

Boundary Matters: Impact of Finite Boundary to Packet Delay Performance in Mobile Data Networks

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Abstract—Although the research of mobile data networks has gained significant attention lately, little have been done to study the impact of finite network boundary in such networks. To understand how finite cell boundary affects the delay performance, we analyze the delay problem from the packet movement point of view. We first divide the network scenarios into three categories based on the value of the expected packet propagation speed \bar{v} and derive the upper and lower bounds for their expected packet delay, respectively. Then, we show that the packet delay scales linearly when $\bar{v} < 0$, while it scales quadratically for the cases $\bar{v} > 0$ and $\bar{v} = 0$ when the boundary effect is absent. Simulation results verify our analysis and show that the boundary effect in fact increases the delay performance for both $\bar{v} > 0$ and $\bar{v} = 0$.

I. INTRODUCTION

The widespread of cloud-based services such as Youtube, Skype, and Facebook have attracted more and more users to access contents on-the-go using mobile data networks. In these services, chunks of information are distributed throughout data centers over the internet, referred as the *cloud*. In contrast to traditional point-to-point networking in which data throughput had been a major concern, cloud-based networking highly considers the effect of delay since the information of interest may be located anywhere in the cloud. A very long delay will result in a *stale* data packet that will not be useful anymore to the user. The customer will be dissatisfied and the service provider could lose revenue. Thus, it is of high interest to study the packet delay performance in mobile data networks.

The entire mobile data network area is served by many base stations, each forwarding the mobile users' packets to and from the cloud service provider. Consequently, the entire network area is divided into smaller cells. To ensure tractability, the analysis of the entire mobile network is reduced to smaller problems covering only one cell area. When an arbitrary node mobility model is used, there might be instances where the node moves away towards a neighboring cell. To preserve the aforementioned one-cell view of the mobile data network, the boundary-crossing movement is re-mapped back into the original cell, causing a *boundary effect*.

The finite boundary is taken into account into the node mobility in various forms. For example, in the random waypoint model [1], the node selects a new destination within the network cell, whenever it changes direction and/or speed. On the other hand, the node movement across the cell boundary may

be either 'reflected' or 'wrapped' back into its original cell in random walk model [1]. This alteration of trajectory may affect the network dynamics to some extent. An extreme example is that a network with finite boundary will have an exponentially-distributed inter-meeting time, instead of power-law as in the case of unbounded region [2]. Hence, to accurately model the network's performance, it is important to study the impact of the finite boundary towards the packet delay.

Currently, there have been studies that characterized the delay performance in mobile ad-hoc [3]–[5] and cognitive radio networks [6]. Most of them, however, assume specific scheduling or routing scheme. There are various well-known routing algorithms ranging from those designed for end-to-end connected networks, such as AODV [7] and DSDV [8], to the ones that are more well suited to sparse networks, such as randomized [9] and epidemic protocols [10]. On the other hand, the mobile nodes move according to arbitrary mobility models ranging from entity models to group mobilities [1]. It will be hard to follow the conventional approach since we will need to analyze each combination of node mobility and routing protocol separately.

In this paper, our objective is to study what is the impact of the finite boundary towards the delay performance of a mobile data network. To do this, we examine a *sparse* mobile network in which store-carry-and-forward mechanism is employed. The total network area is divided into cells with area $l \times l$ and there are $n \gg 1$ α -nodes and one β -node in each cell. Packets ϵ 's are generated at arbitrary α 's to be delivered to the destination β . First, we define the expected propagation speed \bar{v} and divide the network scenarios into three cases: packet tends to 1) move towards ($\bar{v} < 0$); 2) move away from ($\bar{v} > 0$); or 3) stay at a constant distance ($\bar{v} = 0$) from β . Then, we derive the upper and lower analytic bounds of the expected packet delay $E(T)$, the mean time required to deliver ϵ 's to β -node, for all three cases of \bar{v} assuming the finite boundary effect is absent.

