An Optimal Paging Scheme for Minimizing Signaling Costs Under Delay Bounds

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***Abstract*—This letter introduces a method for minimizing paging costs in wireless systems with delay constraints. An optimal partition algorithm is proposed and the corresponding paging procedure is provided. Numerical results demonstrate that the new scheme outperforms other previous schemes for uniform and nonuniform location probability distributions.**

***Index Terms*—Delays, paging costs, wireless systems.**

I. Introduction

LOCATION tracking in wireless systems requires location registration and paging processes. Both procedures cause heavy signaling costs as the demand of wireless services and number of mobile users grow rapidly. Location registration enables a mobile terminal (MT) to register with the system when it enters a new location area (LA) consisting of a number of cells. Consequently, the system is always aware of the current location of a mobile user. Paging is the process in which a system searches for an MT by sending polling messages to the cells within the last reported LA of the MT. Signaling cost of paging, which is measured in terms of cells to be polled before the called user is found, is related to the efficiency of bandwidth utilization and should be minimized under delay bounds [4], [8]. In order to reduce the paging costs, multi-step paging schemes have been suggested to satisfy the delay bounds while reducing the paging costs [2], [6], [7], [10], [11]. In each step, a group of cells called paging area (PA) is searched in one polling cycle, which is the time elapsed between sending a paging message and receiving the response [1], [5]. On the condition of delay bounds, the minimization of paging costs demands the partitioning of an LA into several PA's. However, most of the previous paging schemes could not minimize the signaling costs of paging with delay constraints.

In this letter, a lemma is provided to illustrate the conditions for an optimal partitioning algorithm. Then the corresponding paging procedure is presented for separating an LA into PA's, which guarantees the minimization of paging costs under delay bounds. Compared to other schemes, the new algorithm is very effective in minimizing the paging costs for nonuniform location probability distributions.

Manuscript received April 4, 2000. The associate editor coordinating the review of this letter and approving it for publication was Dr. I. S. Venieris. This work was supported by the National Science Foundation under Grant NCR-97-04393 and by Korea Telecom-Wireless Communications Research Laboratory (WCRL).

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Publisher Item Identifier S 1089-7798(01)02128-7.

II. Partition Algorithm

Assume that each LA consists of the same number of cells N and the worst-case paging delay is considered as the delay bound, $\mathcal{D}$, in terms of polling cycles. For $1 \leq \mathcal{D} \leq N$, the partitioning of an LA requires grouping cells in an LA into $\mathcal{D}$ smaller PA's. The initial state $\mathbf{P}$ is defined as $\mathbf{P} = [p_1, p_2, \cdots, p_j, \cdots, p_N]$, where p_j is the probability of the jth cell to be searched in a decreasing order of probabilities. Each PA is denoted by a triplet $\text{PA}_{\mathcal{P}}(i, q_i, n_i)$ under paging scheme $\mathcal{P}$, in which i is the sequence number of the PA, q_i is the probability of the called MT being found in the ith PA, and n_i is the number of cells contained in the ith PA. Accordingly, the *paging cost* under delay bound $\mathcal{D}$, $E[C(\mathcal{D})]$ and the *average delay*, $E[D(\mathcal{D})]$ can be computed from

$$E[C(\mathcal{D})] = \sum_{i=1}^{\mathcal{D}} q_i \cdot k_i, \quad \text{and} \quad E[D(\mathcal{D})] = \sum_{i=1}^{\mathcal{D}} i \cdot q_i \qquad (1)$$

where $q_i = \sum_{j \in \text{PA}_{\mathcal{P}}(i)} p_j$ and $k_i = \sum_{k=1}^{i} n_k$.

The objective of the optimal paging scheme is to minimize the average number of cells needed to be searched for the called MT under delay bounds.

Lemma: If a paging sequence $\mathcal{P}$ satisfies the following conditions, the *paging cost* $E[C(\mathcal{D})]$ can be minimized:

