Horizon on the Move: Geocast in Intermittently Connected Vehicular Ad Hoc Networks

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***Abstract*—Vehicular ad hoc network (VANET) is one of the most promising large-scale applications of mobile ad hoc networks. VANET applications are rooted in advanced understanding of communication networks because both control messages and data information need to be disseminated in geographic regions (i.e., *Geocast*). The challenges come from highly dynamic environments in VANET. Destination nodes in geocast are *dynamic* over time due to vehicle mobility, which undermines existing results on dissemination latency and information propagation speed with *pre-determined* destinations. Moreover, the affected area by the dissemination, which is referred to as *horizon of message (HOM)*, is critical in geocast as it determines the latency for the message reaching nodes inside the area of interest (AOI), in which the message is relevant to drivers. Therefore, we characterize the HOM in geocast by how far the message can reach within time t (referred as *dissemination distance*) and how long the message takes to inform nodes at certain locations (referred as *hitting time*). Analytic bounds of dissemination distance and hitting time are derived under four types of dissemination mechanisms, which provide insights into the *spatial and temporal limits* of HOM as well as how the numbers of disseminators and geographic information exchanges affect them. Applying analytic and simulation results to two real applications, we observe that geocast with AOI near the source or high reliability requirement should recruit multiple disseminators while geocast with AOI far from the source need to utilize geographic information for fast message propagation.**

I. Introduction

VANET is one of the most promising large-scale applications of mobile ad hoc networks (MANETs). VANET has emerged to facilitate the design of *intelligent transportation systems* (ITS) and Dedicated Short Range Communications (DSRC) service [1] in order to improve road safety, traffic efficiency, and driving convenience. The research on VANETs focuses on applications [2] including prevention of collision [3], real-time detour routes computation, and Internet downloading. Many VANET applications are heavily dependent on the message dissemination in specific geographic regions, which is referred to as *geocast*. For example, an emergency vehicle can disseminate traffic signal preemption message to cars in the area around the intersection (i.e., AOI). Geocast restricts destinations only to vehicles in AOI, which not only helps to avoid the broadcast storm problem [4] but also enables the coexistence of multiple VANET applications.

At the same time, geocast introduces challenges to system design and evaluation, even performance metrics of VANETs. Due to high vehicle mobility and limited transmission range, network can only maintain *intermittent connectivity*, which makes achieving satisfactory performance difficult especially for time critical message dissemination. Moreover, because vehicles can move into and out from AOI, geocast in VANET has *dynamic* destinations (see Fig. 1). Such a *dynamic* group of destinations is different from message dissemination in traditional MANETs, which specifies destinations prior to transmissions. Papers [5, 6] studied the upper bound on information propagation speed in large static wireless networks using broadcast and unicast, which may no longer be valid in VANET geocast. Existing work on dissemination in VANET, such as papers [7, 8], has studied the delivery ratio and delay of dissemination in 1-D (dimensional) static or highway scenario. There still lacks understanding of geocast in realistic vehicular network scenario. Therefore, it motivates us to study geocast in 2-D network with realistic vehicle mobility.

Since geocast targets vehicles in AOI, the location of message dissemination (i.e., HOM) becomes critical to geocast performance. Therefore, we characterize HOM by *how far the dissemination can reach within time t and how long the dissemination takes to inform partial or all nodes located in an area*. Answers to these questions can provide understanding of the spatial and temporal limits of HOM. The results can also yield insights in how the application requirements (such as allowable latency) could be possibly satisfied, thus help network designers choose appropriate dissemination strategy according to the requirements and feasibility of applications.

To proceed, we denote by *disseminators* nodes that are chosen to rebroadcast messages. Messages that are carried by disseminators are referred to as *active messages*. Recipients other than disseminators do not contribute to dissemination. Therefore, only disseminators with active messages will be used to find out the answers for our research questions (i.e., spatial and temporal limits of geocast). We examine the mobility of *active message*, which includes movements of disseminators and jumps incurred by transmissions among disseminators. Message mobility focuses on movements of active message rather than relay nodes on information propagation path, thus can shroud the dynamic destination nodes in geocast and manifest *on the move-HOM due to node mobility and changes of disseminators*.

Based on *active message mobility*, we derive lower and upper bounds for the farthest distance that messages reach at time t (denoted as *dissemination distance* $|D(t)|$) and the

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first time that messages reach distance d from the source location (denoted as *hitting time* $\tau(d)$). Simulation results show that several well known dissemination algorithms, including stateless opportunistic forwarding (SOF) [9] and GPS-based broadcasting (GBB) [10], are well bounded by our analytic bounds. Both analysis and simulation demonstrate that the upper bounds on expectation of $|D(t)|$ increase with the *square root* of t or *linearly* with t depending on how disseminators are chosen. Furthermore, we apply the analytical and simulation results to two real scenarios in VANETs, i.e., *post-crash warning* and *emergency vehicle signal preemption*. We observe that for geocast applications with AOI near the source (such as post-crash warning), dissemination algorithms with multiple disseminators are suitable, while for geocast applications with AOI far from the source (such as emergency vehicle signal preemption), dissemination strategies that utilize geographic information to choose disseminators are preferable.

The remainder of this paper is structured as follows. In Section II we introduce our models and dissemination strategies, and formulate the problem. Lower and upper bounds on dissemination distance and hitting time are derived in Sections III, IV and V. In Section VI, we validate our analytic bounds using simulation results of four dissemination algorithms, and provide guidelines for choosing appropriate dissemination methods. We conclude this paper with Section VII.

II. Network Models and Dissemination Strategies

In this section, we introduce our network and mobility models and dissemination strategies, and formally define dissemination distance and hitting time.

A. Network and Mobility Model

Assume that at time 0, n nodes $\{\mathcal{X}(0)\} = \{X_1(0), \ldots, X_n(0)\}$ are uniformly distributed at random in a 2-D torus $\mathcal{B} = [0, B]^2$ where $B = \sqrt{n/\lambda}$, for some $\lambda > 0$, and $X_i(0)$ denotes the location of node v_i. By definition [11], $\{\mathcal{X}(0)\}$ is a homogeneous Poisson point process. n nodes are Poisson distributed in the network with density $\lambda = n/B^2$ everywhere. In [12], Xue and Kumar have shown that if the average number of neighbors is smaller than $0.074 \log n$, then the network is almost surely disconnected when n is large. In order to study the properties of intermittently connected VANETs, we further assume that λ is small. Two nodes v_i and v_j can communicate with each other if only if their distance is less or equal to transmission range r, i.e., $||X_i(t) - X_j(t)|| \leq r$. Suppose that time is slotted and each node moves according to the following mobility model.

