Information Subspace-Based Fusion for Vehicle Classification

Sally Ghanem, Ashkan Panahi, Hamid Krim
Department of Electrical and Computer Engineering
North Carolina State University
Raleigh, North Carolina

Ryan A. Kerekes
Oak Ridge National Laboratory
Oak Ridge, Tennessee

John Mattingly
Department of Nuclear Engineering
North Carolina State University
Raleigh, North Carolina

***Abstract*—Union of Subspaces (UoS) is a new paradigm for signal modeling and processing, which is capable of identifying more complex trends in data sets than simple linear models. Relying on a bi-sparsity pursuit framework and advanced non-smooth optimization techniques, the Robust Subspace Recovery (RoSuRe) algorithm was introduced in the recent literature as a reliable and numerically efficient algorithm to unfold unions of subspaces. In this study, we apply RoSuRe to prospect the structure of a data type (e.g. sensed data on vehicle through passive audio and magnetic observations). Applying RoSuRe to the observation data set, we obtain a new representation of the time series, respecting an underlying UoS model. We subsequently employ Spectral Clustering on the new representations of the data set. The classification performance on the dataset shows a considerable improvement compared to direct application of other unsupervised clustering methods.**

***Index Terms*—Sparse learning, Classification, Magnetic sensors, Acoustics.**

I. Introduction

Recent developments in sensor technology have provided many possibilities in developing real-time transportation systems technologies. Traffic flow optimization, dynamic traffic management solutions, vehicle counting, travel time estimation, and other traffic modeling studies frequently require classification and identification of streams of vehicles. Moreover, accurate estimation of traffic parameters needs to be performed in real time for decision makers [1] [2]. Conventional vehicle identification methods such as license plate recognition and Radio Frequency Identification Tags (RFID) have been widely used for that purpose for so long [3] [4]. Unfortunately, such image-based methods are not appropriate for studies that require low-power consumption and low cost. Additionally, privacy issues becoming front and center have raised the concern over with image acquisition. On the contrary, further alternatives such as magnetic sensors and microphones are inexpensive and do not raise privacy concerns [5] [6]. Vehicles are primarily of metallic structure that perturb the earth's magnetic field, and hence produce unique magnetic signatures that have been served to discriminate between vehicles [7] [8]. Audio sensors have also been extensively employed in the area of vehicle classification for different applications, and have proven their effectiveness and robustness [9] [10].

The objective of this work is to devise a multi-modal approach to vehicle classification and identification using an ensemble of sensors consisting of a magnetometer and three microphones. We consider a more realistic unsupervised learning scenario, where no training dataset is provided and adopt a data driven approach to determine vehicle signatures utilizing key features extracted from each sensor modality. We subsequently combine the features from each sensor modality to generate a desirable universal feature and increase the classification rate of specific vehicle classes. Despite simplicity in acquisition, magnetic and audio data are challenging to directly exploit due to the dimensionality of the collected data. For feature extraction, we use RoSuRe, where high dimensional data is assumed to lie in a union of low dimensional subspaces capturing underlying common hidden features, albeit possibly adversely affected by errors. Sparse modeling have been extensively utilized in the computer vision and machine learning literature to obtain linear models under the influence of perturbation [11] [12] [13] [14] [15]. We consider the procedure studied in [16], in which a bi-sparse model, known as Robust Subspace Recovery via Bi-sparsity Pursuit, is employed as a framework to recover the union of subspaces in the presence of sparse corruptions. The UoS structure is unveiled by pursuing sparse self-representation of the given data. We employ the bi-sparsity framework to recover the underlying subspace structure in each sensor modality and obtain a finer level of classification by combining them. We also use the resulting UoS structure to classify new observed data points, which illustrates the generalization power of our technique.

The paper is organized as follows. In Section 2, we provide the fundamental concepts of the Bi-sparsity pursuit for RoSuRe. In Section 3, we introduce the different sensing modalities that will be utilized for experimentation along with the pre-processing, feature selection and extraction techniques. In Section 4, we present the experimental results of our approach, while Section 5 provides concluding remarks.