1) *Probability Condition*: The cells must be searched in a decreasing order of the location probabilities. In other words, if u and v are cells with $p_u > p_v$, then the optimal paging sequence $\text{PA}_{\mathcal{P}}$ that minimizes $E[C(\mathcal{D})]$ must satisfy $u \in \text{PA}_{\mathcal{P}}(g, q_g, n_g)$ and $v \in \text{PA}_{\mathcal{P}}(h, q_h, n_h)$ for all $g \leq h$.
2) *Forward Boundary Condition*: It determines the largest probability cell, i.e., with the largest location probability in a PA. Let p_{i+1}^1 be the largest probability cell in the $(i+1)$th PA with n_{i+1} cells. Then, $p_{i+1}^1 \cdot (n_{i+1} - 1)$ must be less than or equal to q_i. This condition implies that the largest probability cell in the $(i+1)$th PA cannot be moved "forward" to the ith PA that is prior to the $(i+1)$th PA.
3) *Backward Boundary Condition*: It chooses the smallest probability cell, i.e., with the smallest location probability in the PA. The backward boundary condition demands that q_i should be less than or equal to $p_i^s \cdot (n_{i+1} + 1)$, where p_i^s is the smallest probability in the ith PA. Thus, the smallest probability cell p_i^s cannot be moved "backward" to the $(i+1)$th PA, which comes after the ith PA.

Proof of Lemma: With regard to the probability condition, suppose the paging scheme $\mathcal{P}$ is optimal, but there exists $u \in \text{PA}_{\mathcal{P}}(g, q_g, n_g)$ and $v \in \text{PA}_{\mathcal{P}}(h, q_h, n_h)$ with $p_u > p_v$ for $g > h$. Let $\mathcal{P}'$ denote the new paging sequence derived from $\mathcal{P}$. In this new sequence, u and v are swapped so that

$u \in \mathrm{PA}_{\mathcal{P}'}(g, q'_g, n_g)$ and $v \in \mathrm{PA}_{\mathcal{P}'}(h, q'_h, n_h)$. As a result

$$\begin{aligned} E[C(\mathcal{D})] - E[C'(\mathcal{D})] &= \left(\sum_{i=1,\, i\neq g,h} k_i \cdot q_i + k_g \cdot q_g + k_h \cdot q_h \right) \\ &\quad - \left(\sum_{i=1, i\neq g,h} k_i \cdot q_i + k_h \cdot q_g + k_g \cdot q_h \right) \\ &= (k_g - k_h)(p_u - p_v) > 0 \end{aligned} \tag{2}$$

where k_g is larger than k_h for $g > h$ according to the definition in (1). This is a contradiction of the assumed optimality of $\mathcal{P}$. Therefore, the probability condition in the Lemma is necessary.

Given that the first probability condition is satisfied, the cells can be organized in a nonincreasing order of probabilities. The *paging cost* C_1 of an MT being found in the $(i+1)$th PA using $\mathcal{P}$ is calculated from

$$C_1 = \overline{n}_{i-1} + q_i \cdot (k_{i-1} + n_i) + q_{i+1} \cdot (k_{i-1} + n_i + n_{i+1}) \tag{3}$$

where $\overline{n}_{i-1} = \sum_{l=1}^{i-1} q_l \cdot k_l$, and k_{i-1} is defined in (1). If the largest probability cell p_{i+1}^1 in the $(i+1)$th partition is moved to the ith partition, the *paging cost* C_2 is determined by

$$\begin{aligned} C_2 = &\overline{n}_{i-1} + (q_i + p_{i+1}^1) \cdot (k_{i-1} + n_i + 1) \\ &+ (q_{i+1} - p_{i+1}^1) \cdot (k_{i-1} + n_i + n_{i+1}). \end{aligned} \tag{4}$$

Since the optimality of the partition must be retained, i.e., $C_1 \leq C_2$, consequently,

$$p_{i+1}^1 \cdot (n_{i+1} - 1) \leq q_i. \tag{5}$$

This is exactly the forward condition in the lemma.

In a similar way, if the smallest probability cell p_i^s in the ith partition is moved backward to the $(i+1)$th partition, the *paging cost* C_3 is determined by

$$\begin{aligned} C_3 = \, &\overline{n}_{i-1} + (q_i - p_i^s) \cdot (k_{i-1} + n_i - 1) \\ &+ (q_{i+1} + p_i^s) \cdot (k_{i-1} + n_i + n_{i+1}). \end{aligned} \tag{6}$$

Due to the optimality requirement, the following formula can be obtained by applying $C_1 \leq C_3$:

$$q_i \leq p_i^s \cdot (n_{i+1} + 1) \tag{7}$$

which is the backward condition in the lemma.

III. The Paging Procedure

Here we present a paging scheme that fulfills the conditions mentioned in the lemma. First of all, an LA is partitioned into a series of PA's in such a way that all PA's consist of approximately the same number of cells [9], followed by testing the boundary conditions described in the previous section.