Definition 1. (Generic Mobility) Given nodes' initial positions $\mathcal{X}(0)$, the spatial distribution $X_i(t)$ of node v_i at time t is around a point x_i^* by a non-increasing and direction-invariant function $\Psi_i(x) = \Psi(x - x_i^*)$. Assume that Ψ_i is non-zero in and only in a region characterized by a constant a; that is, $\Psi_i(x) = \Psi(x - x_i^*) > 0$ when $||x - x_i^*|| < a$ and $\Psi_i(x) = \Psi(x - x_i^*) = 0$ otherwise.

To mimic vehicle mobility, we consider the *constrained vehicle mobility* model by setting $a < \infty$ and $x_i^* = X_i(t-1)$ at time slot t. In this case, $X_i(t)$ is uniformly distributed at random in $\mathcal{A}(X_i(t-1), a)$-the circular region centered at $X_i(t-1)$ with radius $a > 0$, and the positions $X_i(t)$ are mutually independent among all nodes. This reflects that vehicle mobility is constrained by the speed limit and dependent on previous movements. Movement vector of node v_i at time t is denoted by $Y_M(t)$ with origin at $X_i(t)$ and endpoint at $X_i(t+1)$, and $Y_M(t)$ $(t = 1, 2 \ldots)$ are i.i.d. random variables.

Remark 1. We use the *constrained vehicle mobility* model because (i) parameter a (referred as *mobility radius*) reflects the speed limit of vehicles that nodes can jump to adjacent locations with pre-assigned probabilities and each movement step is limited in a circular region around previous location, (ii) it generally accounts for a wide range of realistic mobility processes in vehicular scenarios, including Manhattan mobility [13] and random walk, and (iii) n nodes are Poisson distributed in the network $\mathcal{B}$ with density λ everywhere at all times (proof is omitted due to space limit), thus it is an ergodic and stationary mobility process.

Note that the mobility model in Definition 1 is very general because it covers a wide range of possible scenarios of realistic mobility processes. The case of static nodes uniformly deployed over network area can be obtained by setting $\Psi_i(x) = \delta(x - X_i(0))$. The i.i.d. mobility model in [14] corresponds to the case when $\Psi(x)$ is a constant function independent of x and $a = \infty$. When $a < \infty$ and $x_i^* = X_i(0)$, we obtain the constrained i.i.d. mobility model used in [15]. Hence, our analysis based on the constrained vehicle mobility can be extended to other scenarios.

B. Dissemination Strategies

Dissemination performance, such as information propagation speed and dissemination latency, depends on how many nodes are recruited to disseminate the message (i.e., *number of disseminators*) and *how the disseminators are chosen*. On one hand, full epidemic broadcast achieves high delivery ratio by using as many disseminators as possible, but leads to network congestion. Hence, limited number of disseminators is more feasible in order to save network resources and enable the coexistence of multiple applications. On the other hand, if geographic information can be used for choosing disseminators, it has the potential to speed up information propagation. But geographic information may not be available for all vehicles in the network and exchanging geographic information consumes the already limited network resources. Since the number of disseminators and whether geographic information is used in disseminator selection affect HOM, we classify dissemination strategies according to these two factors, based on which we study geocast performance.

Definition 2. (1-Copy Message Dissemination) Assume that node v_0 initiates a message dissemination and there is only *one disseminator at all times*. Disseminator is selected based on criteria imposed by applications. The disseminator rebroadcasts the message until it finds a succeeding disseminator. This process repeats until the dissemination completes.

1-copy message dissemination is particularly useful in the following situations: 1) network has limited capacity; 2) network load is heavy; 3) nodes are computationally-constrained or energy-constrained devices. In these situations, the network could only support one disseminator in order to save network resources and enable the coexistence of multiple applications.

Definition 3. (L-Copy Message Dissemination) Assume that node v_0 initiates a message dissemination at $t = 0$. First, the message will be spread to L *distinct disseminators*. Then, each disseminator independently disseminates the message according to 1-copy message dissemination in Definition 2.

Note that when $L = n$, this strategy becomes epidemic routing. As multiple disseminators actively rebroadcast the message at the same time, L-copy message dissemination can increase the message propagation speed and enhance the delivery ratio (i.e., dissemination reliability). Thus L-copy message dissemination could be favorable for time critical message dissemination of safety applications in VANETs.

Remark 2. For dissemination strategies that use geographic information to assist disseminator selection such as to enhance dissemination speed, we refer them as *1-copy* and *L-copy geographic-assisted dissemination*; otherwise, for dissemination strategies in which a disseminator chooses its next-hop disseminator isotropically (equally in all directions), we refer them as *1-copy* and *L-copy direction-invariant dissemination*.

C. Problem Formulation

In geocast, HOM is the area in which contains all disseminators and nodes (the green nodes in Fig. 1) are at least partially informed (see Fig. 1). HOM is dynamic due to mobility and changes of disseminators. HOM is closely related to dissemination performance (latency and propagation speed). Hence, we characterize spatial and temporal limits of HOM by how far the disseminators can reach by time t (dissemination distance) and how long the disseminators take to spread the message to certain location (hitting time).

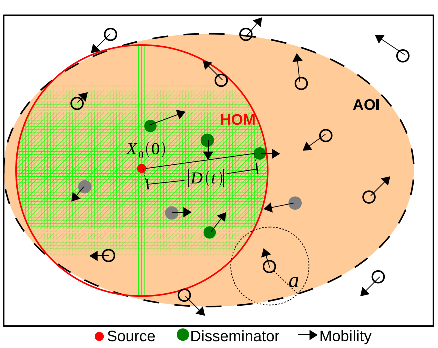

Fig. 1. In geocast, AOI is an area in which the message is relevant to drivers; HOM is a circular region with source at the center and $|D(t)|$ as diameter.

In order to derive performance limits of geocast, we do not consider the effects of buffering or congestion, and assume that a message can be transmitted instantaneously between two nodes in range (i.e., omit the transmission delay). Under these assumptions, we are able to derive spatial and temporal bounds of geocast since they correspond to an ideal scenario with that respect. Actually, previous assumptions have little impact on the accuracy of our results because information transmission occurs much faster than the speed of the mobile nodes and propagation delay is much smaller than the dissemination latency incurred by dynamic topology and intermittent connectivity in VANETs.