From the theoretical derivation, we found that $E(T)$ scales linearly as $\Theta(L_\epsilon(0))$ for **Case 1** ($\bar{v} < 0$), while it grows quadratically as $\Theta(L_\epsilon^2(0))$ for both **Case 2** ($\bar{v} = 0$) and **Case 3** ($\bar{v} > 0$) when the boundary effect is absent. We argue that if the cell side length l is very large, the boundary effect will be small such that these scaling orders will *still* be satisfied. Numerical simulations justify this argument and show that the boundary effect is non-existent and the analytic $E(T)$ bounds hold for **Case 1**. More interestingly, numerical simulations

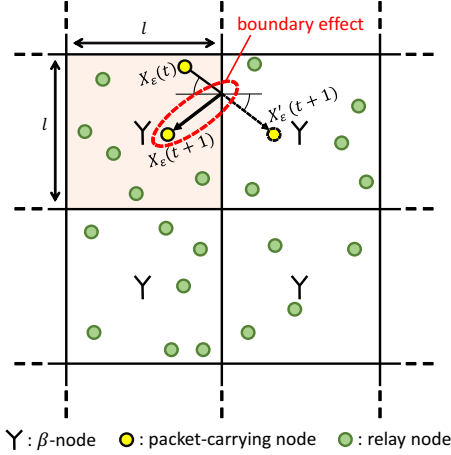


Fig. 1: Structure of the examined mobile data network and the corresponding boundary effect.

for Case 2 and Case 3 indicate that the boundary effect is in fact beneficial because it will induce an upper bound such that $E(T)$ is lower than that predicted through the theoretical scaling order of $\Theta(L_\epsilon^2(0))$.

The rest of the paper is then organized as follows. Section II introduces the mobile network setup as well as the definition of packet delay and boundary effect. Section III discusses the bounds for the expected packet delay and its order of growth, that are further verified by numerical simulations in Section IV. Finally, the paper is concluded in Section V.

II. MODELS AND DEFINITIONS

A. Network and Communication Model

In this paper, infrastructure nodes β 's are uniformly placed throughout the network. More specifically, the entire area of the examined mobile data network is divided into rectangular cells with size $l \times l$ where a β -node is located at the center of the square, as depicted by Fig. 1. In every cell, there are n mobile α -nodes, represented by the set $\mathcal{A} = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$. They portray the mobile users that want to access services on-the-go from the content providers. The α -nodes communicate with the providers by connecting to the nearest β -node in their cell. The data are then forwarded by the β -node to and from the service providers through high-speed backhaul links. Assuming reciprocity within each link and because the wired backhaul links have lower latency than the wireless connectivity portion of the mobile data network, we focus our study to the packet movement trace from an α -node to its associated β -node. Note that we specifically use the term *mobile data network* here, different to *mobile ad-hoc network* [3] in which packets are exchanged between two arbitrary mobile users.

In the network, a packet ϵ is generated at an α -node to be delivered to the corresponding β -node. However, direct transmission may not be feasible between both nodes because

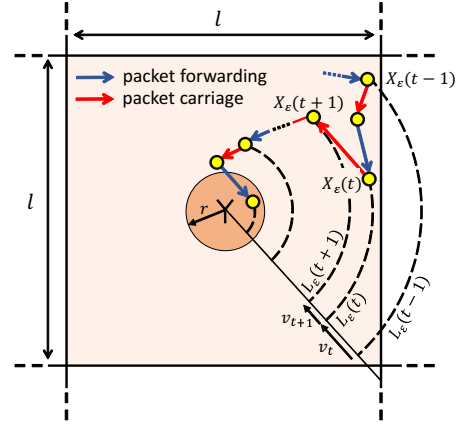


Fig. 2: Packet's movement model in mobile data networks.

the wireless nodes have limited transmission range¹ r due to their physical restrictions [11], [12] and health concerns [13]. Moreover, α -nodes might be sparsely-located such that there may not be any end-to-end path connecting nodes α and β at every time instance. In this case, packet ϵ can be delivered using a *store-carry-and-forward* mechanism in which it is firstly carried around the cell by the movement of the mobile α -node. This represents the packet carriage step. After that, the packet-carrying node chooses whether packet ϵ will be forwarded to another node within its range, signifying the packet forwarding step. In every time slot, both packet carriage and forwarding interleave each other until the packet is delivered to the β -node.