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II. Algorithm: Robust subspace recovery via Bi-sparsity Pursuit

In this Section, we present a summary of the Robust Subspace Recovery via Bi-Sparsity Pursuit (RoSuRe) introduced in [16]. The algorithm assumes that all data samples may be corrupted by additive sparse errors. Therefore, the UoS structure is often corrupted and each data sample deviates from its original subspace. Precisely, considering a set of data samples $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n]$, where $\mathbf{x}_k \in R^m$ is a data point, n corresponds to the number of observations and m specifies the number of variables or features in each observation, the columns of the matrix $\mathbf{X}$ may be partitioned such that each part $\mathbf{X}_I$ is decomposed into a low dimensional subspace and a sparse corruption:

$$\mathbf{X_I} = \mathbf{L_I} + \mathbf{E_I}, I = 1, ..., J, \tag{1}$$

where each $\mathbf{L}_I$ serves as a single low dimensional subspace of the original data, and $\mathbf{L} = [\mathbf{L}_1|\mathbf{L}_2|..........|\mathbf{L}_J]$ is the desired union of subspaces. Furthermore, the partition recovers the clusterings of the original data samples corrupted by the error $\mathbf{E} = [\mathbf{E}_1|\mathbf{E}_2|......|\mathbf{E}_J]$. The objective of this approach is to simultaneously retrieve the subspaces and the noiseless samples from the observed noisy data. The RoSuRe via Bi-Sparsity pursuit is based on the idea of self-representation. In other words, $\mathbf{l}_i$ can be represented by the other samples from the same subspace $S(\mathbf{l}_i)$.

$$\mathbf{l_i} = \sum_{i \neq j, I_j \in S(I_i)} w_{ij} \mathbf{l_j}. \tag{2}$$

The above relation can be represented in a matrix form as follows,

$$\mathbf{L} = \mathbf{LW}. \tag{3}$$

Under a suitable arrangement of the data points, the sparse coefficient matrix $\mathbf{W}$ is an $n \times n$ block-diagonal matrix with zero diagonals provided that each sample is represented by other samples only from the same subspace. More precisely, $W_{ij} = 0$ whenever the indexes i, j correspond to samples from different subspaces. As a result, the majority of the elements in W is equal to zero. After further approximations and relaxations, the problem is formulated as follows,

$$\begin{aligned} &\min_{\mathbf{W},\mathbf{E},\mathbf{L}} \|\mathbf{W}\|_1 + \lambda \|\mathbf{E}\|_1, \\ &\text{s.t. } \mathbf{X} = \mathbf{L} + \mathbf{E},\ \mathbf{L} = \mathbf{LW},\ W_{ii} = 0. \end{aligned} \tag{4}$$

where $\|\|_1$ denotes the l_1 norm, i.e. the sum of absolute values of the argument. The minimum of Eqn.(4) is approximated through linearized Alternating Direction Method of Multipliers ADMM [17] and the sparsity of both $\mathbf{E}$ and $\mathbf{W}$ is traced until convergence. See [16] for more details.

III. Multi-modal sensing

As previously stated, the goal is to recover the union of subspace structure underlying the data measurements from each sensor modality and then integrate the obtained structures to increase the classification rate and support decision making. A roadside sensor system was exploited to collect data from passing vehicles using various sensors, including a camera, microphone, laser range-finder, magnetometer, and low-frequency RF antenna. In this study, we are using the signatures captured using passive magnetic and acoustic sensors. The magnetic signatures are recorded using a single three-axis magnetic sensor, while the acoustic data is collected by a set of three dimensional microphones. The sensors were mounted on a rigid rack for ease of deployment and management. The data collection was conducted in a park environment, with limited public interference. The data is collected for seven different vehicles; two SUVs, one sedan and four trucks. The two SUVs are GMC Yukon and Hyundai Tucson, the sedan is Honda Accord. The four trucks are Chevrolet pickup truck, 14 ft rental moving truck and two Ford F-150s, one has a mounted top on the bed and the other one does not. The seven different vehicles were driven by the system yielding a total of 546 observations per sensor. Our goal is to analyze the dataset and distinguish seven classes where each class corresponds to one car. Furthermore, our goal is to be able to classify a newly observed dataset, using the structure learned through the current unlabeled data. For this purpose, the observations were divided into training and testing as discussed in Table 1. As shown in the table, we used 50 observations for each car in the learning phase, and the rest of the observations for validation. Sample outputs of the magnetometer and microphone are shown in Figs. 1 and 2.

