Scheduling Partition for Order Optimal Capacity in Large-Scale Wireless Networks

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ABSTRACT

The capacity scaling property specifies the changes in network throughput when network size increases and serves as an essential performance evaluation metric for large-scale wireless networks. Existing results have been obtained based on the implicit assumption of negligible overhead in acquiring the network topology and synchronizing the link transmissions. In large networks, however, global topology collection and global link synchronization are infeasible with both the centralized and the distributed link scheduling schemes. This gap between the well-known capacity results and the impractical assumption on link scheduling potentially undermines our understanding of the *achievable* network capacity. Therefore, the following question remains open: can *localized* scheduling algorithms achieve the same order of capacity as their global counterpart? In this paper, we propose the *scheduling partition* methodology by decomposing a large network into many small autonomous scheduling zones, in which localized scheduling algorithms are implemented independently from one another. We prove that any localized scheduling algorithm that satisfies a set of sufficient and necessary conditions can yield the same order of capacity as the widely assumed global scheduling strategy. In comparison to the network dimension $\sqrt{n}$, scheduling partition sizes $\Theta(\sqrt{\log n})$ and $\Theta(1)$ are sufficient for optimal capacity scaling in the random and the arbitrary node placement models respectively. We finally propose an example localized link scheduling algorithm to verify the capacity achieved by scheduling partition. Our results thus provide guidelines on the scheduling algorithm design toward maximum capacity scaling in large-scale wireless networks.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design – Wireless communication; C.2.3 [**Computer-Communication Networks**]: Network Operations – Network management

General Terms

Algorithms, Design, Theory

Keywords

Network design, wireless multihop networks, capacity scaling, network decomposition, link scheduling

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1. INTRODUCTION

The wireless networks have gained tremendous success as a substitute and extension for the traditional wired networks in many places. As the user population increases, deployment of large-scale wireless networks is expected to accommodate the increasing demand for wireless services. The capacity scaling property is one of the most important metrics to evaluate the performance of large wireless networks, which indicates the trend of throughput changes when the network size increases. Large network size helps diversify path selections and balance traffic flows, but it intensifies the wireless interference among users and impacts the network capacity negatively. In order to understand the capacity scaling of large wireless networks, many insightful results have been obtained in the literature [1–11] that provide the limiting upper bounds and the achievable lower bounds on network capacity.

Specifically, motivated by the seminal work [1], many efforts have been made to find the upper and lower capacity bounds for different communication types in various network settings. On one hand, the constraints of wireless interference and multihop relay prevent a large wireless network from reaching arbitrarily high throughput, while on the other hand, thoughtful link scheduling can avoid transmission collisions such that the achievable throughput is within a constant fraction of the upper bound. The work in [1] considered the throughput of unicast communications. The study was extended to broadcast communications in [4–6] and later to multicast communications in [7, 8]. Recently, the capacity results of unicast, broadcast and multicast were unified by introducing the concept of (n, m, k)-casting [9]. Another important discovery on wireless network capacity with random node placement was made by utilizing the percolation method [10] and followed by the improved bounds on multicast capacity [11]. All of these results were obtained by assuming a globally collision-free link scheduling algorithm, which, however, may not be available in large wireless networks.

Collision-free transmissions are implemented in wireless networks via link scheduling schemes. The goal of scheduling is to coordinate transmissions such that every transmission is guaranteed successful and network throughput is maximized, i.e., to find the shortest schedule that fulfills all the link transmission requests. In the approach of centralized scheduling [12–16], a designated entity collects the entire network topology information and computes a schedule that allocates different time slots to the interfering links which otherwise would collide if transmitting concurrently. As network size increases, centralized scheduling clearly becomes infeasible because of the requirement for global topology collection. The distributed scheduling algorithms [17–21] work better in large networks by not collecting global topology. Instead, nodes exchange messages with neighbors to reach a consensus regarding their respective transmission times. As a result of the consensus, a maximal set of collision-free links, called *maximal matching* [22], transmit simultaneously and interfering links transmit at different times. However, distributed scheduling is not a feasible solution for large wireless networks either. The amount of exchanged messages and the latency to determine the transmission schedule increase as network size grows. Furthermore, network-wide clock synchronization is required in order for each link to adhere to the established schedule. Any clock error may result in the failure of correct timing in link transmissions. Besides scheduling, random access [23, 24] has also been studied in the literature as a simple and alternative approach to link coordination, but the capacity bounds with random access are unknown yet in large wireless networks, where the medium access competition among neighboring nodes may lead to high chance of transmission failures.

Therefore, the inapplicability of existing link scheduling schemes in large wireless networks is in sharp contrast to the assumption of a globally collision-free link transmission schedule in the current study of network capacity scaling. The gap between them presents an open question: can localized link scheduling algorithms achieve similar capacity scaling as their global counterpart? The answer to this question is important as it bridges the disconnection between the theoretical capacity results and the practical implementation toward capacity maximization.

In this paper, we propose a *scheduling partition* methodology to address the feasibility of maximum capacity scaling in large-scale wireless networks. We decompose a large wireless network into many small partitions with the links in each partition scheduled independently from other partitions. The network decomposition approach thus significantly reduces the scheduling complexity as compared to the existing algorithms. When designed properly, the scheduling complexity is constant for each partition, regardless of the network size. Note that scheduling partition may introduce link collisions due to the absence of coordination across partitions, which breaches the requirement for collision-free scheduling. Nevertheless, we provide a set of partition and scheduling principles that guarantee the infringement not to jeopardize the objective of maximum capacity scaling. We intend to characterize and present scheduling partition as a general methodology. Meanwhile, we also design a simple localized link scheduling algorithm as an application under this framework and verify the effectiveness of scheduling partition in achieving maximum capacity scaling with constant scheduling complexity.

The rest of this paper is structured as follows. We describe the network models and formulate the scheduling problem in Section 2. Our main results are summarized in Section 3. Sections 4 and 5 present the proofs of our discoveries. In Section 6 we design a simple localized scheduling algorithm to verify the effectiveness of our scheduling partition approach. Lastly, Section 7 concludes this paper.

2. NETWORK MODELS AND PROBLEM FORMULATION

A variety of network models have been used in the literature to represent different scenarios of network expansions, node communications, location distributions and wireless interferences. As we study a generally applicable scheduling method, we consider all these models in this paper.

2.1 Network Models

We consider in this paper the expansion of network size as the well-known *extended network* model, which was initially introduced in [1] and later widely used for wireless network capacity study, e.g., [2–11]. The extended network is characterized by n nodes distributed in a square region $\mathcal{B}$ with area $|\mathcal{B}| = n$. As network scales $n \to \infty$, the node density keeps constant 1. Another popular scaling model, the *dense network*, differs from the extended model by a factor of $\sqrt{n}$. After including this scaling factor, our results in this paper also apply to dense networks. Besides network expansion, we also consider the following models for wireless interferences, node locations and communication scenarios.

2.1.1 Interference Models

Three models are widely used in the literature to represent wireless interference: the *protocol* model, the *physical* model and the *generalized physical* model.

The protocol model ($\mathbb{I}_{\text{prot}}$) [1] specifies a successful transmission from node v_i to node v_j if

$$|X_i - X_j| \le r(n), \tag{1}$$

and for any other simultaneously transmitting node v_k

$$|X_k - X_j| \ge (1+\Delta) r(n), \tag{2}$$

where X_i, X_j and X_k are the locations of v_i, v_j and v_k, $r(n)$ is the critical transmission radius of all nodes, and Δ models the guard zone around v_j in which any simultaneous transmission causes collision at v_j. Whenever a transmission is successful, v_i communicates to v_j at a constant data rate $w_{ij} = W$. Otherwise, whenever collision occurs, $w_{ij} = 0$.