- **Step 1:** Calculate the number of cells in each PA as

$$n_0 = \left\lfloor \frac{N}{\mathcal{D}} \right\rfloor \tag{8}$$

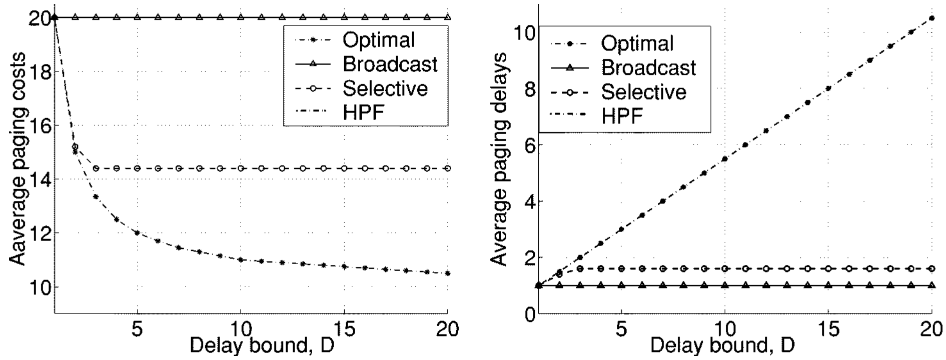
  and determine the variable k as $k = N - n_0\mathcal{D}$.
- **Step 2:** Determine a series of PA's as $\mathrm{PA}^0(1), \mathrm{PA}^0(2), \cdots, \mathrm{PA}^0(\mathcal{D})$ with the location probabilities of $q_1, q_2, \cdots, q_{\mathcal{D}}$, respectively. n_0 cells are allocated to each of the first $(\mathcal{D} - k)$ PA's, and $(n_0 + 1)$ cells are assigned to each of the remaining k PA's. For example, the first PA consists of n_0 cells and the last PA, i.e., $\mathcal{D}$th PA, consists of $n_0 + 1$ cells
- **Step 3:** Test the first PA using the backward boundary condition in Section II. If $q_1 > p_1^s \cdot (n_2 + 1)$, then p_1^s is moved to the second PA. Otherwise, keep the partitions obtained from Step 2. The testing of the first PA continues until the backward condition is satisfied.
- **Step 4:** Test the first PA using the forward boundary condition in Section II. If $q_1 < p_2^1 \cdot (n_2 - 1)$, then p_2^1 will be moved to the first PA. If this movement occurs, it is required to go back to Step 3, in which the backward boundary condition is tested again. This procedure continues iteratively until the forward condition is satisfied, i.e., $q_1 \geq p_2^1 \cdot (n_2 - 1)$.
- **Step 5:** Test the second PA using forward and backward boundary conditions as in Steps 3 and 4. This procedure continues until each PA has been tested and meets the conditions described in the Lemma. The finalized partitions form the optimal sequence that produces the minimum paging costs.
- **Step 6:** The system polls n_1 cells in $\mathrm{PA}_{\mathcal{P}}(1, q_1, n_1)$ first, followed by searching $\mathrm{PA}_{\mathcal{P}}(2, q_2, n_2)$, and so forth. The paging procedure stops when the called MT is found.

IV. Performance Analysis

In this section, numerical results are provided to compare the *paging costs* and *average delays* of the proposed scheme with three other paging schemes: broadcast paging, selective paging, and highest probability first (HPF) scheme for uniform and nonuniform location probability distributions. In one-step or broadcast scheme [2], [3], all cells in the LA are polled simultaneously so that the paging cost is the number of cells in the LA. For the selective paging scheme, one of its simulated cases is chosen. In this case, the LA is divided into three partitions with location probabilities 0.6, 0.2, and 0.2, respectively [1]. For the HPF scheme, an analogous continuous probability density function must be found for the nonincreasing discrete distribution. Nevertheless, how to find this continuous function is not provided in the paper [8]. In order to obtain the numerical results, the paging procedure of the original HPF, which is called the enhanced-HPF (E-HPF) scheme, is designed according to the theoretical description of the HPF.

The *paging costs* and *average delays* versus the delay bounds, $\mathcal{D}$ ($N = 20$), for the uniform distribution are shown in Fig. 1. These results reveal that the new paging scheme produces the minimum paging costs, which are the same as the theoretical results from HPF scheme for the uniform distribution. It is also observed that the average delay increases as the delay bound increases as shown in Fig. 1(b). Note that the paging costs are considered having higher priority than the average delays under the delay bounds.