B. Definition of Packet Delay

To characterize the packet delay performance, we first model how the packet propagates in the mobile data network. The location of packet over time is largely affected by the node mobility and routing algorithm. Because these two components are random processes, respectively, we combine their joint effects into a stochastic process $\{X_\epsilon(t) : t \in \mathbb{N}_0\}$ that represents the packet's location at time slot t . The Euclidean distance between ϵ and the nearest β -node is then given as

$$L_\epsilon(t) = |X_\epsilon(t) - X_\beta|. \quad (1)$$

Now, we are ready to formally state the packet propagation speed as follows.

Definition 1: Packet propagation speed² v_t is defined as the progress of $L_\epsilon(t)$ at every time slot, given by

$$v_t = L_\epsilon(t) - L_\epsilon(t-1). \quad (2)$$

The interplay between $X_\epsilon(t)$, $L_\epsilon(t)$ and v_t within the examined packet movement model itself is depicted in Fig. 2. Note that in every time slot, v_t can be either positive, zero, or negative, depending on the random packet movement.

¹Range r is limited if it is much smaller than $l/\sqrt{2}$, the maximum possible distance between nodes α and β .

²Note that the packet propagation speed v_t is *not* equivalent to the node movement speed in node mobility models [1].

Our objective is to study the performance of mobile data networks from the packet delay perspective. To analytically define the packet delay, let us use (2) to obtain

$$L_\epsilon(t) = L_\epsilon(0) + \sum_{k=1}^t v_k. \quad (3)$$

Here, the packet is generated at $t = 0$ with the initial distance $L_\epsilon(0)$ and moves with the propagation speed of v_k for time slots $k = 1, 2, \dots$ until it arrives within the transmission range r of the β -node. Based on this observation, the instantaneous packet delay can be formalized as follows.

Definition 2: Packet delay is defined as the time required for a packet ϵ generated at $L_\epsilon(0)$ until it lies within the transmission range r of β -node, e.g.,

$$T = \inf_{t \geq 0} \left\{ t : L_\epsilon(0) + \sum_{k=1}^t v_k \leq r \right\}. \quad (4)$$

Because store-carry-and-forward is used, we assume that the transmission, processing, and queueing delays are negligible such that the packet delay is only affected by the node mobility and the packet forwarding algorithm employed.

C. Boundary Effect in Mobile Data Networks

The entire network area is divided into rectangular cells with side length l , such that there are possibilities that a mobile node moves away from its original cell into an adjacent cell. Because all the β -nodes are interconnected through low-latency backhaul, the packet will still be delivered to the provider's network even if it is now delivered to the adjacent cell's β -node. Thus, the node trajectory across the cell boundary is usually mapped back as a 'dual' trajectory into its original cell. This 'dual' trajectory has different treatments, depending on the node mobility used. In a random waypoint mobility model, whenever the mobile node changes speed and/or direction, it chooses a new direction within the cell area. On the other hand, the node movement towards the cell boundary may be either 'wrapped' or 'reflected' back into its original cell area in random walk mobility.

Either way, the 'dual' trajectory will ensure that the node moves only within its own cell. This, however, will alter the instantaneous values of v_t in (2), especially when the packet tends to move towards the boundary. We refer the impact of the finite boundary to the overall network dynamics as *boundary effect*, as depicted in Fig. 1.

III. EXPECTED PACKET DELAY PERFORMANCE IN MOBILE DATA NETWORKS

The packet delay T is a random process because $X_\epsilon(t)$ is also a stochastic process. Thus, it is more practical to evaluate the mobile network's performance with respect to the expected packet delay $E(T)$, instead of the instantaneous T . Unfortunately, deriving the general closed-form expression for $E(T)$ is not trivial due to the complex interplay between the packet carriage and forwarding components. Instead, we aim to derive the upper and lower bounds of $E(T)$ in this section.