The physical model ($\mathbb{I}_{\text{phy}}$) [1] requires a minimum Signal-to-Interference-plus-Noise-Ratio (SINR) at v_j in order for a transmission from v_i to v_j to be successful, as shown below.

$$\frac{\frac{P_i}{|X_i - X_j|^\alpha}}{BN_0 + \sum_{k \ne i} \frac{P_k}{|X_k - X_j|^\alpha}} \ge \beta, \tag{3}$$

where v_k is any simultaneously transmitting node, $P_{\min} \le P_i, P_k \le P_{\max}$ are the transmission powers, B is the spectrum bandwidth, N_0 is the spectrum density of ambient noise, $\alpha > 2$ is the path loss exponent, and β is a constant. We assume that all the transmissions occur within radius $r(n)$. In order to overcome the singularity problem [25] that occurs when v_i and v_j are arbitrarily close and the received signal power at v_j is amplified unrealistically by the path loss model, we assume a minimum distance $r_0(n) = \varepsilon r(n)$ ($0 < \varepsilon < 1$) for every transmission. In summary, $r_0(n) \leq |X_i - X_j| \leq r(n)$. Besides, we assume the ambient noise is non-negligible as compared to the received signal power, i.e., $\gamma_1 BN_0 \leq P_{\min} r^{-\alpha}(n) \leq P_{\max} r^{-\alpha}(n) \leq \gamma_2 BN_0$ where $\gamma_1, \gamma_2 > 0$ are constants. The data rate is $w_{ij} = W$ for successful transmissions or $w_{ij} = 0$ for failed transmisssions.

The generalized physical model ($\mathbb{I}_{\text{gen}}$) [6] differs from the physical model in the data rate w_{ij}, which is determined as

$$w_{ij} = B \log_2 \Big(1 + \frac{\frac{P_i}{|X_i - X_j|^\alpha}}{BN_0 + \sum_{k \neq i} \frac{P_k}{|X_k - X_j|^\alpha}}\Big). \tag{4}$$

As in the physical model, we assume $P_{\min} \leq P_i, P_k \leq P_{\max}$, $\alpha > 2$, $r_0(n) \leq |X_i - X_j| \leq r(n)$, and $\gamma_1 BN_0 \leq P_{\min} r^{-\alpha}(n)$ $\leq P_{\max} r^{-\alpha}(n) \leq \gamma_2 BN_0$. Note that due to non-negligence of BN_0, the variable link data rate is upper bounded by

$$w_{ij} \leq B \log_2(1 + \frac{P_{\max}(\varepsilon r(n))^{-\alpha}}{BN_0}) \leq B \log_2(1 + \gamma_2 \varepsilon^{-\alpha}). \tag{5}$$

2.1.2 Location Models

We consider two prevailing node location models: *random* and *arbitrary*. In random networks ($\mathbb{L}_{\text{rand}}$) [1], node locations are distributed in a random Poisson point process. As $n \to \infty$, $r(n) = \Theta(\sqrt{\log n})$ is required for network connectivity. In arbitrary networks ($\mathbb{L}_{\text{arbi}}$) [1], node locations are assigned in need. One example is to place the nodes on a grid with equal distance between neighbors. In this case, as $n \to \infty$, $r(n) = \Theta(1)$ is sufficient for network connectivity.

2.1.3 Communication Models

Three communication models are studied in the literature: *unicast*, *broadcast* and *multicast*. The unicast model ($\mathbb{C}_{\text{uni}}$) [1–3] assumes n source-destination pairs. Every node in the network is a source and it selects another node randomly as its destination. In the broadcast model ($\mathbb{C}_{\text{bro}}$) [4,5], each node disseminates its packets to all the other $n-1$ nodes. As the transition between unicast and broadcast, the multicast model ($\mathbb{C}_{\text{mul}}$) [6–8] disseminates the packets from each node to $k-1$ randomly chosen destinations. It is equivalent to unicast if $k = 2$ and to broadcast if $k = n$.

From a single transmission point of view, these communication models differ in the number of receivers during one transmission. In unicast there is only one receiver for each transmission, while in broad/multi-cast there could be multiple receivers at the branching points of the broad/multicast tree. For presentation convenience, we define the group of links sharing the same branching point in a broad/multicast tree as *broad/multi-cast branches* when the branching node sends the same packet along these links. In addition, we define the number of communication sessions that traverse a node v_i as the *node session degree* ζ_i and the number of sessions that traverse a link l_{ij} (l_{ij} is directional) as the *link session degree* ζ_{ij}. We further define $\eta_{ij} = \frac{\zeta_{ij}}{\zeta_i}$ to denote the percentage of time spent on l_{ij} when v_i is transmitting. Finally, we assume the packet length L bounded by $L_{\min} \leq L \leq L_{\max}$.

2.2 Problem Formulation

In this paper, we propose a scheduling partition methodology for achieving the optimal capacity scaling. We will show that, by proper network partitioning and link scheduling, network capacity scales on the same order as the theoretical results [1–11], but with significantly reduced scheduling complexity that can be as small as a constant. Before problem formulation, we define a few relevant concepts.

DEFINITION 1. *Asymptotic bounds. (i) $f(n) = O(g(n))$ means there exists a constant c such that $f(n) \leq cg(n)$ as $n \to \infty$; (ii) $f(n) = o(g(n))$ means $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$; (iii) $f(n) = \Omega(g(n))$ means $g(n) = O(f(n))$; (iv) $f(n) = \omega(g(n))$ means $g(n) = o(f(n))$; (v) $f(n) = \Theta(g(n))$ means $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$. (See [3].)*

DEFINITION 2. *Scheduling diameter. The diameter of a scheduling algorithm $\mathcal{S}(t) = \{l_{ij}(t) : l_{ij}$ is active at time $t\}$ is defined as $\Phi(\mathcal{S}(t)) = \max_{\{l_{i_1 j_1}, l_{i_2 j_2} \in \mathcal{S}(t)\}} \{|X_{i_1} - X_{i_2}|\}$.*

DEFINITION 3. *Scheduling localization. In extended networks, algorithm $\mathcal{S}(t)$ is global if $\Phi(\mathcal{S}(t)) = \Theta(\sqrt{n})$ and algorithm $\mathcal{S}(t)$ is localized if $\Phi(\mathcal{S}(t)) = o(\sqrt{n})$.*

DEFINITION 4. *Scheduling partition. A scheduling partition is a geographic region in which the complete topology information is collectable for collision-free link scheduling. This paper considers convex polygonal partitions only.*

DEFINITION 5. *Scheduling complexity. We consider centralized scheduling algorithm in each partition. The scheduling complexity is the number of computational steps required in one run of the algorithm.*

All the previous scheduling algorithms assumed in the capacity scaling study [1–11] are global, thus facing the implementability problem in large-scale networks. Our objective is to achieve the same order optimal capacity with manageable scheduling complexity. By taking the partition approach, the scheduling task is completed independently in individual partitions, thus avoiding network-wide topology collection or information exchange. The challenge for the partition approach is the simultaneous satisfaction of two goals: maximum capacity (at least in the order sense) and minimum complexity (constant, if possible). Our scheduling partition problem is hence formulated as follows.

DEFINITION 6. *Network capacity. A data rate $\lambda(n)$ is the network capacity if there exists a joint packet routing and link scheduling scheme such that every node in the network can send data at rate $\lambda(n)$ to its destinations losslessly, but not with any rate higher than $\lambda(n)$.*

Note that $\lambda(n)$ is determined jointly by the packet routing protocol and the link scheduling algorithm. As our research focus is the possibility of using localized scheduling algorithms to achieve order optimal capacity, we assume that the same routing protocol (the specific protocol selection is insignificant here) is applied when we compare the performance of global and localized scheduling algorithms.

Denoting $\lambda_g(n)$ as the $\lambda(n)$ when *any* of the global scheduling algorithms assumed in [1–11] is used, and $\lambda_l(n)$ as the $\lambda(n)$ when we partition the network and implement a localized scheduling algorithm in each partition, we will prove

$$\lambda_l(n) = \Theta(\lambda_g(n)) \tag{6}$$

is possible if we partition the network and design the localized scheduling algorithms appropriately. In other words, given $\lambda_g(n) = \Theta(f(n))$, we will demonstrate the way to achieve $\lambda_l(n) = \Theta(f(n))$, regardless of the function $f(n)$ that depends on the particular assumptions on the interference, location and communication models.

3. MAIN RESULTS

The key question in this study is *whether* and *how* scheduling partition can achieve the order optimal capacity in large-scale wireless networks. To facilitate our study, we characterize the scheduling partition methodology by using three parameters and define a class of localized scheduling algorithms $\{\mathcal{S}_{l(\rho,\delta,\xi)}(t)\}$ that satisfy three specifications.

S1: Each partition is a convex polygon disjoint from others. It contains a disk of radius $\rho(n)$ and is contained in a larger co-centric disk of radius $\sigma\rho(n)$ (constant $\sigma > 1$). The disk center is the center of the partition.