TABLE I
The dataset description

Vehicle	Training points	Testing points
Chevrolet Truck	50	29
Ford F-150 (Topper)	50	19
Ford F-150	50	31
GMC Yukon	50	24
Honda Accord	50	41
Hyundai Tuscon	50	20
Uhaul Truck	50	32

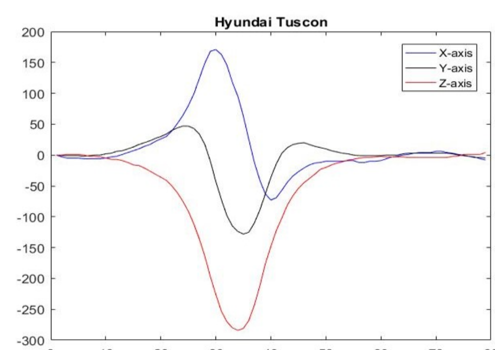

Fig. 1. Sample output for magnetometer data.

Acoustic sensors have been analyzed in various applications related to automatic transportation systems [18] [19] [20].

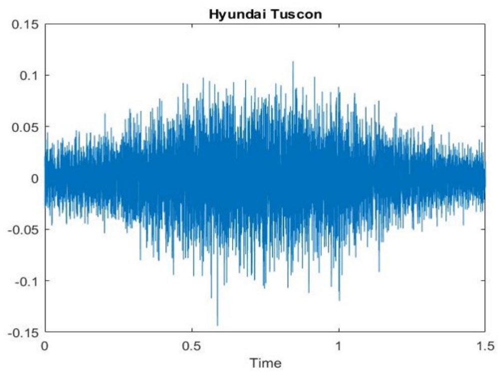

Fig. 2. Sample output for audio data.

Mel Frequency Cepstral Coefficients [21] are widely used in automatic speech recognition literature. They were introduced by Davis and Mermelstein in the 1980's, and have been the state-of-the-art ever since. The mel-frequency cepstrum (MFC) is a representation of the short-term power spectrum of a sound, based on a linear cosine transform of a log power spectrum on a nonlinear mel scale of frequency. We extract and process MFCCs from our audio data. In our experiments, a low pass filter was applied to the audio signals to remove noise. Signals were downsampled from 92 kHz to 64 kHz. The audio signals were divided into windows of size 0.025 seconds with a step size of 0.01 seconds to allow some overlap between the frames, and get a reliable spectral estimate. Finally, the MFFCs were extracted for each window and averaged to result in 26 log filterbank energies for each observation.

Magnetic sensors operate by detecting the variation in the magnetic inductance. Magnetic signatures can be characteristic of the vehicle of interest. Earth's magnetic field distortion can be used not only for the detection, but also for the classification and recognition of transport vehicles [22] [23] [7] [8]. The three-axis system exploited is capable of producing up to 154 Hz and outputs 16-bit values with 67 Gauss resolution. In our experiment, a sample rate of 40 Hz has been used. For calibration, the magnetic signatures were extracted from the magnetic signals by subtracting the value of the local magnetic field, which is measured when no car passed by the sensor. Afterwards, the beginning and the ending of the signal are determined. Each observation is then normalized and re-sampled to get a normalized length of 100 samples per axis and a total of 300 samples per observation. The X, Y and Z signal amplitudes are re-scaled to be in the [-1,1] interval.

IV. EXPERIMENTAL RESULTS

In the following, we use the RoSuRe technique to recover the subspace structure embedded in the data associated with each magnetic or audio observation. The sparse solution of the problem in Eqn.(4), $\mathbf{W}$, provides important information about the relations among data points, which may be used to split data into individual clusters residing in a common subspace. Observations from each car can be seen as data points from one subspace.