1) Dissemination Distance: Denote by $\mathcal{V}(t)$ the set of disseminators at time t and $|\mathcal{V}(t)| \leq L$. Let us place a Cartesian coordinate system in the network with its origin at the source location. The *dissemination vector* $D(t)$ is the vector from source point $X_0(0)$ to the location of the farthest disseminator at time t. The length of dissemination vector is called *Dissemination Distance*, which is defined as

$$|D(t)| \triangleq \max\{v_k \in \mathcal{V}(t) : ||X_k(t) - X_0(0)||\}. \qquad (1)$$

To avoid specifying the relay nodes, we study $D(t)$ through the mobility of active messages. In 1-copy message dissemination, denote by $Y(t) = D(t) - D(t-1)$ the progress of active message from time $t-1$ to t ($t = 1, 2, \ldots.$). Suppose v_i is the disseminator at time $t-1$, (i) if v_i is also the disseminator at time t, the active message moves with v_i, which means that $Y(t)$ equals the movement step $Y_M(t)$ of v_i; (ii) if node v_j ($i \neq j$) is selected as the next disseminator at time t, the active message jumps from v_i to v_j and moves with v_j, which means that $Y(t)$ equals the propagation vector $Y_P(t)$ plus the movement step $Y_M(t)$.

In L-copy message dissemination, we number each active message from 1 to L and denote by $|D_k(t)|$ the farthest distance reached by the k^{th} active message. Denote by $|D^L(t)|$ the dissemination distance, which is the maximum of $|D_i(t)|$ ($1 \leq i \leq L$). We assume that each active message is independently disseminated. Hence, the progress of the k^{th} active message $Y_k(t)$ has the same distribution as $Y(t)$.

Remark 3. For *direction-invariant* dissemination, $E(Y_P^x(t)) = E(Y_P^y(t)) = 0$, where $Y_P^x(t)$ and $Y_P^y(t)$ are x, y-components of $Y_P(t)$, respectively. For *geographic-assisted* dissemination that utilizes geographic information to increase dissemination distance $|D(t)|$, $E(Y_P^x(t)) \geq 0$ when the x-component of dissemination distance $D_x(t) \geq 0$ and $E(Y_P^x(t)) \leq 0$ when $D_x(t) < 0$ (the same for the y-component).

2) Hitting Time: In this paper, we define the hitting time as the first time that process $D(t)$ hits the region outside the circular region centered at $X_0(0)$ with radius $d > 0$, which is equivalent to the following definition:

$$\tau(d) \triangleq \inf_{t>0}\{t : |D(t)| \geq d\}. \qquad (2)$$

Hitting time is well studied in the mathematics literature, under a variety of contexts. Our interest in hitting time stems from the fact that it has a close connection with dissemination latency. For example, if $d = \max\{s \in AOI : ||s - X_0(0)||\}$, $\tau(d)$ becomes the first exit time when a disseminator stops message dissemination.

Dissemination distance and hitting time manifest the spatial and temporal limits of HOM, respectively. Dissemination distance reveals the size of HOM in which nodes are at least partially informed by the message. Hitting time uncovers the minimum latency of HOM reaching nodes at certain locations. Putting together, they can be used to determine whether the geocast has reached vehicles in the AOI and whether it can possibly satisfy the time requirements of time critical safety applications in VANETs. As different dissemination strategies beget different performance in dissemination distance and hitting time, we also expect our results to provide guidelines on choosing appropriate dissemination methods according to application requirements.

III. Lower bound on $|D(t)|$ and $\tau(d)$

Our objective is to characterize the dynamic HOM. We start by deriving a lower bound on dissemination distance and hitting time under a baseline message dissemination method that the source will be the only disseminator. Originally, the source initiate a geocast and $|D(0)| = 0$. Only the source node actively spreads the message while all recipients do not retransmit the message. Thus, the dissemination is solely determined by the mobility of the source node. In other words, $Y(k)$ equals $Y_M(k)$ and $D(t) = \sum_{k=1}^{t} Y_M(k)$.

A. Lower bound on dissemination distance

In order to find lower bound on $|D(t)|$, we need examine mobility vector $Y_M(t)$.

Lemma 1. *Under constrained vehicle mobility, $Y_M(t)$ satisfies that $E(|Y_M(t)|) = \frac{2a}{3}$ and $E\left\{|Y_M(t)|^2\right\} = \frac{a^2}{2}$, where $|Y_M(t)|$ is the length of $Y_M(t)$.*

Proof: (Sketch) Based on constrained vehicle mobility, it is not difficult to obtain the result in this lemma (proof details are omitted due to space limit). ■

Based on Lemma 1, we have

$$E(|D(t)|^2) = \sum_{k=1}^{t} E(|Y_M(k)|^2) = \frac{a^2 t}{2}. \tag{3}$$

Since better designed dissemination algorithm can spread out the message faster, $a^2t/2$ can serve as the lower bound of the mean square displacement (MSD) of dissemination distance.

B. Lower bound on hitting time

Probability distribution of hitting time $\tau(d)$ satisfies the following theorem.

Theorem 1. $P(\tau(d) < t) \geq \max\{0, 1 - \frac{4(d+a)^2}{a^2 t}\}$

Proof: Denote the x-component and y-component of dissemination distance vector $D(t)$ as $D_x(t)$ and $D_y(t)$, respectively. Then,

$$\tau(d) \leq \tau_x(d) \triangleq \inf_{t>0}\{t : |D_x(t)| \geq d\}.$$

And $D_x(t) = \sum_{k=1}^{t} Y_M^x(k)$, where $Y_M^x(k)$ is the x-component of $Y_M(k)$. Based on constrained vehicle mobility model,

$$E((Y_M^x(k))^2) = 2\int_0^a x^2 \frac{2\sqrt{a^2 - x^2}}{\pi a^2} dx = \frac{a^2}{4}.$$

Clearly, the probability distribution of $Y_M^x(k)$ is an even function, thus $E(Y_M^x(k)) = 0$. $D_x(t)$ is a generalized 1-D random walk with independent and mean-zero increments $Y_M^x(k)$. According to the definition of martingale [16], $D_x(t)$ is a martingale process with respect to $(Y_M^x(k), k \geq 0)$. Upon Wald's Second Inequality, the stopping time of $D_x(t)$

$$E(\tau_x(d)) = \frac{E(D_x^2(\tau_x(d)))}{E((Y_M^x(k))^2)} \leq \frac{4(d+a)^2}{a^2}.$$

Based on Markov inequality $P(\tau_x(d) < t) \geq 1 - \frac{E(\tau_x(d))}{t}$,

$$P(\tau(d) < t) \geq P(\tau_x(d) < t) \geq \max\{0, 1 - \frac{4(d+a)^2}{a^2 t}\}.$$

■

Eqs. (3) and Theorem 1 show that the lower bounds for dissemination distance and hitting time increase as mobility radius increases. Interestingly, *the higher mobility the vehicles have, the higher spatial and temporal limits of HOM are.*

IV. Upper bounds on 1-Copy Message Dissemination

It is easier to characterize the dynamic HOM with the *1-copy message dissemination strategy* defined in Definition 2.