Again, v_t is a random variable that depends on the instantaneous realization of packet carriage and forwarding at every

time slot. Thus, we are interested in its expected value $E(v_t)$, which captures the long-term packet movement tendency. For simplicity, we assume v_t 's are independent and identically-distributed (i.i.d.) such that $\bar{v} = E(v_t)$ for any $t \geq 0$. Based on this model, the packet movement can be divided into three cases as follows. The packet ϵ tends to move towards β -node ($\bar{v} < 0$) in **Case 1**. On the other hand, ϵ drifts away from β -node ($\bar{v} > 0$) in **Case 2**, while it tends to stay at a constant distant ($\bar{v} = 0$) in **Case 3**.

We assume that any arbitrary combination of node mobility and packet routing algorithm can be categorized into one of the three aforementioned cases such that the corresponding analysis below can be performed accordingly.

A. Expected Packet Delay for Case 1

First of all, we examine the case of $\bar{v} < 0$, in which packet ϵ have the tendency to move towards the β -node. Intuitively, ϵ will be delivered faster than the other two cases of \bar{v} . We justify this by deriving the upper and lower bounds for $E(T)$ and show that it scales linearly with respect to the packet's initial distance $L_\epsilon(0)$.

Theorem 1: Given any mobile data network in which $\bar{v} < 0$, the expected packet delay is bounded by

$$\frac{L_\epsilon(0) - r}{-\bar{v}} \leq E(T) \leq \frac{L_\epsilon(0)}{-\bar{v}}. \quad (5)$$

Proof: Let $L'_\epsilon(t) = L_\epsilon(t) - H_t$, where $L_\epsilon(t)$ is supermartingale [14] and $H_t = \sum_{k=1}^t E(v_k | \mathcal{F}_{k-1})$. Here, $\{\mathcal{F}_t : t = 0, 1, \dots\}$ is a filtration that captures all information about the history up to time slot t . We have

$$\begin{aligned} E(L'_\epsilon(t) | \mathcal{F}_{t-1}) &= E((L_\epsilon(t) - H_t) | \mathcal{F}_{t-1}) \\ &= L_\epsilon(t-1) - H_{t-1} \\ &= L'_\epsilon(t-1). \end{aligned} \quad (6)$$

Hence, the sequence $\{L'_\epsilon(t)\}$ is martingale. By using the optional stopping theorem [14], [15], we have

$$E(L_\epsilon(0) - L_\epsilon(T)) = -H_T. \quad (7)$$

Because $0 \leq L_\epsilon(T) \leq r$, we further have

$$L_\epsilon(0) - r \leq -H_T \leq L_\epsilon(0). \quad (8)$$

Then, Eq. (5) is obtained because $\bar{v} = E(v_t)$ for any $t \geq 0$ such that $H_T = \bar{v}E(T)$. ■

Corollary 1: For any mobile data network with $\bar{v} < 0$, the expected packet delay grows in the order of $E(T) = \Theta(L_\epsilon(0))$.³

The corollary above can be easily obtained by applying the definition of $\Theta(\cdot)$ ³ to eq. (5).

³Here, $f(n) = \Theta(g(n))$ indicates that there exist positive constants c_1 , c_2 , and n_0 such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0$.

B. Expected Packet Delay for Case 2

In the second case, we have $\bar{v} > 0$, which means packet ϵ have the long-term inclination to move away from the destination. Intuitively, this scenario will result to the worst $E(T)$, among all three possible cases of \bar{v} 's. To justify this assumption, we first derive the lower and upper bounds of $E(T)$ for the case where the boundary effect is absent. Then, we argue that when the boundary effect is small, $E(T)$ grows with the order of $\Theta(L_\epsilon^2(0))$, verifying that the expected packet delay for **Case 2** grows much faster than in **Case 1** ($\bar{v} < 0$).

