoretical result in Section III-C that information can be disseminated to the entire network given $\lambda > \lambda_c$. It can also be observed that the average dissemination radius scales approximately linearly with time t. Five independent simulations of propagation speed $\mathcal{S}(d)$ are shown in Fig. 6(c), which shows that the speed $\mathcal{S}(d)$ approaches to some constant as the transmission distance d increases. Fig. 6(b) and 6(c) validate our theoretical analysis in Theorem 3 that when $\lambda_c < \lambda < \lambda_{c,c}$, information can be disseminated to the entire network at some constant speed. In addition, Fig. 6(b) and 6(c) show that information will be disseminated faster as the number of channel m increases or the interference range R_I of primary users decreases. This is because, as mentioned above, the increase of m and the decrease of R_I can provide more spectrum opportunities for secondary users.

VI. Conclusions

In this paper we have studied the information dissemination radius and propagation speed to understand how far and how fast messages are transmitted in large multi-channel cognitive radio networks. We have identified the sufficient and necessary conditions under which information can and cannot be disseminated to the entire network, depending on the spatial densities of primary and secondary users and the number of channels. When information can reach an *infinite* area, we find that the propagation speed is no lower than a constant κ. When information cannot percolate to the entire network, our analysis shows that the farthest information dissemination distance is statistically dominated by an exponential distribution and information cannot reach most destinations in the network. Our results are validated through extensive simulations.

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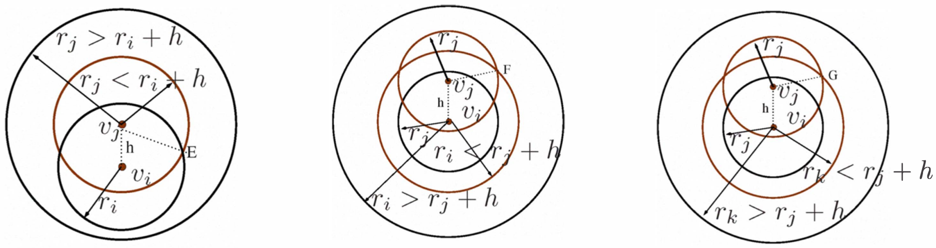

(a) Case I: $r_i \leq r_j \leq r_k$ (b) Case II: $r_j < r_i < r_k$ (c) Case III: $r_j < r_k < r_i$

Fig. 7. Link Correlation Coefficient $\mathcal{C}_{LCC}$ Calculation.

Appendix

A. The Area Of $\mathcal{B}_{s_i} \cap \mathcal{B}_{s_j}$: $\alpha = \|\mathcal{B}_{s_i} \cap \mathcal{B}_{s_j}\|$

From the elementary geometry, we have $\alpha = 2\pi\varpi^2 - 2\arccos(\frac{\|e\|}{2\varpi})\varpi^2 + \varpi^2 \sin(2\arccos\frac{\|e\|}{2\varpi})$ with $\varpi = R_I + \|e\|$.

B. A Sufficient Bond Open Probability $\mathbb{P}_c$ for Dependent Percolation Models on The Triangular Lattice

Denote $\mathcal{D}$ and $\mathcal{D}_d$ as a triangular lattice and its dual. The derivation of the sufficient condition for percolation on $\mathcal{D}$ is based on the observation that the origin of $\mathcal{D}$ belongs to an infinite open cluster if and only if it lies in the interior of no closed circuit of $\mathcal{D}_d$ (see [16], [18] for details). Let $\rho(\iota)$ be the number of circuits of length ι of $\mathcal{D}_d$ and $\rho(\iota) \leq 3\iota 2^{\iota-2}$, which follows from the fact that any circuit of length ι C_ι surrounding the origin contains a path of length $\iota - 1$ starting from some point $(k\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{6}, 1/2)$ for $0 \leq k < \iota$. Let q be the probability of edge being closed and $\Phi(\iota)$ be the probability that C_ι is closed. Then $\sum_m \mathbb{P}(C_\iota \text{is closed}) \leq \sum_{\iota=1}^{\infty} \rho(\iota)\Phi(\iota)$. For independent bond models, $\Phi(\iota) = q^\iota$.

For our model, the bonds within a circle with radius $2(R_I + \|e\|)$ are dependent by definition. Note that the area of each hexagon is $\frac{\sqrt{3}}{2}\|e\|^2$. Thus ignoring the edge effect, each circle contains at most $\Upsilon = \lceil \frac{4\pi(R_I+\|e\|)^2}{\frac{\sqrt{3}}{2}\|e\|^2} \rceil$ hexagons and thus approximately 3Υ bonds. Thus, if we partition C_ι with such circles, there exist at least $\theta = \lfloor \frac{\iota}{3\Upsilon} \rfloor$ independent bonds and thus $\Phi(\iota) < q^\theta$. Thus $\sum_m \mathbb{P}(C_\iota \text{is closed})$ converges if $q < \frac{1}{2}^{3\Upsilon}$, which implies that $\mathbb{P}_c = 1 - \frac{1}{2}^{3\Upsilon}$ is enough for percolation.