A. 1-Copy Message Dissemination Distance

Originally, $D(0) = 0$. At time t, $D(t) = \sum_{k=1}^{t} Y(k)$, where $Y(k)$ equals either $Y_M(k)$ or $Y_M(k) + Y_P(k)$. We have studied movement vector $Y_M(k)$ in Lemma 1. Now we study propagation vector $Y_P(k)$ in Lemma 2.

Lemma 2. *Propagation vector $Y_P(t)$ satisfies that*

$$E(|Y_P(t)|) \leq r(1 - e^{-\lambda\pi r^2}),$$

$$E\left\{|Y_P(t)|^2\right\} \leq r^2 - \frac{1}{\lambda\pi}(1 - e^{-\lambda\pi r^2}),$$

where $|Y_P(t)|$ is the length of propagation vector $Y_P(t)$.

Proof: (Sketch) Clearly, $|Y_P(t)|$ is stochastically dominated by the farthest distance from a dissemination to its neighbors. As constrained vehicle mobility exhibits Poisson node distribution in the network at all times, we can prove this lemma. (Details are omitted due to space limit.) ■

Lemma 2 shows that the upper bound of $E(|Y_P(t)|)$ increases as r and λ increase, which provide higher probability for long transmissions between two disseminators.

Due to the high vehicle mobility and limited transmission range, VANETs often have intermittent connectivity. Active message may not be propagated on a path from source to destinations. The active message likely travels a journey in the network area through movements and transmissions of disseminators. Hence, dissemination distance $|D(t)|$ is determined not only by distributions of $Y_M(k)$ and $Y_P(k)$ but also by number of active message transmissions within time t. Denote by $Z(k)$ the event of active message transmission between two disseminators (i.e., $Y(k) = Y_M(k) + Y_P(k)$). Number of active message transmissions $\mathcal{N}(t) = \sum_{k=1}^{t} 1_{Z(k)}$. As a jump of an active message occurs when the preceding disseminator meets its succeeding disseminator, $\mathcal{N}(t)$ is determined by intermittent connectivity of VANETs as well as dissemination algorithms. Rather than limiting our study on specific algorithms, we focus on the impact of intermittent connectivity and derive an upper bound for $\mathcal{N}(t)$.

Lemma 3. *$\mathcal{N}(t)$ is stochastically dominated by Poisson process with parameter α and $E(\mathcal{N}(t)) \le \alpha t$, where α is a constant determined by mobility model and $\alpha = 1/2$ in 2-D constrained vehicle mobility.*

Proof: See Appendix. ■

Equipped with previous results on $Y_M(t)$, $Y_P(t)$ and $\mathcal{N}(t)$, we are ready to analyze dissemination distance for both *direction-invariant* dissemination and *geographic-assisted* dissemination that are presented in Remarks 2 and 3.

Theorem 2. *$E(|D(t)|)$ is upper bounded by $\sqrt{tf_1(r,\lambda,a,\alpha)}$ in 1-copy direction-invariant dissemination, by $\sqrt{tf_2(r,\lambda,a,\alpha,t)}$ in 1-copy geographic-assisted dissemination, where $f_1(r,\lambda,a,\alpha)$ and $f_2(r,\lambda,a,\alpha,t)$ are in Eqs. (4) and (5).*

Proof: Denote dissemination vector $D(t) = (D_x(t), D_y(t))$ by its x and y components.

(i) Direction-invariant dissemination

Because $E(Y_M^x(k)) = E(Y_M^y(k)) = 0$ and $E(Y_P^x(k)) = E(Y_P^y(k)) = 0$, and $Y_M(k)$ and $Y_P(k)$ are independent,

$$E\{D_x^2(t)\} = tE\left\{|Y_M^x(k)|^2\right\} + E(\mathcal{N}(t))E\left\{|Y_P^x(k)|^2\right\}.$$

Similar results can be obtained for $E\{D_y^2(t)\}$. Accordingly,

$$E(|D(t)|^2) = tE\left\{|Y_M(k)|^2\right\} + E(\mathcal{N}(t))E\left\{|Y_P(k)|^2\right\}.$$

Based on results in Lemmas 1, 2, and 3, we get

$$E^2(|D(t)|) \le E(|D(t)|^2) \le tf_1(r,\lambda,a,\alpha),$$

thus $E(|D(t)|) \le \sqrt{tf_1(r,\lambda,a,\alpha)}$, where

$$f_1(r,\lambda,a,\alpha) = a^2/2 + \alpha r^2 - \frac{\alpha}{\lambda\pi}(1 - e^{-\lambda\pi r^2}). \quad (4)$$

(ii) Geographic-assisted dissemination, which differs from direction-invariant dissemination by $E(Y_P^x(k)) = E(Y_P^y(k)) \ne 0$. Using the same methodology above, we have

$$E(|D(t)|^2) \le tE\left\{|Y_M(k)|^2\right\} + \left(E(\mathcal{N}(t)) + \frac{t(t-1)}{2}\right)E\left\{|Y_P(k)|^2\right\}.$$

In view of Lemmas 1, 2, and 3, we have

$$E^2(|D(t)|) \le E(|D(t)|^2) \le tf_2(r,\lambda,a,\alpha,t),$$

thus $E(|D(t)|) \le \sqrt{tf_2(r,\lambda,a,\alpha,t)}$, where

$$f_2(r,\lambda,a,\alpha,t) = \frac{a^2}{2} + (\alpha + (t-1)/2)\left(r^2 - \frac{1 - e^{-\lambda\pi r^2}}{\lambda\pi}\right). \quad (5)$$

■

Theorem 2 shows that the upper bound of $E(|D(t)|)$ depends on node *velocity* (indicated by maximum movement step a per time slot), *mobility model* (represented by α), node *transmission range* r, and node *density* λ.

Remark 4. The expected dissemination distance can at most increase with the *square root* of t in *direction-invariant* dissemination while approximately *linearly* with t in *geographic-assisted* dissemination. Comparing Eqs. (4) and (5), we find that $f_2(r,\lambda,a,\alpha,t) - f_1(r,\lambda,a,\alpha)$ is a function of t, This means that comparing to direction-invariant dissemination, the increase in dissemination distance of utilizing geographic information accumulates as time goes by.