Theorem 2: Given any mobile data network in which $\bar{v} > 0$ and the boundary effect is non-existent, the expected packet delay is bounded by

$$\frac{(L_\epsilon(0) - r)^2}{\sigma^2 + (\bar{v})^2 + 2l\bar{v}} \leq E(T) \leq \frac{(L_\epsilon(0) - r)^2}{\sigma^2 + (\bar{v})^2 - 2l\bar{v}}. \quad (9)$$

Proof: From (3), for any positive integer t , we have the following decomposition.

$$\begin{aligned} \sum_{i=1}^t v_i^2 &= (L_\epsilon(t) - L_\epsilon(0))^2 - 2 \sum_{1 \leq i < j \leq t} v_i v_j \\ &= (L_\epsilon(t) - L_\epsilon(0))^2 - 2 \sum_{i=2}^t v_i (L_\epsilon(i-1) - L_\epsilon(0)) \\ &= (L_\epsilon(t) - L_\epsilon(0))^2 \\ &\quad - 2 \sum_{i=2}^t v_i L_\epsilon(i-1) + 2 \sum_{i=2}^t v_i L_\epsilon(0). \end{aligned} \quad (10)$$

Let us set $t = T$, the packet delivery instance. By applying expectations to both sides of (10), we obtain

$$\begin{aligned} \sum_{i=1}^T E(v_i^2) &= E^2(L_\epsilon(T) - L_\epsilon(0)) \\ &\quad - 2 \sum_{i=2}^T E(v_i)(L_\epsilon(i-1)) + 2 \sum_{i=2}^T E(v_i)L_\epsilon(0). \end{aligned} \quad (11)$$

The left hand-side of (9) is proved as follows. First of all, we define the variance of the packet propagation speed as

$$\sigma_t^2 = E(v_t^2) - E^2(v_t). \quad (12)$$

By assuming $E(v_t) \geq 0$ and $L_\epsilon(0) \geq 0$, we obtain

$$\begin{aligned} E^2(L_\epsilon(0) - L_\epsilon(T)) &\leq \sum_{i=1}^T E(v_i^2) + 2 \sum_{i=2}^T L_\epsilon(i-1)E(v_i) \\ &\leq \sum_{i=1}^T E(v_i^2) + 2l \sum_{i=1}^T E(v_i) \end{aligned} \quad (13)$$

$$= E(T) [E(v_t^2) + 2lE(v_t)] \quad (14)$$

$$= E(T) [\sigma^2 + (\bar{v})^2 + 2l\bar{v}]. \quad (15)$$

Here, (13) applies because $L_\epsilon(i-1) \leq l$, eq. (14) is obtained by applying another expectation to (13), while (15) holds by employing (12) and assuming σ_t^2 's and v_t 's are i.i.d. such that $\sigma_t^2 = \sigma^2$ and $E(v_t) = \bar{v}$. The right hand side of (9) can then easily be obtained from (15) using the fact $L_\epsilon(T) \leq r$.

To prove the right hand-side of (9), we use (11) and derive the upper bound as follows.

$$E^2(L_\epsilon(0) - L_\epsilon(T)) \geq \sum_{i=1}^T E(v_i^2) - 2 \sum_{i=2}^T L_\epsilon(0)E(v_i) \quad (16)$$

$$\geq E(T) [E(v_t^2) - 2lE(v_t)] \quad (17)$$

$$= E(T) [\sigma^2 + (\bar{v})^2 - 2l\bar{v}]. \quad (18)$$

In the equations above, (16) holds because $\bar{v} \geq 0$ and $L_\epsilon(i-1) \geq 0$, (17) is obtained by applying another expectation to the line above it and because $L_\epsilon(0) \leq l$, while (18) applies by using (12) and assuming i.i.d. v_i 's. From (18), we can easily get the left hand side of (9). ■

In this subsection, the boundary effect matters since $\bar{v} > 0$ and packet ϵ tends to move away from the β -node, located at the center, towards the cell boundary. Unfortunately, the results above holds only for the case in which the boundary effect is non-existent. For this to happen, the sidelength size of the network cell must be $l = \infty$. Practically, as in our assumption depicted by Fig. 1, the cell sidelength is finite such that the boundary effect will also exist. In this case, the bounds in eq. (9) do not hold in general. In some cases, however, a packet ϵ may be generated at an initial distance $L_\epsilon(0)$ that is much smaller than l . Because ϵ starts near the destination β -node, there is a higher probability that it will hit β before arriving at the cell boundary. Here, we can assume that the boundary effect is very small such that the delay will still scale in the order equivalent to the theoretical $E(T)$ in (9). We then have the following observation.