S2: In each partition, the links (excluding those in the same group of broad/multi-cast branches) scheduled for concurrent transmissions must have a minimum interspace $\delta(n)$, which is defined as the Euclidean distance between the transmitters.

S3: Given any location in a partition, at any time at least one link must be scheduled for transmission within radius $\xi(n)$ from that location. The location of a link is the location of the transmitter.

The above specifications prescribe the partition dimension via $\rho(n)$ and the density of concurrent transmissions via $\delta(n)$ and $\xi(n)$. In practice, a large network can be partitioned using Voronoi tessellation [26] by placing a set of schedulers at strategic locations. Each node in the network contacts its nearest scheduler and follows the scheduling instructions from the chosen scheduler. When $\xi(n) > \delta(n)$, **S2** and **S3** can co-exist without conflict. According to Definition 4, $\mathcal{S}_{l(\rho,\delta,\xi)}(t)$ guarantees collision-free link transmissions within each individual partition, but does not preclude transmission failures due to cross-partition interference. Our results will demonstrate that the impact of cross-partition interference can be bounded in a way that the network capacity scales on the same order as using global link scheduling. For conciseness, we abbreviate $\mathcal{S}_{l(\rho,\delta,\xi)}(t)$ as $\mathcal{S}_l(t)$ from now on, and denote any global scheduling algorithm as $\mathcal{S}_g(t)$.

Next, we provide the sufficient and necessary conditions on $\rho(n)$, $\delta(n)$ and $\xi(n)$ for $\mathcal{S}_l(t)$ to achieve order optimal capacity. The following theorem summarizes our main result.

THEOREM 1. *In large-scale wireless networks, $\forall \mathbb{I} \in \{\mathbb{I}_{\text{prot}}, \mathbb{I}_{\text{phy}}, \mathbb{I}_{\text{gen}}\}$, $\forall \mathbb{L} \in \{\mathbb{L}_{\text{rand}}, \mathbb{L}_{\text{arbi}}\}$, $\forall \mathbb{C} \in \{\mathbb{C}_{\text{uni}}, \mathbb{C}_{\text{bro}}, \mathbb{C}_{\text{mul}}\}$, a localized scheduling algorithm $\mathcal{S}_l(t)$ achieves $\lambda_l(n) = \Theta(\lambda_g(n))$ if and only if the following conditions are satisfied:*

- *Partition dimension: $\rho(n) = \Omega(r(n))$, the partition size should scale at least on the same order of the critical transmission radius $r(n)$.*
- *Minimum link separation: $\delta(n) = O(r(n))$, the minimum separation between concurrent transmissions should scale at most on the same order of the critical transmission radius $r(n)$.*
- *Maximum link separation: $\xi(n) = \Omega(r(n))$, the maximum separation between concurrent transmissions should scale at least on the same order of the critical transmission radius $r(n)$.*

Theorem 1 reveals that, in order for $\mathcal{S}_l(t)$ to achieve the order optimal capacity, the smallest acceptable partition dimension is $\Theta(r(n))$, which equals $\Theta(\sqrt{\log n})$ for random node locations and $\Theta(1)$ for arbitrary node locations. As $\mathcal{S}_g(t)$ is a special case of $\mathcal{S}_l(t)$ with partition size $\Theta(\sqrt{n})$, Theorem 1 demonstrates a significant reduction of scheduling complexity by using partition, while still allows $\mathcal{S}_l(t)$ to achieve the same order optimal capacity as $\mathcal{S}_g(t)$. In addition, our result also indicates $r(n)$ as the correct order of link separation for optimal capacity. Scheduling lower (i.e., $\delta(n) = \omega(r(n))$) or higher (i.e., $\xi(n) = o(r(n))$) density of concurrent transmissions will yield less network capacity by either wasting opportunities for parallel communications or introducing excessive link collisions.

Theorem 1 in fact consists of two statements regarding the capacity comparison of localized and global scheduling algorithms under conditions $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$:

P1: $\lambda_l(n) = O(\lambda_g(n))$, the localized algorithms cannot achieve higher capacity than the global algorithms;

P2: $\lambda_l(n) = \Omega(\lambda_g(n))$, the localized algorithms can however achieve at least a constant fraction of capacity of the global algorithms.

We further have the following theorems that prove the above statements in Theorem 1.

THEOREM 2. *Given a localized scheduling algorithm $\mathcal{S}_l(t)$ that achieves capacity $\lambda_l(n)$, there exists a global scheduling algorithm that achieves $\lambda_g(n) \geq \lambda_l(n)$.*

PROOF. See Appendix 9.1. □

The proof of Theorem 2 is also the proof for statement **P1** of Theorem 1. To prove statement **P2**, we have the next theorem that transforms the problem on network capacity measured by the maximum source data rate to an equivalent problem on the data transmission rate on each wireless link in the network. The latter problem is easier to solve.

THEOREM 3. *Denoting $w_{l,ij}(n)$ and $w_{g,ij}(n)$ as the supported data transmission rates on link l_{ij} with localized and global scheduling algorithms respectively, $\lambda_l(n) = \Omega(\lambda_g(n))$ if and only if $w_{l,ij}(n) = \Omega(w_{g,ij}(n))$ for any l_{ij}.*

PROOF. See Appendix 9.2. □

Given Theorem 3, the problem described in statement **P2** becomes comparison of $w_{l,ij}(n)$ and $w_{g,ij}(n)$, on which we have the following result.

THEOREM 4. *In large-scale wireless networks, $\forall\mathbb{I}, \forall\mathbb{L}, \forall\mathbb{C}$, a localized link scheduling algorithm $\mathcal{S}_l(t)$ achieves $w_{l,ij}(n) = \Omega(w_{g,ij}(n))$ for any link l_{ij} if and only if $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$.*

Theorems 2, 3 and 4 together constitute the proof for Theorem 1. In the next two sections, we will present the proof for Theorem 4, which includes the *sufficiency* and the *necessity* of conditions $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$ for obtaining $w_{l,ij}(n) = \Omega(w_{g,ij}(n))$. The main idea of proof is to compare the respective bounds for $w_{l,ij}(n)$ and $w_{g,ij}(n)$ under these scheduling conditions.

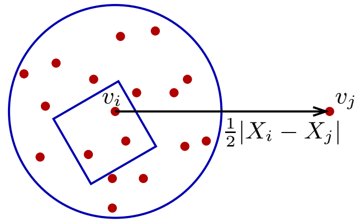


Figure 1: Density determination of the concurrent senders in the physical model. The disk has radius $\frac{1}{2}|X_i - X_j|$. The square has edge length $\frac{1}{2\sqrt{2}}|X_i - X_j|$.

4. PROOF OF CONDITION SUFFICIENCY

We prove in this section the sufficient condition in Theorem 4, which states that $w_{l,ij}(n) = \Omega(w_{g,ij}(n))$ if $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$. Our proof has two steps: i) we bound the link transmission rate of $\mathcal{S}_g(t)$ as $w_{g,ij}(n) = \Theta(\frac{1}{r^2(n)})$, and ii) we bound the corresponding link transmission rate of $\mathcal{S}_l(t)$ as $w_{l,ij}(n) = \Omega(\frac{1}{r^2(n)})$. The combination immediately gives $w_{l,ij}(n) = \Omega(w_{g,ij}(n))$.

4.1 Link Rate in Global Scheduling

We have the following theorem on bounding the link transmission rate in global scheduling.

THEOREM 5. *In large-scale wireless networks, $\forall\mathbb{I}, \forall\mathbb{L}, \forall\mathbb{C}$, the maximum achievable link transmission rate in global scheduling is $w_{g,ij}(n) = \Theta(\frac{1}{r^2(n)})$.*

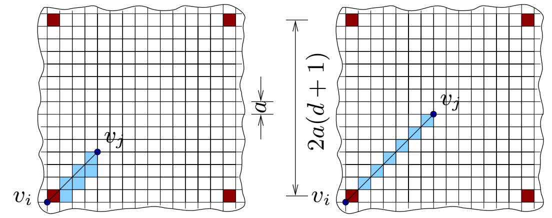
PROOF. We discuss the interference models $\mathbb{I}_{\text{prot}}$, $\mathbb{I}_{\text{phy}}$ and $\mathbb{I}_{\text{gen}}$ separately. As this proof does not make any assumption regarding node locations and communication types, it applies to all the models of $\mathbb{L}$ and $\mathbb{C}$. The foundation of this proof is the fact that links do not collide by using $\mathcal{S}_g(t)$.