In addition, the *paging costs* and *average delays* are investigated for nonuniform probability distributions. For example, in case A, the location probabilities are as follows: 0.36, 0.31, 0.05, 0.05, 0.045, 0.045, 0.04, 0.04, 0.03, and 0.03. In another case B, the location probabilities are created as 0.28, 0.26, 0.08, 0.08, 0.05, 0.05, 0.05, 0.05, 0.05, and 0.05. The *paging costs* and *average delays* of cases A and B are demonstrated in Table I.



(a) (b)

Fig. 1. Paging costs and delays for uniform distribution.

TABLE I
THE COMPARISON OF PAGING COSTS AND DELAYS

Partitions (PAs)		$PA(i, q_i, n_i)$	$E[C(\mathcal{D})]$	$E[D(\mathcal{D})]$
Case A $\mathcal{D}=3$	Optimal	(1, 0.36, 1); (2, 0.36, 2): (3, 0.28, 7)	**4.24**	1.92
	Selective	(1, 0.55, 5): (2, 0.31, 1): (3, 0.14, 4)	6.01	1.59
	E-HPF	(1, 0.77, 4): (2, 0.13, 3): (3, 0.10, 3)	4.99	*1.33*
Case A $\mathcal{D}=4$	Optimal	(1, 0.36, 1); (2, 0.31, 1): (3, 0.19, 4): (4, 0.14, 4)	**3.52**	2.11
	Selective	(1, 0.72, 3); (2, 0.14, 3); (3, 0.08, 2)	6.01	1.59
	E-HPF	(1, 0.72, 3); (2, 0.14, 3); (3, 0.08, 2); (4, 0.06, 2)	4.28	*1.48*
Case A $\mathcal{D}=5$	Optimal	(1, 0.36, 1); (2, 0.31, 1): (3, 0.10, 2): (4, 0.13, 3): (5, 0.10, 3)	**3.29**	2.03
	Selective	(1, 0.55, 5): (2, 0.31, 1): (3, 0.14, 4)	6.01	*1.59*
	E-HPF	(1, 0.67, 2); (2, 0.10, 2); (3, 0.09, 2); (4, 0.08, 2); (5, 0.06, 2)	3.52	1.76
Case B $\mathcal{D}=3$	Optimal	(1, 0.54, 2); (2, 0.26, 4): (3, 0.20, 4)	**4.64**	1.66
	Selective	(1, 0.62, 3): (2, 0.18, 3): (3, 0.20, 4)	4.94	1.58
	E-HPF	(1, 0.70, 4); (2, 0.15, 3); (3, 0.15, 3)	5.35	*1.45*
Case B $\mathcal{D}=4$	Optimal	(1, 0.54, 2); (2, 0.16, 2): (3, 0.15, 3): (4, 0.15, 3)	**4.27**	1.91
	Selective	(1, 0.62, 3): (2, 0.18, 3): (3, 0.20, 4)	4.94	*1.58*
	E-HPF	(1, 0.62, 3); (2, 0.18, 3); (3, 0.10, 2); (4, 0.10, 2)	4.74	1.68
Case B $\mathcal{D}=5$	Optimal	(1, 0.54, 2); (2, 0.16, 2): (3, 0.10, 2): (4, 0.10, 2): (5, 0.10, 2)	**4.12**	2.06
	Selective	(1, 0.62, 3): (2, 0.18, 3): (3, 0.20, 4)	4.94	*1.58*
	E-HPF	(1, 0.54, 2); (2, 0.16, 2); (3, 0.10, 2); (4, 0.10, 2); (5, 0.10, 2)	**4.12**	2.06
Broadcast Scheme		(1,1,10)	10	1

The details of each PA, such as the paging sequence, the location probability, and the number of cells under $\mathcal{D} = 3, 4, 5$, are also presented in Table I. The minimum paging costs and delays are indicated in bold and italics, respectively. It can be seen that the optimal paging scheme always results in the minimum paging costs. In addition, the new scheme is simple to implement and significantly reduces the paging costs for various probability distributions.

V. CONCLUSION

An optimal paging scheme which is capable of minimizing paging costs under delay bounds has been presented. This scheme is simple to implement and it is applicable to arbitrary location distributions. In particular, when the location probability distribution is nonuniform, which is not considered in other paging schemes, the proposed algorithm is still very effective in minimizing the paging costs.

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