Corollary 2: For any mobile data network with $\bar{v} > 0$ in which the boundary effect is arbitrarily small, the expected packet delay scales in the order of $E(T) = \Theta(L_\epsilon^2(0))$.

If the delay scaling above does not hold, then we say that the packet movement experiences *significant* boundary effect.

C. Expected Packet Delay for Case 3

Here, we observe that $\bar{v} = 0$ such that this case serves as the boundary between **Case 1** ($\bar{v} < 0$) and **Case 2** ($\bar{v} > 0$). Thus, it is of high interest to know whether the mobile network with $\bar{v} = 0$ exhibits delay performance that is more similar to the former or the latter case. In the followings, we show that it shares more resemblance with **Case 2**.

Theorem 3: Given any mobile data network in which $\bar{v} = 0$ and the boundary effect is non-existent, the expected packet delay is bounded by

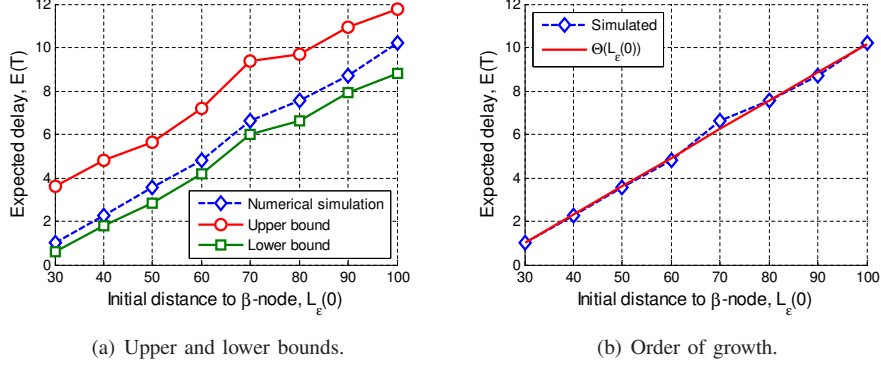
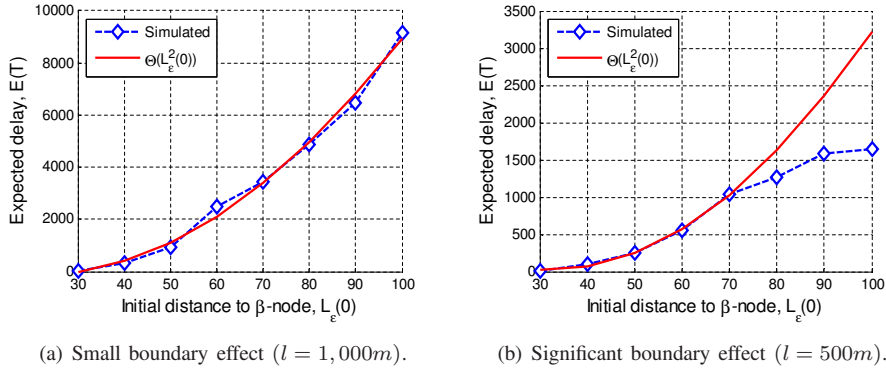
$$\frac{(L_\epsilon(0) - r)^2}{\sigma^2} \leq E(T) \leq \frac{(L_\epsilon(0))^2}{\sigma^2}. \quad (19)$$

Proof: We let $Y_t = (L_\epsilon(0) - L_\epsilon(t))^2 - S_t$, where $L_\epsilon(t)$ is martingale [14] and $S_t = \sum_{k=1}^t \sigma_k^2$. We have

$$\begin{aligned} E(Y_t | \mathcal{F}_{t-1}) &= E(((L_\epsilon(0) - L_\epsilon(t))^2 - S_t) | \mathcal{F}_{t-1}) \\ &= E(((L_\epsilon(0) - L_\epsilon(t-1))^2 - S_{t-1} \\ &\quad - 2v_t(L_\epsilon(0) - L_\epsilon(t-1)) \\ &\quad + v_t^2 - \sigma_t^2) | \mathcal{F}_{t-1}). \end{aligned} \quad (20)$$