C. $\mathcal{C}_{LCC}$ of $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$

To determine the *Link Correlation Coefficient* $\mathcal{C}_{LCC}$ of the CRN $G(\mathcal{H}_\lambda, F_r, \mathcal{M})$, i.e., the conditional probability that the link $v_j v_k$ exists given that both links $v_i v_k$ and $v_i v_j$ exist, we consider three possible cases: $r_i \leq r_j \leq r_k$, $r_j \leq r_i \leq r_k$ and $r_j \leq r_k \leq r_i$, where r_i, r_j and r_k are the transmission ranges of v_i, v_j and v_k respectively (see Fig. 7). Denote $h = \|v_i - v_j\|$ as the Euclidean distance between v_i and v_j and $\mathcal{B}_{f,g}$ as the circle centering at node v_f with radius r_g for $f,\ g = i,\ j,\ k$ and $\|\mathcal{B}_{f,g}\|$ denote its area. Denote $\mathbb{P}_{sd}$ as the conditional probability that some channel is *available* within $\mathcal{B}_{j,j} \cap \mathcal{B}_{k,k}$ given some channel is *available* within $\mathcal{B}_{i,i} \cap \mathcal{B}_{k,k}$ and $\mathcal{B}_{j,j} \cap \mathcal{B}_{i,i}$.

For Case I (see Fig. 7(a)), the conditional probability is $\mathbb{P}_{sd}\frac{\|\mathcal{B}_{j,j} \cap \mathcal{B}_{i,i}\|}{\|\mathcal{B}_{i,i}\|}$. Thus if $r_j > r_i + h$, the conditional probability is $\mathbb{P}_{sd}$. If $r_j < r_i + h$, this probability is $b_1(h) = \mathbb{P}_{sd}[\pi(r_i - h)^2 + \int_{r_i-h}^{r_j} 2\theta_1 x dx]/[\pi r_i^2]$, where $\theta_1 = \angle v_i v_j E = cos^{-1}(\frac{x^2+h^2-r_i^2}{2xh})$. Let $f_{i,j,k} = f_r(r_i)f_r(r_j)f_r(r_k)$ be the joint distribution of r_i, r_j and r_k. Thus the conditional probability $\mathbb{P}_1$ for Case I is:

$$\mathbb{P}_1 = \int_0^{1/2}\int_h^{1-h}\int_{r_i+h}^{1}\int_{r_j}^{1} 2\mathbb{P}_{sd} h f_{i,j,k} dr_k dr_j dr_i dh + \int_0^1\int_h^1\int_{r_i}^{r_i+h}\int_{r_j}^{1} 2\mathbb{P}_{sd} b_1(h) h f_{i,j,k} dr_k dr_j dr_i dh.$$

For Case II (see Fig. 7(b)), the conditional probability is $\mathbb{P}_{sd}\frac{\|\mathcal{B}_{j,j} \cap \mathcal{B}_{i,i}\|}{\|\mathcal{B}_{i,i}\|}$. Thus if $r_i > r_j + h$, this probability is $\mathbb{P}_{sd}\frac{r_j^2}{r_i^2}$. If $r_i < r_j + h$, this probability is $b_2(h) = \mathbb{P}_{sd}\frac{\pi(r_i-h)^2 + \int_{r_i-h}^{r_j} 2\theta_2 x dx}{\pi r_i^2}$, with $\theta_2 = \angle v_i v_j F = cos^{-1}(\frac{x^2+h^2-r_i^2}{2xh})$. Hence for Case II, the conditional probability $\mathbb{P}_2$ is

$$\mathbb{P}_2 = \int_0^{1/2}\int_h^{1}\int_{r_j+h}^{1}\int_{r_i}^{1} 2h\mathbb{P}_{sd}\frac{r_j^2}{r_i^2} f_{i,j,k} dr_j dr_i dr_k dh + \int_0^1\int_h^1\int_{r_j+h}^{1}\int_{r_i}^{1} 2h b_2(h) f_{i,j,k} dr_k dr_i dr_j dh.$$

For Case III, the conditional probability is $\mathbb{P}_s\frac{\|\mathcal{B}_{j,j} \cap \mathcal{B}_{i,k}\|}{\|\mathcal{B}_{i,k}\|}$. Thus if $r_k > r_j + h$, this probability is $\mathbb{P}_s\frac{r_j^2}{r_k^2}$. If $r_k < r_j + h$, this probability is $b_3(h) = \mathbb{P}_s\frac{\pi(r_k-h)^2 + \int_{r_k-h}^{r_j} 2\theta_3 x dx}{\pi r_i^2}$, with $\theta_3 = \angle v_i v_j G = cos^{-1}(\frac{x^2+h^2-r_k^2}{2xh})$. Hence for Case II, the conditional probability $\mathbb{P}_3$ is

$$\mathbb{P}_3 = \int_0^{1/2}\int_h^{1}\int_{r_j+h}^{1}\int_{r_k}^{1} 2h\mathbb{P}_s\frac{r_j^2}{r_k^2} f_{i,j,k} dr_i dr_k dr_j dh + \int_0^1\int_h^1\int_{r_j+h}^{1}\int_{r_k}^{1} 2h b_3(h) f_{i,j,k} dr_i dr_k dr_j dh.$$

Finally, by the symmetry of v_j and v_k, we have $\mathcal{C}_{LCC} = 2(\mathbb{P}_1 + \mathbb{P}_2 + \mathbb{P}_3)$. Note that the calculation of the conditional probability $\mathbb{P}_{sd}$ that some channel is *available* for v_j and v_k is very complicated and thus make $\mathcal{C}_{LCC}$ intractable. Since we only need a lower bound of $\mathcal{C}_{LCC}$ to determine $\lambda_{c,c}$ in Eq. (2), we can use the unconditional probability $\mathbb{P}_s(d)$ (see Section II-A) that some channel is *available* for for v_j and v_k to replace $\mathbb{P}_{sd}$ as approximation.