For $\mathbb{I}_{\text{prot}}$, any two concurrently transmitting nodes must be separated at least by distance $\Delta r(n)$. Otherwise, both transmissions fail due to the violation of Eq. (2). Therefore, at any time $\mathcal{S}_g(t)$ can schedule at most one sender in any arbitrary square region with edge $\frac{\Delta r(n)}{\sqrt{2}}$. Given node density 1, we bound the time-average data transmission rate of link l_{ij} as $w_{g,ij}(n) \leq \eta_{ij}\frac{W}{(\Delta r(n)/\sqrt{2})^2} = \frac{2\eta_{ij}W}{\Delta^2 r^2(n)}$. We obtain the lower bound on $w_{g,ij}(n)$ by observing that senders spaced by $(2+\Delta)r(n)$ do not collide, i.e., $w_{g,ij}(n) \geq \frac{\eta_{ij}W}{(2+\Delta)^2 r^2(n)}$.

For $\mathbb{I}_{\text{phy}}$, a transmission is successful if Eq. (3) holds. As shown in Fig. 1, when v_i sends a packet to v_j, the number of concurrent senders y within distance $\frac{1}{2}|X_i - X_j|$ from v_i must satisfy the following inequality in order for v_j to receive the packet correctly

$$\frac{P_{\max}}{|X_i - X_j|^\alpha} \geq \beta y \frac{P_{\min}}{(\frac{3}{2}|X_i - X_j|)^\alpha}, \tag{7}$$

from which we obtain $y \leq \frac{(3/2)^\alpha P_{\max}}{\beta P_{\min}}$. By taking an arbitrary square region with edge $\frac{1}{2\sqrt{2}}|X_i - X_j|$ that covers v_i, as illustrated in Fig. 1, and noting that $|X_i - X_j| \geq r_0(n)$, we show that the number of concurrent senders within any square region with edge $\frac{r_0(n)}{2\sqrt{2}}$ cannot exceed $\frac{(3/2)^\alpha P_{\max}}{\beta P_{\min}} + 1$. Otherwise, all these transmissions fail due to their mutual interference. So, we have $w_{g,ij}(n) \leq \eta_{ij}\frac{(3/2)^\alpha P_{\max}/(\beta P_{\min})+1}{(r_0(n)/(2\sqrt{2}))^2}W = \frac{8((3/2)^\alpha P_{\max}/(\beta P_{\min})+1)\eta_{ij}W}{\varepsilon^2 r^2(n)}$. To find the lower bound on $w_{g,ij}(n)$, we cite a relevant result. It is pointed out in [1] that there is a large enough constant Δ_1 such that all the transmissions are successful in the physical model as long as the concurrent senders are separated by distance $(2+\Delta_1)r(n)$. We thus obtain easily $w_{g,ij}(n) \geq \frac{\eta_{ij}W}{(2+\Delta_1)^2 r^2(n)}$.



(a) $r(n) \geq (\frac{d-1}{2}+1)\sqrt{2}a$ (b) $r(n) \leq (d+1)\sqrt{2}a$

Figure 2: Determination of the bounds on $r(n)$ in the generalized physical model. In this fraction of network, a node transmits in each of the four corner cells that are spaced by Euclidean distance $2a(d+1)$, as in [4] and [10]. Considering all the possibilities of d (regardless of its parity) and the arbitrary locations of v_i and v_j in their respective cells, $r(n) \geq (\frac{d-1}{2}+1)\sqrt{2}a$ is necessary to guarantee the connection between v_i and v_j, as illustrated in (a). Besides, given Manhattan distance d, $r(n) \leq (d+1)\sqrt{2}a$ since the Euclidean distance between v_i and v_j does not exceed $(d+1)\sqrt{2}a$, as shown in (b).

For $\mathbb{I}_{\text{gen}}$, by dividing the network into small cells with edge a, link l_{ij} can transmit successfully at rate $R(a,d)$ when scheduled [4, 10], where nodes v_i and v_j are separated by Manhattan distance d and function $R(a,d)$ is a constant under our assumptions on $\mathbb{I}_{\text{gen}}$. The Manhattan distance is defined as the minimum number of contiguous cells to connect v_i and v_j. Two cells are contiguous in [4] if they share an edge, while in [10] they are contiguous if they share either an edge or a vertex. We consider both definitions here. We find that the critical node transmission radius $r(n)$ must satisfy the following relations with the cell size a and the Manhattan distance d: $(\frac{d-1}{2}+1)\sqrt{2}a \leq r(n) \leq (d+1)\sqrt{2}a$, as explained in Fig. 2. Since at any time the scheduling algorithm $\mathcal{S}_g(t)$ presented in [4] and [10] allows only one node to transmit within any square region of area $4a^2(d+1)^2$ and the bounds on $r(n)$ give $2r^2(n) \leq 4a^2(d+1)^2 \leq 8r^2(n)$, we have $\frac{\eta_{ij}R(a,d)}{8r^2(n)} \leq w_{g,ij}(n) \leq \frac{\eta_{ij}R(a,d)}{2r^2(n)}$.

By defining $c_1 = \min\{\frac{\eta_{ij}W}{(2+\Delta)^2}, \frac{\eta_{ij}W}{(2+\Delta_1)^2}, \frac{\eta_{ij}R(a,d)}{8}\}$ and $c_2 = \max\{\frac{2\eta_{ij}W}{\Delta^2}, \frac{8((3/2)^\alpha P_{\max}/(\beta P_{\min})+1)\eta_{ij}W}{\varepsilon^2}, \frac{\eta_{ij}R(a,d)}{2}\}$, we summarize the bounds as $\frac{c_1}{r^2(n)} \leq w_{g,ij}(n) \leq \frac{c_2}{r^2(n)}$ in all the three interference models. Thus, $w_{g,ij}(n) = \Theta(\frac{1}{r^2(n)})$. □

4.2 Link Rate in Localized Scheduling

We next determine the lower bound on the link transmission rate when $\mathcal{S}_l(t)$ is used. Before presenting the theorem on this lower bound, we have the following two lemmas that will be used in the proof of the theorem.

LEMMA 1. *Assume that $\kappa_1, \kappa_2 > 0$ are constants. Given a disk of radius $\kappa_1 r(n)$, if $\rho(n) \geq \kappa_2 r(n)$, the number of partitions that overlap with the disk is at most a constant.*

PROOF. See Appendix 9.3. □

LEMMA 2. *Assume that constants κ_1, κ_2 and κ_3 satisfy $0 < \kappa_1 < \kappa_2 < \kappa_3 \leq 2\kappa_1$. Given two co-centric disks of* *radius $\kappa_1 r(n)$ and $\kappa_3 r(n)$ respectively, define an annulus as the area between the disk circumferences. If $\rho(n) \geq \kappa_2 r(n)$, there exists a constant $c > 0$ such that a partition that overlaps with the smaller disk must overlap with the annulus by at least an area of $cr^2(n)$.*

PROOF. See Appendix 9.4. ☐

It is worth noted that Lemma 2 is still true if $\kappa_3 > 2\kappa_1$. We do not elaborate since the current form of Lemma 2 is enough for us to prove a lower bound on $w_{l,ij}(n)$.

THEOREM 6. *In large-scale wireless networks, $\forall\mathbb{I}, \forall\mathbb{L}, \forall\mathbb{C}$, a localized link scheduling algorithm $\mathcal{S}_l(t)$ achieves $w_{l,ij}(n) = \Omega(\frac{1}{r^2(n)})$ for any link l_{ij} if $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$.*

PROOF. By the definition of asymptotic bounds, we need to find constants c, c_ρ, c_δ and c_ξ such that $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ if $\rho(n) \geq c_\rho r(n)$, $\delta(n) \leq c_\delta r(n)$ and $\xi(n) \geq c_\xi r(n)$. We present the proof for $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ in two steps:

1: $\rho(n) = c_\rho r(n)$, $\delta(n) = c_\delta r(n)$, $\xi(n) = c_\xi r(n)$.

2: $\rho(n) \geq c_\rho r(n)$, $\delta(n) \leq c_\delta r(n)$, $\xi(n) \geq c_\xi r(n)$.