Furthermore, $\lim_{t\to\infty} f_2(r,\lambda,a,\alpha,t)/t$ converges to a constant (a function of λ and r). In other words, *geocast is upper bounded by a constant propagation speed*, which is consistent with existing results in [5, 6, 17].

B. 1-Copy Message Dissemination Hitting Time

After studying the spatial limit of HOM, we move to study the temporal limit of HOM. As dissemination distance vector $D(t) = \sum_{k=1}^{t} Y(k)$, i.e., sum of i.i.d. random variables, we use martingale theory to study the hitting time $\tau(d)$.

Lemma 4. *Dissemination distance $\{|D(t)|^2\}_{t\in\mathbb{N}}$ is a submartingale with respect to Filtration $\mathcal{F}_t$, which is the σ-algebra generated by $\{D(k); k \le t\}$ for every $t \in \mathbb{N}$.*

Proof: See Appendix. ■

Based on Lemma 4, we have the following theorem.

Theorem 3. *In a geocast, hitting time $\tau(d)$ satisfies,*
i) for direction-invariant dissemination,

$$P(\tau(d) \le t) \le \frac{t}{d^2} f_1(r,\lambda,a,\alpha); \quad (6)$$

ii) for geographic-assisted dissemination,

$$P(\tau(d) \le t) \le \frac{t}{d^2} f_2(r,\lambda,a,\alpha,t). \quad (7)$$

Proof: We proceed to find distribution of hitting time $\tau(d)$ using *Doob's Submartingale Maximal Inequality*, which is that for $(|D(k)|^2)_{k\in\mathbb{N}}$ being a non-negative sub-martingale with respect to a filtration $(\mathcal{F}_k)_{k\in\mathbb{N}}$, for any $d > 0$,

$$P(\max_{1\le k\le t}|D(k)|^2 \ge d^2) \le \frac{1}{d^2}E(|D(t)|^2). \quad (8)$$

Based on definitions of dissemination distance in Eq. (1) and hitting time in Eq. (2), we have $\{\max_{1\le k\le t}|D(k)|^2 \ge d^2\} = \{\tau(d) \le t\}$. Applying Doob's Submartingale Maximal Inequality to the submartingale $|D(t)|^2$, we have

$$P(\tau(d) \le t) \le \frac{1}{d^2}E(|D(t)|^2).$$

(i) For direction-invariant dissemination, the proof of Theorem 2 shows that $E[|D(t)|^2] \le tf_1(r,\lambda,a,\alpha)$. (ii) For geographic-assisted dissemination, the proof of Theorem 2 shows that $E(|D(t)|^2) \le tf_2(r,\lambda,a,\alpha,t)$. In view these two cases, we can prove Eq. (6) and (7), respectively. ■

Remark 5. The probability that a 1-copy message dissemination reaches nodes or infrastructures located distance d from the source within time t is upper bounded by a function proportional to $E(|D(t)|^2)$ and inversely proportional to d^2.

V. Upper Bounds on L-Copy Message Dissemination

Upon results on 1-copy message dissemination, we extend the HOM analysis to general L-copy message dissemination.

A. L-Copy Message Dissemination Distance

We find the following upper bound of L-copy message dissemination distance $D^L(t)$.

Theorem 4. *For L-copy direction-invariant dissemination, $E(|D^L(t)|) \leq (\sqrt{L-1}+1)\sqrt{tf_1(r,\lambda,a,\alpha)}$; for L-copy geographic-assisted dissemination, $E(|D^L(t)|) \leq (\sqrt{L-1}+1)\sqrt{tf_2(r,\lambda,a,\alpha,t)}$.*

Proof: To analyze $|D^L(t)|$, which equals the maximum of several random variables, we introduce Aven's upper bound on the mean of the maximum of a number of random variables $\{Z_i, 1 \leq i \leq L\}$ with general distributions (not necessarily independent and identically distributed) [18].

$$E(\max_{1\leq i\leq L} Z_i) \leq \max_{1\leq i\leq L} E(Z_i) + \sqrt{\frac{L-1}{L}\left(\sum_{i=1}^{L}\text{Var}(Z_i)\right)^{1/2}}. \quad (9)$$

Applying the above equation to $|D^L(t)|$, we have

$$E(|D^L(t)|) = E(\max_{v_i\in\mathcal{V}(t)}\{|D_i(t)|\})$$
$$\leq \max_{1\leq i\leq L} E(|D_i(t)|) + \sqrt{\frac{L-1}{L}\left(\sum_{i=1}^{L}E(|D_i(t)|^2)\right)^{1/2}}$$

Based on the results of Theorem 2, we completes the proof. ■

Remark 6. Theorem 4 shows that the upper bounds on the expected dissemination distances in L-copy message dissemination are $\sqrt{L-1}$ times larger than their corresponding 1-copy message dissemination distances in Theorem 2.

Regarding 1-copy direction-invariant dissemination as a base line, the upper bounds of dissemination distance increase $\sqrt{L-1}$ times under L-copy direction-invariant dissemination, increase approximately $\sqrt{t}$ times under 1-copy geographic-assisted dissemination. This means that multiple disseminators benefit dissemination at a constant rate while the benefit of geographic information tends to accumulate with time. This seems to suggest that *multiple disseminators should be recruited for geocast with AOI near the source, while geographic information should be used for geocast with AOI far from the the source*. We will further investigate this in Section VI.

B. L-Copy Message Dissemination Hitting Time

The hitting time $\tau^L(d)$, i.e., the first time that $|D^L(t)|^2 \geq d^2$, satisfies the following theorem.