We know $E((L_\epsilon(t) - L_\epsilon(t-1)) | \mathcal{F}_{t-1}) = 0$ because $L_\epsilon(t)$ is a martingale, such that from Eq. (20) we obtain

$$\begin{aligned} E(Y_t | \mathcal{F}_{t-1}) &= E(((L_\epsilon(0) - L_\epsilon(t-1))^2 - S_{t-1}) | \mathcal{F}_{t-1}) \\ &= Y_{t-1}. \end{aligned} \quad (21)$$

Fig. 3: Packet delay $E(T)$ for Case 1.Fig. 4: Packet delay $E(T)$ for Case 2.

Therefore, the stochastic process $\{Y_t\}$ is also martingale. By invoking the Optional Stopping Theorem [14], and denoting T as the time instance when the packet is delivered, we obtain $E(Y_T) = E(Y_1) = 0$. Therefore,

$$E((L_\epsilon(0) - L_\epsilon(T))^2) = S_T. \quad (22)$$

Also, according to our definition in (2), when the packet is delivered we have

$$0 \leq L_\epsilon(T) \leq r. \quad (23)$$

From (22) and (23) we obtain

$$(L_\epsilon(0) - r)^2 \leq S_T \leq (L_\epsilon(0))^2. \quad (24)$$

By assuming σ_t^2 's and v_t 's for $t = 0, 1, \dots$ are i.i.d., we can easily obtain (19) by plugging $\sigma_t^2 = \sigma^2$ and $E(v_t | \mathcal{F}_{t-1}) = \bar{v}$ into eq. (24). ■

Here, $\bar{v} = 0$ equivalently means that packet ϵ have the same probability of moving towards and away from β -node. Thus, there will be non-negligible probability that ϵ will move towards the cell's periphery and experience boundary effect. As in the previous subsection, however, l may be very large such that the effect of boundary will be small. Thus, we have the following corollary. Here, we can see that $E(T)$ scales as $\Theta(L_\epsilon^2(0))$, similar to that of Case 2 given in Corollary 2.

Corollary 3: For any mobile data network with $\bar{v} = 0$ in which the boundary effect is arbitrarily small, the expected packet delay scales as $E(T) = \Theta(L_\epsilon^2(0))$.

IV. SIMULATION RESULTS

In this section, we verify our analytic findings through numerical simulations using OmNeT++'s Inetmanet 2.0 framework. Unless specified otherwise, the followings are used. The α -nodes move with a uniformly-distributed speed between $[v_{min}, v_{max}] = [1, 10]m/s$. We employ the cell side length, number of α -nodes per cell, and transmission range of $l = 1,000m$, $n = 100$, and $r = 25m$, respectively. All results are obtained after simulating over 10,000 network realizations. To verify the order of growth in Corollaries 1, 2, and 3, we specifically select the combinations of node mobilities [1] and packet forwarding algorithms [9], [10] that results in fixed \bar{v} 's and σ^2 's for varying $L_\epsilon(0)$. In the figures, the graph representing $\Theta(L_\epsilon(0))$ and $\Theta(L_\epsilon^2(0))$ are obtained using least-square curve fitting technique.