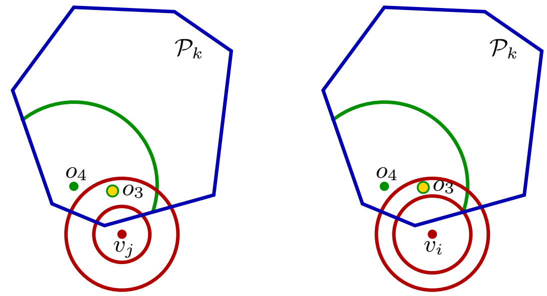
Step 1 is the special case in which we show that $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ when we configure $\rho(n)$, $\delta(n)$ and $\xi(n)$ correctly, and step 2 is the generalized case in which $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ is still true if $\rho(n)$ and $\xi(n)$ are configured larger while $\delta(n)$ is configured smaller than our choices. Next, we present the proof for all the models of $\mathbb{I}$, $\mathbb{L}$ and $\mathbb{C}$.

Step 1, for $\mathbb{I}_{\text{prot}}$. We first consider location model $\mathbb{L}_{\text{rand}}$ and communication model $\mathbb{C}_{\text{uni}}$. We demonstrate that if we choose constants $c_\rho = \frac{3}{2}(1+\Delta)$, $c_\delta = 3(1+\Delta)$, $c_\xi = 4(1+\Delta)$ and some constant c_u, we will have $w_{l,ij}(n) \geq \frac{c_u}{r^2(n)}$ when $\rho(n) = c_\rho r(n)$, $\delta(n) = c_\delta r(n)$ and $\xi(n) = c_\xi r(n)$. By $\mathbb{I}_{\text{prot}}$, a transmission from v_i to v_j is successful when there are no other concurrent senders within distance $(1+\Delta)r(n)$ from v_j. Since $\mathcal{S}_l(t)$ guarantees non-collision within individual partitions, any sender that fails the transmission on link l_{ij}, if any, must reside in a neighbor partition that overlaps with disk $D(v_j, (1+\Delta)r(n))$. By setting $\kappa_1 = 1+\Delta$ and $\kappa_2 = \frac{3}{2}(1+\Delta)$, Lemma 1 shows that $D(v_j, (1+\Delta)r(n))$ overlaps with at most a constant c_1 number of partitions. In addition, by setting $\kappa_3 = 2(1+\Delta)$, Lemma 2 tells us that each of these overlapping partitions must overlap with the annulus $D(v_j, 2(1+\Delta)r(n)) \setminus D(v_j, (1+\Delta)r(n))$ at least by an area $c_2 r^2(n)$, where c_2 is a constant, as depicted in Fig. 3(a). In any neighbor partition, for example $\mathcal{P}_k$, because specification **S3** dictates at least one active sender within any arbitrarily located disk of radius $\xi(n)$, for example $D(o_4, 4(1+\Delta)r(n))$, the probability of finding an active sender inside $D(o_3, \epsilon)$ is bounded as

$$\Pr[\text{active sender in } D(o_3, \epsilon)] \geq \frac{c_2 r^2(n)}{\pi(4(1+\Delta)r(n))^2} = c_3. \tag{8}$$

Since $\delta(n) = 3(1+\Delta)r(n)$, when a node in $D(o_3, \epsilon)$ is active in transmission, $\mathcal{P}_k$ does not schedule any concurrent senders in $D(v_j, (1+\Delta)r(n))$. Hence we obtain

$$\Pr[\mathcal{P}_k \text{ does not fail reception at } v_j \text{ at a given time}] \geq c_3. \tag{9}$$



(a) Unicast (b) Broadcast/Multicast

Figure 3: Proof of Theorem 6 in the protocol interference model. In unicast, we define disks $D(v_j, (1+\Delta)r(n))$, $D(v_j, 2(1+\Delta)r(n))$, $D(o_3, \epsilon)$ and $D(o_4, 4(1+\Delta)r(n))$. In broad/multi-cast, we define disks $D(v_i, (2+\Delta)r(n))$, $D(v_i, 2(1+\Delta)r(n))$, $D(o_3, \epsilon)$ and $D(o_4, 4(1+\Delta)r(n))$. The area of $D(o_3, \epsilon)$ is at least $c_2 r^2(n)$. The polygon denotes a neighbor partition.

As $\mathcal{P}_k$ may reschedule up to $\lfloor \frac{L_{\max}}{L_{\min}} \rfloor + 2$ times during the packet transmission from v_i to v_j, we further have

$$\Pr[\mathcal{P}_k \text{ does not fail reception at } v_j] \geq c_3^{\lfloor \frac{L_{\max}}{L_{\min}} \rfloor + 2} = c_4. \tag{10}$$

Finally, considering all the neighbor partitions, we arrive at

$$\Pr[\text{successful reception at } v_j] \geq c_4^{c_1} = c_5. \tag{11}$$

Note that at least one sender is active within any square region with edge $2\xi(n)$ and the node density is 1, so v_i has a chance of at least $\frac{1}{64(1+\Delta)^2 r^2(n)}$ to be scheduled. Thus, we can bound the time-average transmission rate of l_{ij} as

$$w_{l,ij}(n) \geq \frac{c_5 \eta_{ij} W}{64(1+\Delta)^2 r^2(n)} = \frac{c_u}{r^2(n)}, \tag{12}$$

where $c_u = \frac{c_5 \eta_{ij} W}{64(1+\Delta)^2}$.

Step 1, for $\mathbb{I}_{\text{prot}}$ (cont'd). We then consider location model $\mathbb{L}_{\text{rand}}$ with communication models $\mathbb{C}_{\text{bro}}$ and $\mathbb{C}_{\text{mul}}$. By $\mathbb{C}_{\text{bro}}$ and $\mathbb{C}_{\text{mul}}$, the packet transmissions from v_i along the broad/multi-cast branches are successful if there are no concurrent senders within distance $(2+\Delta)r(n)$ from v_i. Similar to unicast, we prove the existence of a constant c_6 such that

$$\Pr[\text{successful reception at all receivers}] \geq c_6, \tag{13}$$

by setting $\rho(n) = (2+\frac{3}{2}\Delta)r(n)$, $\delta(n) = (4+3\Delta)r(n)$, $\xi(n) = 4(1+\Delta)r(n)$, $\kappa_1 = 2+\Delta$, $\kappa_2 = 2+\frac{3}{2}\Delta$ and $\kappa_3 = 2(1+\Delta)$, as shown in Fig. 3(b). We further prove

$$w_{l,ij}(n) \geq \frac{c_6 \eta_{ij} W}{64(1+\Delta)^2 r^2(n)} = \frac{c_b}{r^2(n)}, \tag{14}$$

where $c_b = \frac{c_6 \eta_{ij} W}{64(1+\Delta)^2}$.

Combining the analysis of $\mathbb{C}_{\text{uni}}$, $\mathbb{C}_{\text{bro}}$, $\mathbb{C}_{\text{mul}}$ and defining $c = \min\{c_u, c_b\}$, we have proven $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ for random node locations in the protocol interference model.

Step 1, for $\mathbb{I}_{\text{prot}}$ (cont'd). We next consider location model $\mathbb{L}_{\text{arbi}}$ in which node locations are assigned in need. A popular assignment is to place the nodes on a grid with equal distance between neighbors [1,8]. When we partition the network, we align the node locations into a grid form in each partition. The grids are however not aligned across partitions due to their independence from one another, as

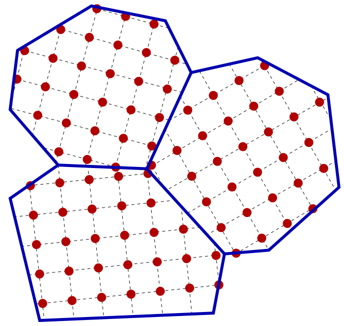


Figure 4: Node locations on grids.