A. Processing Algorithm

First, we extract the principal components of the data corresponding to each sensor [24]. The largest 100 principal values for magnetometer data and the largest 20 principal components for audio data are selected to serve as representatives of the data in the principal component space. More precisely, we identify a lower dimensional space whose corresponding basis vectors maximize the variability of the data. The sparse coefficient matrix $\mathbf{W}_m$ $\{m=$ audio, magnetometer$\}$ is computed, from RoSuRe, for each sensor modality by solving (4), taking $\mathbf{X}$ as the PCA representation of the data points. Next, we threshold $\mathbf{W}_m$ by its median value. We exploit the resulting $\mathbf{W}_m$ to evaluate an affinity matrix. The affinity matrix is computed by,

$$\mathbf{A}_m = \mathbf{W}_m + \mathbf{W}_m^T. \tag{5}$$

Subsequently, the spectral clustering method in [25] is utilized for data clustering. The method can be summarized as follows, a matrix $\mathbf{D}$ is defined to be a diagonal matrix whose i^{th} diagonal element is the degree of the i^{th} node, $i.e.$ the sum of i^{th} row in $\mathbf{A}_m$. The standard graph Laplacian matrix is next constructed as follows,

$$\mathbf{L} = \mathbf{D}^{-1/2}\mathbf{A}_m\mathbf{D}^{-1/2}.$$

Next, the eigenvectors $\mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_k$ of $\mathbf{L}$ corresponding to the largest k eigenvalues are computed, where k is the desired number of clusters. Then, the matrix $\mathbf{S} = [\mathbf{s}_1\mathbf{s}_2..........\mathbf{s}_k]$ is formed by stacking the eigenvectors in columns. Treating each row of $\mathbf{S}$ as a point in $\mathbb{R}^k$, k-means is then used to cluster the rows of $\mathbf{S}$. Finally, the original point $\mathbf{x}_i$ is assigned to cluster j iff row i of the matrix $\mathbf{S}$ was assigned to cluster j. The sparse coefficient matrices for magnetic and acoustic sensors are respectively illustrated in Figs. 3 and 4. The block-diagonal structure can be clearly seen from either of the matrices.

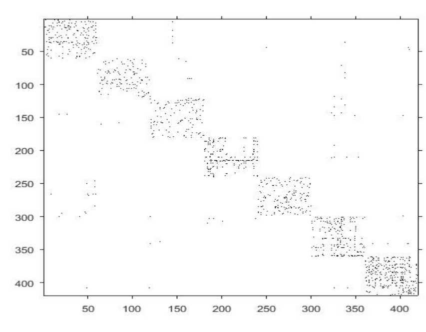

Fig. 3. The sparse coefficient matrix for magnetometer data ($\mathbf{W}_{magnetic}$).

TABLE II
THE CLUSTERING PERFORMANCE FOR DIFFERENT CLUSTERING METHODS

	RoSuRe	kmeans	GMM	HCA
Magnetomter data	86.71%	82.29%	77.14%	64.57%
Audio data	86.57%	52.1%	62.57%	40%
Fused	98.29%	82.29%	77.14%	64.57%

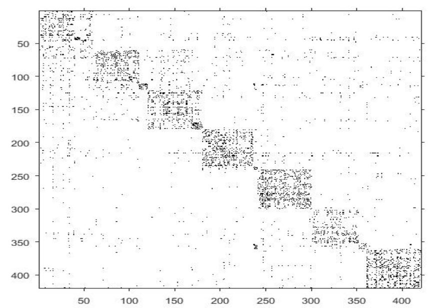

Fig. 4. The sparse coefficient matrix for audio data ($\mathbf{W}_{audio}$).