Theorem 5. *For L-copy direction-invariant dissemination, $P(\tau^L(d) \leq t)$ is upper bounded by*

$$\frac{t}{d^2}\left(f_1(r,\lambda,a,\alpha) + \sqrt{L-1}(r+a)\sqrt{f_1(r,\lambda,a,\alpha)}\right);$$

for L-copy geographic-assisted dissemination, $P(\tau(d) \leq t)$ is upper bounded by

$$\frac{t}{d^2}\left(f_2(r,\lambda,a,\alpha,t) + \sqrt{L-1}(r+a)\sqrt{f_2(r,\lambda,a,\alpha,t)}\right).$$

Proof: According to L-copy message dissemination in Definition 3, the message is first spread to L distinct disseminators and then each of disseminators independently disseminates the message according to 1-copy message dissemination. Define $|D^{L^*}(t)|$ as the dissemination distance that L disseminators start to independently disseminate the message from $t=0$. As $|\mathcal{V}(t)| \leq L$, $|D^L(t)|^2 = \max_{i\in\mathcal{V}(t)}\{|D_i(t)|^2\} \leq |D^{L^*}(t)|^2 = \max_{i=1}^{L}\{|D_i^*(t)|^2\}$. Thus,

$$\tau^L(d) = \inf_{t\geq 1}\{|D^L(t)| \geq d\} \geq \tau^{L^*}(d) = \inf_{t\geq 1}\{|D^{L^*}(t)| \geq d\} \quad (10)$$

Upon Lemma 4, $\{|D_i^*(t)|^2, 1 \leq i \leq L\}$ are independent sub-martingales. Hence, $|D^{L^*}(t)|^2 = \max_{i=1}^{L}\{|D_i(t)|^2\}$ is a sub-martingale. Based on Doob's Submartingale Maximal Inequality in Eq. (8),

$$P(\tau^{L^*}(d) \leq t) \leq \frac{1}{d^2}E(|D^{L^*}(t)|^2). \quad (11)$$

Using Aven's [18] upper bound on the mean of the maximum of a number of random variables in Eq. (9), we have

$$E(|D^{L^*}(t)|^2) = E(\max_{1\leq i\leq L}\{|D_i^*(t)|^2\})$$
$$\leq \max_{1\leq i\leq L} E(|D_i^*(t)|^2) + \sqrt{\frac{L-1}{L}\left(\sum_{i=1}^{L}\text{Var}(|D_i^*(t)|^2)\right)^{1/2}}.$$

Denote $Z = \frac{|D_i^*(t)|^2}{(r+a)^2 t}$. Clearly, $0 \leq Z \leq 1$. Thus,

$$\text{Var}(Z) = E(Z^2) - E^2(Z) \leq E(Z)(1-E(Z)) \leq E(Z),$$
$$\text{Var}(|D_i^*(t)|^2) = (r+a)^4 t^2 \text{Var}(Z) \leq (r+a)^2 tE(|D_i^*(t)|^2). \quad (12)$$

Based on the results of Theorem 2 and combining Eqs. (10), (11), and (12), we complete our proof. ■

Compared with 1-copy message dissemination, L-copy message dissemination can reduce dissemination latency as it can increase the probability of reaching nodes or infrastructures located distance d from the source within time t.

VI. Simulations and Applications

Previous research work on geocast schemes for vehicular networks mostly proposed various flooding schemes, which can cause network congestion [4]. Although limiting number of disseminators and geographic information exchanges can reduce network load, there is a trade-off between dissemination performance and network load. It is not clear *how many disseminators are needed and whether geographic information should be exchanged* for selecting disseminators in order to satisfy application requirements. Our theoretical analysis suggests that geocast with AOI near the source prefers mechanism with multiple disseminators while geocast with AOI far from the source prefers geographic-assisted dissemination strategy. In this section, we perform simulations along with two real applications to verify our analysis.

A. Dissemination Algorithms

In *stateless opportunistic forwarding (SOF)* [9], a disseminator will choose its succeeding disseminator from its available neighbors at random. Stateless opportunistic forwarding has been suggested to be useful in intermittently connected networks [19–21]. It is particularly useful in vehicular ad hoc network as its global network topology is not known and rapidly varying due to high vehicle mobility and the presence or availability of the next-hop neighbors is not easily controllable. SOF chooses next disseminator isotropically, thus is a type of *direction-invariant* dissemination. The SOF with one disseminator at each time slot is referred to as *1-copy SOF*. Similarly, dissemination algorithm that first sprays active messages to L disseminators and then each disseminator performs SOF independently, is referred to as *L-copy SOF*.

In *GPS-based broadcasting (GBB)* [10], a disseminator will choose its farthest neighbor as next disseminator so that the message can be spread out as fast as possible to certain locations (e.g., police station). GBB is useful for disseminating time-critical message (such as emergency warning) in VANETs. Apparently, GBB is an example of *geographic-assisted* dissemination. The GBB with one disseminator is referred as *1-copy GBB*. Similarly, in *L-copy GBB*, source node first sprays active messages to L disseminators and then each disseminator performs GBB independently.

B. Simulation Results

In a 10 km×10 km area, 5000 nodes move according to constrained vehicle mobility. Each time slot is 1 second and the maximum movement length $a = 20$ per time slot, which means that speed limit is about 40 miles/hour. The transmission range of a node is $R = 200$ meters. Node density is 5×10^{-5} vehicles per square meter. $L = 4$ for L-copy message dissemination.

As shown in Fig. 2, average dissemination distances of 1-copy SOF and 1-copy GBB are bounded by Eq. (3) and upper bounds of expected dissemination distances of 1-copy direction-invariant and geographic-assisted dissemination in Theorem 2, respectively. Similarly, Fig. 3 shows that average dissemination distance of 4-copy SOF and 4-copy GBB are bounded by Eq. (3) and upper bounds of expected dissemination distances of L-copy direction-invariant and geographic-assisted dissemination in Theorem 4, respectively. In a word, *the average dissemination distances of above four algorithms are well bounded by their corresponding analytic bounds.*

Figs. 2 and 3 show that the upper bounds of expected dissemination distance under direction-invariant dissemination are tight, while there is a wide gap between the performance of L-copy GBB and the upper bounds under L-copy geographic-assisted dissemination. The gap is because of the low transmission success probability at the furthest neighbor due to long transmission distance and the interference caused by simultaneous transmissions of multiple disseminators. The gap can be lessened by more sophisticated algorithms, such as those choosing nodes that move away from the message source as disseminators and scheduling transmissions of disseminators to avoid interference.

In addition, Figs. 2 and 3 show that the expected dissemination distances of direction-invariant dissemination algorithms (i.e., 1-copy and 4-copy SOF) exhibit increase with $\sqrt{t}$, while geographic-assisted dissemination algorithms (i.e., 1-copy and 4-copy GBB) achieve approximately *linear* increase as time

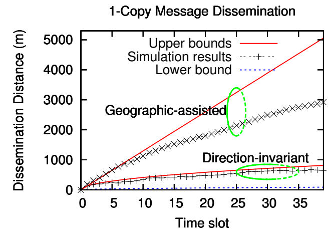

Fig. 2. Dissemination distance $|D(t)|$ of 1-copy message dissemination.