shown in Fig. 4. The node interspace in each partition is set to 1 to satisfy the constant node density of 1. Given an arbitrary point on the partition boundary, because there is at least one node within distance $\sqrt{2}$ in each neighbor partition, any constant $r(n) \geq 2\sqrt{2}$ is large enough to guarantee the network connectivity. The proof for $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ is almost the same as that for the $\mathbb{L}_{\text{rand}}$ model. For conciseness, we only highlight the differences. In Eq. (8), we have shown that the probability for a neighbor partition to schedule a sender in the annulus area is at least a constant c_3, where c_3 is defined as the ratio between two areas: $D(o_3, \epsilon)$ versus $D(o_4, 4(1+\Delta)r(n))$. To be accurate, c_3 should be the ratio between the numbers of nodes therein instead of the areas. These two metrics are equivalent for random node locations since $r(n) = \Theta(\sqrt{\log n}) \to \infty$ as $n \to \infty$ and the constant node density can be equivalently scaled to infinity if $r(n)$ is normalized to 1. For arbitrary node locations, $r(n) = \Theta(1)$ and the two ratios are not interchangeable. We next determine the ratio between the numbers of nodes. Recall that the area of $D(o_3, \epsilon)$ is at least $c_2 r^2(n)$. By choosing $r(n) \geq \max\{\sqrt{\frac{\pi}{c_2}}, 2\sqrt{2}\}$, we show that $D(o_3, \epsilon)$ contains at least one node. In general, there are at least $\lfloor\sqrt{\frac{c_2}{\pi}} r(n)\rfloor^2$ nodes inside $D(o_3, \epsilon)$. As every node placed inside $D(o_4, 4(1+\Delta)r(n))$ occupies a unit square area in a slightly larger disk $D(o_4, 4(1+\Delta)r(n) + \frac{\sqrt{2}}{2})$, a partition can place at most $\pi(4(1+\Delta)r(n) + \frac{\sqrt{2}}{2})^2$ nodes in $D(o_4, 4(1+\Delta)r(n))$. By setting $c_3' = \frac{\lfloor\sqrt{\frac{c_2}{\pi}} r(n)\rfloor^2}{\pi(4(1+\Delta)r(n)+\frac{\sqrt{2}}{2})^2}$, where $r(n)$ is constant for arbitrary node locations, we can rewrite Eq. (8) as

$$\Pr[\text{active sender in } D(o_3,\epsilon)] \geq \frac{\lfloor\sqrt{\frac{c_2}{\pi}} r(n)\rfloor^2}{\pi(4(1+\Delta)r(n)+\frac{\sqrt{2}}{2})^2} = c_3'. \tag{15}$$

Following the same reasoning as (9), (10) and (11), we have

$$\Pr[\text{successful reception at } v_j] \geq c_5'. \tag{16}$$

Because at least one node is active in any square region with edge $2\xi(n)$, which contains no more than $(8(1+\Delta)r(n) + \sqrt{2})^2$ nodes, v_i has a chance of at least $\frac{1}{(8(1+\Delta)r(n)+\sqrt{2})^2}$ to be scheduled. We hence obtain

$$w_{l,ij}(n) \geq \frac{c_5' \eta_{ij} W}{(8(1+\Delta)r(n)+\sqrt{2})^2} = \frac{c_u'}{r^2(n)}, \tag{17}$$

where $c_u' = \frac{c_5' \eta_{ij} W r^2(n)}{(8(1+\Delta)r(n)+\sqrt{2})^2}$. Similarly, by setting $c_b' = \frac{c_6' \eta_{ij} W r^2(n)}{(8(1+\Delta)r(n)+\sqrt{2})^2}$ in broadcast and multicast, we have

$$w_{l,ij}(n) \geq \frac{c_6' \eta_{ij} W}{(8(1+\Delta)r(n)+\sqrt{2})^2} = \frac{c_b'}{r^2(n)}. \tag{18}$$

We have thus proven the case for arbitrary node locations in the protocol interference model by setting $c = \min\{c_u', c_b'\}$.

Step 1, for $\mathbb{I}_{\text{phy}}$. Similar to the proof technique used in [1], we transform the physical model into an equivalent protocol model. It is shown in [1] that every node can receive packets correctly in the physical model as long as a minimum space $(2+\Delta_1)r(n)$ is enforced between neighbor senders, where Δ_1 is a constant. Distance $(2+\Delta_1)r(n)$ demarcates *close-in* and *far-away* regions, and the aggregate interference from the far-away region is insignificant to correct packet reception. Similar result is also found in [12]. In the following, we extend this finding to network partition and prove the existence of another constant Δ_2. Specifically, we show that if $\rho(n) \geq r(n)$ and the concurrent senders in each partition are separated at least by $(2+\Delta_2)r(n)$, a transmission from v_i is successful if no other nodes transmit within radius $(2+\Delta_2)r(n)$ from v_i.

In the physical interference model, the SINR at a receiver v_j must exceed threshold β in order to receive a packet successfully. To determine the interference imposed on v_j, we divide the network into non-overlapping square belt regions and consider the aggregate interference from one of them, as illustrated in Fig. 5. We cover this square belt region seamlessly with disks of radius $\frac{(2+\Delta_2)r(n)}{2}$, which are interspaced evenly at distance $\frac{\sqrt{2}(2+\Delta_2)r(n)}{2}$. The disks needed is at most $(\frac{2b}{\sqrt{2}(2+\Delta_2)r(n)/2}+1)^2 - (\frac{2a}{\sqrt{2}(2+\Delta_2)r(n)/2}-1)^2$, if $\frac{\sqrt{2}(2+\Delta_2)r(n)}{2}$ divides $2a$ and $2b$. Note that a partition cannot schedule more than one sender in a disk at any time because of the required minimum separation $(2+\Delta_2)r(n)$ between concurrent senders. The number of concurrent senders inside a disk thus does not exceed the number of partitions that overlap with this disk, which is at most $\frac{\pi((2+\Delta_2)r(n)/2+2\sigma\rho(n))^2}{\pi\rho^2(n)} \leq (\frac{2+\Delta_2}{2} + 2\sigma)^2 \leq (2+\Delta_2)^2$. Here we have assumed $\Delta_2 \geq 4\sigma - 2$. Letting $a = m\frac{\sqrt{2}(2+\Delta_2)r(n)}{4}$ and $b = (m+1)\frac{\sqrt{2}(2+\Delta_2)r(n)}{4}$, where $m = 1, 2, \cdots$, and noting that the interference from any sender inside the square belt zone does not exceed $P_{\max} a^{-\alpha}$, we bound the total interference I from all the concurrent senders outside the square $a = \frac{\sqrt{2}(2+\Delta_2)r(n)}{4}$ as

$$I \leq \sum_{m=1}^{\infty} 3(2m+1)(2+\Delta_2)^2 P_{\max}\Big(m\frac{\sqrt{2}(2+\Delta_2)r(n)}{4}\Big)^{-\alpha}$$

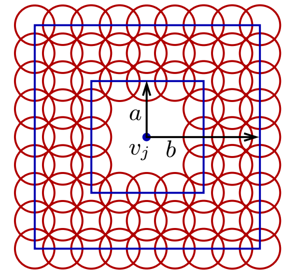


Figure 5: Aggregate interference from the region bounded between distance a and b.

$$\leq 3(2+\Delta_2)^2 P_{\max}\Big(\frac{\sqrt{2}(2+\Delta_2)r(n)}{4}\Big)^{-\alpha}\frac{3\alpha^2-6\alpha+2}{\alpha^2-3\alpha+2}$$
$$= 3(2+\Delta_2)^{2-\alpha} P_{\max}\Big(\frac{\sqrt{2}r(n)}{4}\Big)^{-\alpha}\frac{3\alpha^2-6\alpha+2}{\alpha^2-3\alpha+2}. \quad (19)$$

The SINR at v_j is then bounded by

$$\text{SINR} \geq \frac{P_{\min} r^{-\alpha}(n)}{BN_0 + 3(2+\Delta_2)^{2-\alpha} P_{\max}\big(\frac{\sqrt{2}r(n)}{4}\big)^{-\alpha}\frac{3\alpha^2-6\alpha+2}{\alpha^2-3\alpha+2}}. \quad (20)$$

After some computations, we show SINR $\geq \beta$ if

$$\Delta_2 \geq \Big(\frac{3(2\sqrt{2})^\alpha(3\alpha^2-6\alpha+2)\beta\gamma_1 P_{\max}}{(\alpha^2-3\alpha+2)(\gamma_1-\beta)P_{\min}}\Big)^{\frac{1}{\alpha-2}} - 2. \quad (21)$$

Eq. (21) tells an important fact regarding localized scheduling in the physical interference model: if concurrent senders in each partition are separated at least by $(2+\Delta_2)r(n)$ and constant $\Delta_2 \geq \max\{(\frac{3(2\sqrt{2})^\alpha(3\alpha^2-6\alpha+2)\beta\gamma_1 P_{\max}}{(\alpha^2-3\alpha+2)(\gamma_1-\beta)P_{\min}})^{\frac{1}{\alpha-2}} - 2, 4\sigma-2\}$, packet reception at v_j is always successful as long as there are no other senders in the square close-in region $a = \frac{\sqrt{2}(2+\Delta_2)r(n)}{4}$, which is contained in disk $D(v_i, (2+\Delta_2)r(n))$. We have thus proven our statement that a transmission from v_i is successful if the other nodes within distance $(2+\Delta_2)r(n)$ from v_i keep silent. This result applies to unicast, broadcast and multicast.