B. Fusing the Two Modalities

In order to improve the performance of our method, the two sparse matrices $\mathbf{W}_{audio}$ and $\mathbf{W}_{magnetic}$ are added to produce one sparse matrix for both modalities, $\mathbf{W}_{total}$. By doing so, we reinforce the contribution of similar representations that exist in both modalities. The overall sparse matrix, $\mathbf{W}_{total}$ is displayed in Fig. 5. Observations belonging to one car are clustered as one subspace in which the contribution of each sensor is embedded in the entries of the $\mathbf{W}_{total}$. For clustering by $\mathbf{W}_{total}$, we applied the same spectral clustering approach that we previously explained. As a result, the classification accuracy improved to 98.29% as highlighted in Table 2. The performance of RoSuRe was compared against three widely used unsupervised clustering algorithms, namely, k-means, the Gaussian mixture model and hierarchical cluster analysis (HCA). k-means clustering, also referred to as the Lloyd-Forgy algorithm, is a computationally efficient method for cluster analysis in data mining [26]. k-means clustering aims to partition n observations into k clusters in which each observation belongs to the cluster with the nearest mean, serving as a representative of the cluster. A Gaussian mixture model is a probabilistic model which assumes all the data points generated from a mixture of a finite number of Gaussian distributions with unknown parameters. Mixture models can be considered as a generalization for k-means clustering to incorporate information about the covariance structure of the data as well as the centers of the latent Gaussians. Mixture models are in general less sensitive to the initialization of centroids. They have been used for feature extraction from speech data and object tracking [27] [28] [29]. Hierarchical clustering is a technique which aims to build a hierarchy of clusters [30]. For our experiment, we used a bottom-up approach where all observations start in their own cluster, pairs of clusters are subsequently merged together according to their closeness. The euclidean distance, $d(\mathbf{x}_i, \mathbf{x}_j) = \|\mathbf{x}_i - \mathbf{x}_j\|_2$, was used as a proximity measure between each pair of data points. We used complete-linkage criterion to measure the distance between clusters where the distance $D(X, Y)$ between clusters X and Y is described as follows: $D(X, Y) = \max_{\mathbf{x}\in X, \mathbf{y}\in Y} d(\mathbf{x}, \mathbf{y})$ The results are displayed in Table 2. As shown in the table, RoSuRe has the highest classification accuracy for both audio and magnetometer data. Moreover, after fusing the two data modalities, RoSuRe shows a significant enhancement in the classification performance.

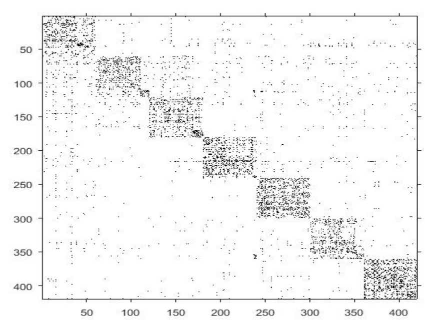

Fig. 5. The overall sparse coefficient matrix ($\mathbf{W}_{total}$)

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Additionally, we compared the RoSuRe fusion performance with the other unsupervised clustering methods through linking the two modalities features. More precisely, we concatenated both magnetometer and audio observations in one vector and we then clustered the new representation of the data. The results in Table 3 show that, by concatenating the data, we are not gaining extra information. Moreover, the classification accuracy after concatenation is the same as that of the magnetometer data because of the higher dimensionality of magnetometer observations as compared to audio observations. Therefore, the results were biased towards the former modality. Whereas, by integrating the sparse coefficient matrix corresponding to each modality, we have obviously boosted the performance of RoSuRe from approximately 86% to 98.29%.

C. Experimental Validation of Classification

After learning the structure of the data clusters, we validate our results on the test data. We extract the principal components (eigen vectors of the covariance matrix) of each cluster in the original (training) dataset, to act as a representative subspace of its corresponding class. We subsequently project each new test point onto the subspace corresponding to each cluster, spanned by its principal components. The l_2 norm of the projection is then computed, and the class with the largest norm is selected to be the class of this test point. For the RoSuRe algorithm, we used the coefficient matrix $\mathbf{W}_{total}$ to cluster the test data points for both magnetometer and audio data. However, classification on the test data is separately performed for each data modality. The simulation results are listed in Table 3. From the results, it is clear that the RoSuRe technique for the fused data remarkably outperforms the other clustering methods.

V. CONCLUSION

In this paper, we proposed a novel approach to fuse passive signal acquired by low power instruments through recovering the underlying subspaces of data samples from measured data corrupted by sparse errors. One advantage of using passive

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TABLE III
THE VALIDATION PERFORMANCE FOR THE DIFFERENT CLUSTERING METHODS

	RoSuRe	kmeans	GMM	HCA
Magnetomter data	90.31%	71.43%	61.73%	57.65%
Audio data	90.31%	66.33%	65.82%	53.06%

sensors is the preservation of privacy and lower cost. The RoSuRe method is used to reliably recover the subspace for different modalities. It also provides a natural way to fuse the data by employing the RoSuRe self representation matrix as an embedding in a shared domain. Experiments on real data are presented to demonstrate the effectiveness of our method in solving the problem of subspace fusion with sparsely corrupted unlabeled data. We also show that the use of multiple sensing methods enhances the performance, and offers more flexibility.

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