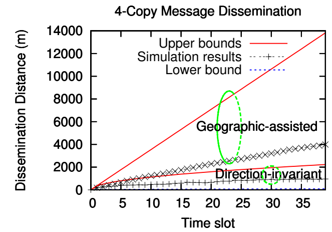

Fig. 3. Dissemination distance $|D(t)|$ of L-copy message dissemination.

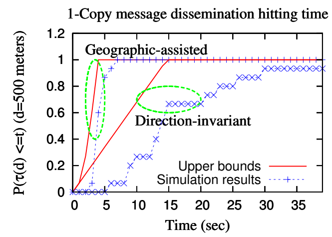

Fig. 4. Hitting time $P(\tau(d) \le t)$ in 1-copy message dissemination.

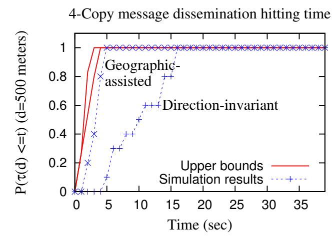

Fig. 5. Hitting time $P(\tau(d) \le t)$ in L-copy message dissemination.

eclipses. Figs. 2 and 3 reveal that comparing to direction-invariant dissemination, geographic-assisted dissemination significantly increases dissemination distance by exploiting geographic information. *Increasing number of disseminators, although benefits the dissemination reliability, is less effective than utilizing geographic information in enhancing long-term dissemination distance or propagation speed.*

Figs. 4 and 5 show that simulation results of $P(\tau(d) \le t)$ of four dissemination algorithms are well bounded by corresponding analytic bounds in Theorems 1, 3 and 5. Both figures demonstrate benefits of geographic-assisted dissemination in reducing hitting times. But, geographic information can reduce hitting time more effectively in 1-copy message dissemination than in L-copy message dissemination, and increasing number of disseminators reduces hitting time more dramatically in direction-invariant dissemination than in geographic-assisted dissemination. In other words, *geographic information is more useful when number of disseminators is small while large number of disseminators is more beneficial when geographic information is unavailable.*

In the following, we further demonstrate how our results serve guidelines to choose dissemination strategy such as to satisfy application requirements in real applications.

C. Applications

Two important applications in VANETs are *post-crash warning* and *emergency vehicle signal preemption*. The application requirements are obtained from vehicle safety communication project report [2] by National Highway Traffic Safety Administration in Department of Transportation of US. In the following, we assume that time slot interval, node density, and node transmission range are the same as our simulation settings in the previous subsection.

1) Post-Crash Warning: In the application of post-crash warning, a disabled vehicle (due to an accident or mechanical breakdown) will warn approaching vehicles of its position and will stop broadcasting when the accident is cleared. According to report [2], the allowable latency for this application is approximately 5 time slots. Suppose vehicle speed is about 20m/s and the the radius of AOI is 200 meters, which reserve 5 seconds for drivers to change lane, slow down, or brake.

Simulation results (Figs. 2 and 3) show that in 5 time slots, dissemination distances are about 252m, 396m, 660m, and 669m for 1-copy and 4-copy SOF, 1-copy and 4-copy GBB, respectively. Since more sophisticated algorithms could achieve better performance than SOF and GBB algorithms, both direction-invariant and geographic-assisted algorithms are fast enough to reach any location in AOI.

When AOI is close and propagation speed is sufficiently fast, dissemination strategy should focus on achieving high reliability in this safety application. Multiple disseminators' simultaneous rebroadcasting can enhance the probability of vehicles receiving this warning, hence *L-copy message dissemination strategy is a better candidate in post-crash warning*.

2) Emergency Vehicle Signal Preemption: Emergency vehicle signal preemption allows the emergency vehicles to override traffic signals. When an emergency vehicle is approaching an intersection, it initiates a geocast targeting vehicles around that intersection. After receiving the message and verifying that the request has been made by an authorized source, the vehicles around the intersection should prepare to stop and provide the right of way to the emergency vehicle.

Assume that the geocast targets vehicles in the circular region around the intersection with radius 150m. Suppose the emergency vehicle starts a message dissemination when it is 350m from the center of the AOI and it moves at speed about 20m/s. Hence. the allowable latency for this application is approximately 10 time slots in order to hit the farthest locations (500m away) of targeted region before entering it.

From Figs. 4 and 5, the upper bounds of the probability of reaching 500 meters in 10 time slots are about 70% for 1-copy direction-invariant dissemination while 100% for other three dissemination strategies. That means that 1-copy direction-invariant message dissemination is incapable of serving this application, while dissemination methods assisted by geographic information or using multiple disseminators could satisfy requirements for this application scenario. Furthermore, we can see that the probability of reaching 500 meters in 10 time slots is about 30% and 40%, 100% and 100% for 1-copy and 4-copy direction-invariant dissemination, 1-copy and 4-copy geographic-assisted dissemination, respectively. Therefore, *geographic-assisted dissemination better serves this application with AOI far from the source*.

Remark 7. Dissemination strategies that use multiple disseminators are suitable for applications like post-crash warning, which AOI is near the source location and requires high dissemination reliability. Dissemination strategies that utilize geographic information to choose relays are needed for applications like emergency vehicle signal preemption, which AOI is far from the source location.

VII. Conclusion

In this paper, we study the spatial and temporal limits of HOM in VANET geocast. By focusing on message mobility rather than specifying relays on information propagation paths, we derive lower and upper bounds for $|D(t)|$ and $\tau(d)$. We find that $E(|D(t)|)$ can at most increase with $\sqrt{t}$ in direction-invariant dissemination while approximately linearly with t in geographic-assisted dissemination, and L-copy message dissemination can increase $E(|D(t)|)$ by $\sqrt{L-1}$ times. Simulation results of four dissemination algorithms validate the analytic bounds. Applying our results in two real applications, we observe that dissemination algorithms with multiple disseminators are suitable for geocast with AOI near the source or high reliability requirement while dissemination algorithms assisted by geographic information are suitable for geocast with AOI far from the source.

Appendix

Proof of the Lemma 3.

Proof: Assume node density λ is constant, which requires infinitely large network area as number of nodes n goes to infinity. According to Definition 1, the position of a node v at time t can be written as $X_v(t) - X_v(0) = \sum_{k=1}^{t} Y_M^v(k) = \sum_{k=1}^{t} A_v(k)e^{i\theta_v(k)}$, where $A_v(k)$ equals $|Y_M(k)| \in [0, a]$ and $0 \leq \theta_v(k) \leq 2\pi$. Define by $C(t) = X_u(t) - X_v(t)$ the difference vector between the positions of nodes u and v at time t. Assume $||C(0)|| > d$, then first passage time becomes $T_F = \inf_{t>0}\{||C(t)|| \leq d\}$.