Given the equivalence of physical and protocol interference models established above, it is straightforward to prove $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ in the physical model. The proof follows the same line of reasoning as presented in the protocol model except for replacing Δ with Δ_2. Note that the two conditions needed for the model equivalence, $\rho(n) \geq r(n)$ and the minimum separation $(2+\Delta_2)r(n)$ between concurrent senders in each partition, are satisfied when we choose appropriate values for c_ρ and c_δ.

Step 1, for $\mathbb{I}_{\text{gen}}$. Our proof for the physical model has shown that every successful transmission satisfies SINR $\geq \beta$. Hence, the same transmission must be able to achieve constant rate $B\log_2(1+\beta)$ in the generalized physical model. The bound $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ is then proven trivially by replacing the constant W used in the physical model with the new constant $B\log_2(1+\beta)$.

Step 2. Till now, we have demonstrated $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ for all the models of $\mathbb{I}$, $\mathbb{L}$ and $\mathbb{C}$ when $\rho(n) = c_\rho r(n)$, $\delta(n) = c_\delta r(n)$ and $\xi(n) = c_\xi r(n)$, if we configure constants c_ρ, c_δ and c_ξ properly. Lastly, we generalize this result to obtain the sufficient scaling conditions on $\rho(n)$, $\delta(n)$ and $\xi(n)$.

When $\rho(n) \geq c_\rho r(n)$, there are two possibilities. If $\rho(n) = \Theta(r(n))$, each partition is geographically bounded between inner radius $\rho'(n) = c_\rho r(n)$ and outer radius $\sigma'\rho'(n)$ ($\sigma' = \frac{\sigma\rho(n)}{\rho'(n)}$ is constant). The proof for $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ is the same as before except for replacing σ with σ'. If $\rho(n) = \omega(r(n))$, we break down each partition into smaller partitions that are parameterized by inner radius $\rho'(n) = c_\rho r(n)$ and outer radius $\sigma\rho'(n)$. Since $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$ in every smaller partition, we can merge these partitions back into the original partition and retain $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$.

We note that $\delta(n)$ (resp. $\xi(n)$) describes the minimum (resp. maximum) separation between concurrently scheduled link transmissions. For $\mathcal{S}_l(t)$ with $\delta(n) \leq c_\delta r(n)$ and $\xi(n) \geq c_\xi r(n)$, we can always choose $\delta'(n) = c_\delta r(n) \geq \delta(n)$ and $\xi'(n) = c_\xi r(n) \leq \xi(n)$ to achieve $w_{l,ij}(n) \geq \frac{c}{r^2(n)}$. $\square$

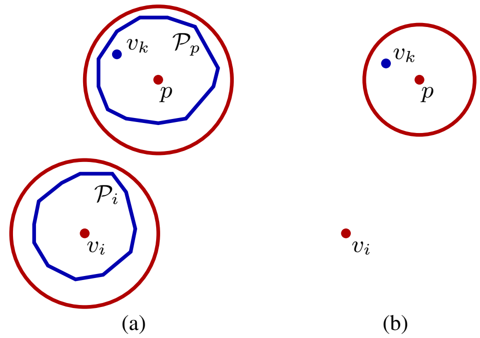


Figure 6: Proof of Theorem 7 in the protocol interference model. Case (a) proves $\rho(n) = \Omega(r(n))$, in which we define disks $D(v_i, 2\sigma\rho(n))$ and $D(p, 2\sigma\rho(n))$. Case (b) proves $\xi(n) = \Omega(r(n))$, in which we define disk $D(p, \xi(n))$. In both cases, $|\overline{v_i p}| = \frac{\Delta r(n)}{2}$.

5. PROOF OF CONDITION NECESSITY

We next prove the necessary condition in Theorem 4, which states that $w_{l,ij}(n) = \Omega(w_{g,ij}(n))$ only if $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$. Given our previous result $w_{g,ij}(n) = \Theta(\frac{1}{r^2(n)})$ obtained in Theorem 5, it is hence required to prove the necessity of $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$ for $w_{l,ij}(n) = \Omega(\frac{1}{r^2(n)})$. Before proceeding with the proof, we present two lemmas.

LEMMA 3. *Given sequences $\{f(n)\}$ and $\{g(n)\}$ that satisfy $f(n) \neq \Omega(g(n))$, there exist subsequences $\{f(n_k)\} \subseteq \{f(n)\}$ and $\{g(n_k)\} \subseteq \{g(n)\}$ such that $f(n_k) = o(g(n_k))$, and vice versa.*

PROOF. See Appendix 9.5. $\square$

LEMMA 4. *Given sequences $\{f(n)\}$ and $\{g(n)\}$ that satisfy $f(n) \neq O(g(n))$, there exist subsequences $\{f(n_k)\} \subseteq \{f(n)\}$ and $\{g(n_k)\} \subseteq \{g(n)\}$ such that $f(n_k) = \omega(g(n_k))$, and vice versa.*

PROOF. See Appendix 9.6. $\square$

We then have the next theorem for the necessity of conditions on $\rho(n)$, $\delta(n)$ and $\xi(n)$ to obtain $w_{l,ij}(n) = \Omega(\frac{1}{r^2(n)})$.

THEOREM 7. *In large-scale wireless networks, $\forall\mathbb{I}, \forall\mathbb{L}, \forall\mathbb{C}$, a localized link scheduling algorithm $\mathcal{S}_l(t)$ achieves $w_{l,ij}(n) = \Omega(\frac{1}{r^2(n)})$ for any link l_{ij} only if $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$.*

PROOF. We use contradiction, i.e., $w_{l,ij}(n) \neq \Omega(\frac{1}{r^2(n)})$ if $\rho(n) \neq \Omega(r(n))$ or $\delta(n) \neq O(r(n))$ or $\xi(n) \neq \Omega(r(n))$. The three interference models are discussed separately. As no assumption is made on node locations and communication types, the proof applies to all the models of $\mathbb{L}$ and $\mathbb{C}$.

For $\mathbb{I}_{\text{prot}}$, if $\rho(n) \neq \Omega(r(n))$, Lemma 3 states that there exists $\rho(n_k) = o(r(n_k))$. We choose a point p such that $|\overline{v_i p}| = \frac{\Delta r(n)}{2}$ and define disks $D(v_i, 2\sigma\rho(n))$ and $D(p, 2\sigma\rho(n))$, as shown in Fig. 6(a). Since the partition diameter does not exceed $2\sigma\rho(n)$, the partition where v_i resides, denoted as $\mathcal{P}_i$, must be contained in $D(v_i, 2\sigma\rho(n))$ and the partition where p is located, denoted as $\mathcal{P}_p$, must be contained in $D(p, 2\sigma\rho(n))$. Given $\rho(n_k) = o(r(n_k))$, we obtain $\frac{2\sigma\rho(n_k)}{|\overline{v_i p}|} = \frac{2\sigma\rho(n_k)}{\Delta r(n_k)/2} \to 0$ as $n_k \to \infty$. Thus, $\exists n^*$ such that $\forall n_k > n^*$,

$\frac{2\sigma\rho(n_k)}{|\overline{v_i p}|} < \frac{1}{2}$, showing that $D(v_i, 2\sigma\rho(n_k))$ and $D(p, 2\sigma\rho(n_k))$ do not overlap and that $\mathcal{P}_i$ and $\mathcal{P}_p$ are separated. Since there is at least one sender v_k in $\mathcal{P}_p$ and $|\overline{v_i v_k}| < \Delta r(n_k)$, collision must occur between v_i and v_k. Consequently, $w_{l,ij}(n_k) = 0$ when $n_k > n^*$. In other words, $w_{l,ij}(n_k) = o(\frac{1}{r^2(n_k)})$ or equivalently $w_{l,ij}(n) \neq \Omega(\frac{1}{r^2(n)})$.