First of all, we verify the upper and lower bounds as well as the growth order of $E(T)$ for Case 1. We employ a random waypoint node mobility in conjunction with epidemic routing protocol that represents the case of $\bar{v} < 0$. The pause time between waypoints is uniformly-distributed within the range $t_p \in [3, 8]s$. The expected packet delay $E(T)$ as well as its

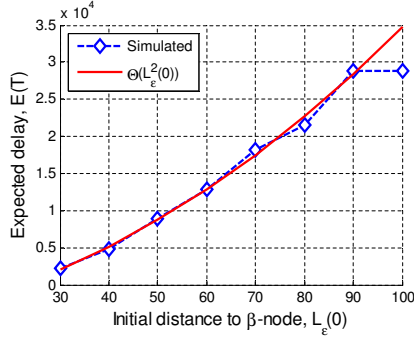


Fig. 5: Packet delay $E(T)$ for Case 3.

analytic upper and lower bounds are given in Fig. 3(a). The figure indicates that the bounds calculated using (5) is valid throughout the examined range of $L_\epsilon(0)$. Next, the growth of $E(T)$ is given in Fig. 3(b). This figure verifies that $E(T)$ grows linearly with respect to $L_\epsilon(0)$ for mobile networks with $\bar{v} < 0$, as outlined in Corollary 1.

Next, we employ a random walk node mobility in conjunction with randomized routing algorithm representing the mobile networks in Case 2 ($\bar{v} > 0$). As mentioned before, the mobile node will tend to move towards the cell boundary such that the packet movement will be governed by the boundary effect. Thus, the bounds in (19) does not hold and we can only verify Corollary 2, instead. In Fig. 4(a), the cell side length $l = 1,000m$ is much larger than the examined $L_\epsilon(0)$'s such that the boundary effect is arbitrarily small and the packet delay scales as $E(T) = \Theta(L_\epsilon^2(0))$. If the side length is reduced to $l = 500m$, the boundary effect becomes *significant*, indicated by the deflection that happens at $L_\epsilon(0) = 80m$ in Fig. 4(b). Both Figs. 4(a) and 4(b) indicates that $E(T)$ will scale as $\Theta(L_\epsilon^2(0))$ as long as the boundary effect can be neglected, justifying Corollary 2 above.

Figs. 4(a) and 4(b) have two important observations. Firstly, as l decreases from $1,000m$ to $500m$, the node density per cell $\frac{n}{l^2}$ will increase such that the opportunity of finding another node to help relay packet ϵ is also increased. Thus, the case $l = 500m$ will have better delay performance than that of $l = 1,000m$, represented by lower $E(T)$. Secondly, the boundary effect will result in a lower $E(T)$ than that predicted by $\Theta(L_\epsilon^2(0))$, translating to better performance. This is due to the fact that the node trajectory near the boundary tends to result in a 'dual' movement towards the destination β at the center of the cell, thus accelerating the packet delivery.

Finally, for Case 3, we verify that $E(T)$ scales as $\Theta(L_\epsilon^2(0))$, as given by Corollary 3. Because there is no scenario, among the examined node mobilities and forwarding algorithms, that leads to the case of $\bar{v} = 0$, we employ a synthetic packet mobility with fixed absolute movement speed $|v_{node}| = 5m/s$ in Fig. 5. The cell sidelength $l = 10,000m$, much larger than the examined $L_\epsilon(0)$'s, is chosen to minimize the boundary effect. From this figure, we can see that $E(T) = \Theta(L_\epsilon^2(0))$ up to the point $L_\epsilon(0) = 90m$. Beyond

this, the boundary effect becomes significant and the quadratic $E(T)$ profile cannot be observed anymore.

Remark 1: In Case 1, packets tend to move towards the destination, such that the boundary effect can be neglected and *both* the bounds in (5) and the scaling in Corollary 1 are satisfied. On the other hand, in Case 2 (resp. Case 3) the boundary effect occurs such that v_t 's are not i.i.d. and (9) (resp. (5)) does not hold. However, the delay scaling in Corollary 2 (resp. Corollary 3) *still* holds.

V. CONCLUSIONS

In this paper, we studied the effect of cell boundary to the packet delivery delay performance in mobile data networks. We first divide the delay problem into three cases based on the packet's movement tendency with respect to the destination. The upper and lower bounds for all three mobility regions are derived. Finally, analytic and numerical results show that the packet delay scales as $\Theta(L_\epsilon(0))$ when the packet tends to move towards the destination, while quadratic growth $\Theta(L_\epsilon^2(0))$ holds for that when the packet is inclined to stay at a constant distance or move away from its destination, when the boundary effect is small.

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