Place a Cartesian coordinate system in the network with its origin at $X_v(0)$, x-axis connecting $C(0)$ and the origin as shown in Figure 6, and y-axis accordingly. Under 2-D constrained vehicle mobility, we observe that $C(t) = \sum_{k=1}^{t}(A_u(k)e^{i\theta_u(k)} - A_v(k)e^{i\theta_v(k)})$. Hence, $[C(t)]_x = \sum_{k=1}^{t}(A_u(k)cos\theta_u(k) - A_v(k)cos\theta_v(k))$. Since $A_u(k)$, $A_v(k)$ are all i.i.d. and so are $\theta_v(k)$, $\theta_v(k)$, $[C(t)]_x$ is 1-D random walk (sum of random variables) with each step distributed as $A_u cos(\theta_u) - A_v cos(\theta_v)$, which is symmetric and continuous (because uniform distribution is continuous).

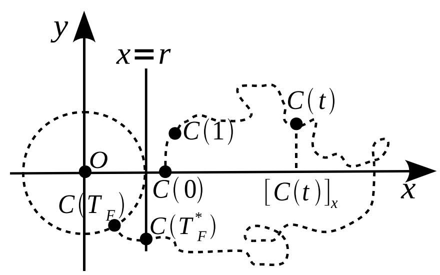

Fig. 6. T_F^* is a lower bound on the first passage time T_F as $C(t)$ must cross the line $x = r$ before crossing the circle of radius r.

Define by T_F^* the first passage time (FPT) of $[C(t)]_x$ to line $x = r$ (the vertical line tangent to the circle at $(r, 0)$). Suppose that the two nodes meet (or equivalently, $C(t)$ crosses the circle) for the first time at $t = T_F$ since $t = 0$ as shown in Fig. 6. Then, it is clear that $C(t)$ must first cross the vertical line $x = d$ before it crosses the circle, i.e., $T_F \geq T_F^*$.

Theorem 6. *[Sparre-Andersen (S-A) Theorem in [22]]: For 1-D discrete time random walk process that starts at $x_0 > 0$ with each step chosen from a continuous, symmetric but otherwise arbitrary distribution, the First Passage Time Density (FPTD) to the origin, i.e., the probability that the random walk first crosses the origin and hits a point on the negative axis, asymptotically decays as $t^{-3/2}$ with the number of steps t.*

According to the Sparre-Andersen (S-A) Theorem, T_F^* asymptotically decays as $\sim t^{-3/2}$ with t, which means that the complementary cumulative distribution function (ccdf) of T_F^* decays as $P(T_F^* > t) \sim t^{-1/2}$. In view of $T_F \geq T_F^*$, we have $P(T_F > t) \geq Ct^{-\frac{1}{2}}$, for all sufficiently large t.

Denote by T the time interval that a disseminator transmits the active message to its next-hop disseminator. Clearly, random variable T stochastically dominates their first passage time T_F, which means $P(T > t) \geq P(T_F > t) \geq Ce^{-\alpha t}$, for all sufficiently large t. Therefore, $\mathcal{N}(t)$ is stochastically dominated by Poisson process with parameter α, and $E(\mathcal{N}(t)) \leq \alpha t$ accordingly, where $\alpha = 1/2$ under 2-D constrained vehicle mobility.

Note that T_F has the equilibrium distribution of inter-meeting time T_I [23] and T_I has shown to exhibit exponential tail decay under many mobility models (such as random waypoint and Brownian Motion) in a bounded domain, while power-law decay in empirical traces as well as infinite domain [23]. Therefore, this result also holds for other mobility models and in realistic traces with α varying with node mobility. ■

Proof of the Lemma 4.

Proof: We prove that $|D(t)|^2$ is a submartingale through proving that $(D_x^2(t), D_y^2(t))$ is a 2-D sub-martingale according to definition in [16]. A sub-martingale is defined as an integer-time stochastic process $\{Z_n; n \geq 1\}$ that $E[|Z_t|] < \infty$ for all $t \geq 1$ and $E[Z_t|Z_{t-1}, \dots, Z_1] \geq Z_{t-1}$ for all $t \geq 2$.

(i) Due to limited transmission range r and movement step a, $|Y_x(k)| \leq r + a < \infty$. Then, for any $k \in \mathbb{N}$.

$$|D_x(t)| \leq |Y_x(1)| + \cdots + |Y_x(t)| \leq t \times (r + a) < \infty.$$

Similarly, $|D_y(t)| \leq t \times (r + a) < \infty$.

(ii) Assume node v_i is the disseminator at time $t - 1$. Denote filtration $\mathcal{F}$ of process $\{D(t)\}$ as $\mathcal{F}_t = \sigma$-algebra generated by $\{D(k); k \leq t\}$ for every t. First, for any $t \in \mathbb{N}$, it holds that $E(D_x(t)|\mathcal{F}_{t-1}) = D_x(t - 1) + E(Y_x(t))$ and $E(D_y(t)|\mathcal{F}_{t-1}) = D_y(t - 1) + E(Y_y(t))$.

(a) When $Y(t) = Y_M(t)$, $E(Y_x(t)) = E(Y_M^x(t)) = 0$ and $E(Y_y(t)) = E(Y_M^y(t)) = 0$ in constrained vehicle mobility.

(b) When $Y(t) = Y_P(t) + Y_M(t)$, for direction-invariant dissemination, $E(Y_P^x(t)) = E(Y_P^y(t)) = 0$; for geographic-assisted dissemination, $E(Y_P^x(t)) \geq 0$ if $D_x(t) \geq 0$ and $E(Y_P^x(t)) \leq 0$ if $D_x(t) \leq 0$ (the same for $Y_P^y(t)$).

From (a) and (b), when $D_x(t) \geq 0$, $E(D_x(t)|\mathcal{F}_{t-1}) \geq D_x(t - 1)$, which proves that $D_x(t)$ is submartingale. When $D_x(t) < 0$, $E(-D_x(t)|\mathcal{F}_{t-1}) \geq -D_x(t - 1)$, which means that $-D_x(t)$ is submartingale. As $D_x^2(t) = D_x(t) * D_x(t) = (-D_x(t)) * (-D_x(t))$ and square function is convex, $D_x^2(t)$ is a submartingale. Similarly $D_y^2(t)$ is also a submartingale. Therefore, $|D(t)|^2 = D_x^2(t) + D_y^2(t)$ is a submartingale. ■

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