For $\mathbb{I}_{\text{prot}}$, if $\delta(n) \neq O(r(n))$, by Lemma 4, there exists $\delta(n_k) = \omega(r(n_k))$. As node density is 1 and a partition cannot schedule multiple concurrent senders in any square region with edge $\frac{\delta(n)}{\sqrt{2}}$, the chance for v_i to be scheduled is at most $\frac{2}{\delta^2(n)}$. The time-average transmission rate of l_{ij} is thus bounded by $w_{l,ij}(n) \leq \frac{2\eta_{ij}W}{\delta^2(n)}$, which gives $\frac{w_{l,ij}(n_k)}{1/r^2(n_k)} \leq \frac{2\eta_{ij}Wr^2(n_k)}{\delta^2(n_k)} \to 0$ as $n_k \to \infty$. Hence, $w_{l,ij}(n_k) = o(\frac{1}{r^2(n_k)})$ and $w_{l,ij}(n) \neq \Omega(\frac{1}{r^2(n)})$.

For $\mathbb{I}_{\text{prot}}$, if $\xi(n) \neq \Omega(r(n))$, by Lemma 3, there exists $\xi(n_k) = o(r(n_k))$. We choose a point p such that $|\overline{v_i p}| = \frac{\Delta r(n)}{2}$ and define disk $D(p, \xi(n))$, as shown in Fig. 6(b). Because $\frac{\xi(n_k)}{|\overline{v_i p}|} = \frac{\xi(n_k)}{\Delta r(n_k)/2} \to 0$ as $n_k \to \infty$, there must exist n^* such that $\forall n_k > n^*$, $\frac{\xi(n_k)}{|\overline{v_i p}|} < 1$ and v_i is outside $D(p, \xi(n_k))$. Since there is at least one sender v_k in $D(p, \xi(n_k))$ and $|\overline{v_i v_k}| < \Delta r(n_k)$, v_k collides with v_i. Therefore, $w_{l,ij}(n_k) = 0$ when $n_k > n^*$. In other words, $w_{l,ij}(n_k) = o(\frac{1}{r^2(n_k)})$ and $w_{l,ij}(n) \neq \Omega(\frac{1}{r^2(n)})$.

For $\mathbb{I}_{\text{phy}}$, from Eq. (3) we have $\frac{P_{\max}}{|X_i - X_j|^\alpha} \geq \beta \frac{P_{\min}}{|X_k - X_j|^\alpha}$ for any node v_k that transmits concurrently with v_i, which leads to $|X_k - X_j| \geq (\frac{\beta P_{\min}}{P_{\max}})^{\frac{1}{\alpha}} |X_i - X_j| \geq (\frac{\beta P_{\min}}{P_{\max}})^{\frac{1}{\alpha}} r_0(n) = (\frac{\beta P_{\min}}{P_{\max}})^{\frac{1}{\alpha}} \varepsilon r(n)$. Hence, when a transmission from v_i to v_j is successful, no senders other than v_i can be scheduled within distance $(\frac{\beta P_{\min}}{P_{\max}})^{\frac{1}{\alpha}} \varepsilon r(n)$ from v_j. Along the same line of reasoning as in the protocol model, we prove the necessity of $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$ by replacing v_i with v_j and $\Delta r(n)$ with $(\frac{\beta P_{\min}}{P_{\max}})^{\frac{1}{\alpha}} \varepsilon r(n)$.

For $\mathbb{I}_{\text{gen}}$, we first consider $\rho(n) \neq \Omega(r(n))$, which implies $\rho(n_k) = o(r(n_k))$. Given a transmission from v_i to v_j, there must exist a node v_k transmitting concurrently within distance $4\sigma\rho(n)$ from v_j in a nearby partition, because the partition diameter does not exceed $2\sigma\rho(n)$. The SINR at v_j is then upper bounded by $\frac{P_{\max}(\varepsilon r(n))^{-\alpha}}{P_{\min}(4\sigma\rho(n))^{-\alpha}}$ and $w_{l,ij}(n_k) \leq \frac{2\eta_{ij} B \log_2(1+\text{SINR})}{(2+\Delta_2)^2 r^2(n_k)} \to \frac{2\eta_{ij} B P_{\max}(\varepsilon r(n_k))^{-\alpha}}{(\ln 2)(2+\Delta_2)^2 r^2(n_k) P_{\min}(4\sigma\rho(n_k))^{-\alpha}}$. Further, $\frac{w_{l,ij}(n_k)}{1/r^2(n_k)} \leq \frac{2\eta_{ij} B P_{\max}(\varepsilon r(n_k))^{-\alpha}}{(\ln 2)(2+\Delta_2)^2 P_{\min}(4\sigma\rho(n_k))^{-\alpha}} \to 0$ as $n_k \to \infty$, showing $w_{l,ij}(n_k) = o(\frac{1}{r^2(n_k)})$ and $w_{l,ij}(n) \neq \Omega(\frac{1}{r^2(n)})$. Hence, $\rho(n) = \Omega(r(n))$ is necessary for $w_{l,ij}(n) = \Omega(\frac{1}{r^2(n)})$. The necessity of condition $\xi(n) = \Omega(r(n))$ is proven similarly. Lastly, if $\delta(n) \neq O(r(n))$, then from Eq. (5) we have $\frac{w_{l,ij}(n_k)}{1/r^2(n_k)} \leq \frac{2\eta_{ij} B \log_2(1+\gamma_2 \varepsilon^{-\alpha}) r^2(n_k)}{\delta^2(n_k)} \to 0$, showing $w_{l,ij}(n_k) = o(\frac{1}{r^2(n_k)})$ and $w_{l,ij}(n) \neq \Omega(\frac{1}{r^2(n)})$. Hence $\delta(n) = O(r(n))$ is also a necessary condition for $w_{l,ij}(n) = \Omega(\frac{1}{r^2(n)})$. □

6. LOCALIZED ALGORITHM

Till now, we have proven Theorem 1, the main result of this paper. The capacity relation $\lambda_l(n) = \Theta(\lambda_g(n))$ implies that the impact of cross-partition collisions is bounded when using localized scheduling such that any failed transmission can eventually get through after a constant number of retries, thus allowing localized scheduling to achieve the same order of capacity as global scheduling. Although our study has focused on the limiting case $n \to \infty$, the result is applicable to finite network sizes too. As long as the parameters $\rho(n)$, $\delta(n)$ and $\xi(n)$ are chosen to the scale of $r(n)$, the network capacity is some constant fraction of that achieved by the ideal global scheduling strategy.

Localized Scheduling Algorithm: LSA

Input: a set of links $\mathcal{L}$ requesting transmission
Output: a schedule $\mathcal{S}_l(t)$ satisfying **S1**, **S2** and **S3**
1 $\mathcal{S}_l(t) \leftarrow \emptyset$
2 **while** $\mathcal{L} \neq \emptyset$
3 select a link l_{ij} randomly from $\mathcal{L}$
4 $\mathcal{S}_l(t) \leftarrow \mathcal{S}_l(t) \cup \{l_{ij}\}$
5 $\mathcal{L} \leftarrow \mathcal{L} \setminus \{l_{ij}\}$
6 $\mathcal{L} \leftarrow \mathcal{L} \setminus \{l_{i'j'} : |X_i - X_{i'}| < \delta(n)\}$
7 **return** $\mathcal{S}_l(t)$

Figure 7: A localized scheduling algorithm that runs in each partition and in each time slot t.

Next, we apply our analytical result to design a simple localized scheduling algorithm that serves as our solution for practically acquiring optimal capacity scaling. As our objective is to achieve two goals together: maximum capacity and minimum complexity, we configure the partitions to the smallest acceptable size to minimize scheduling complexity, which is $\Theta(r(n))$ according to Theorem 1.

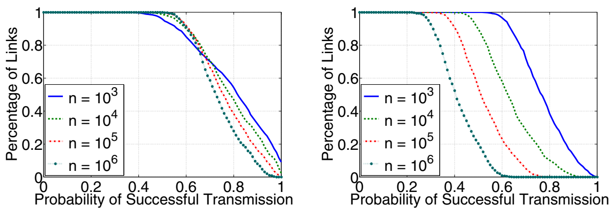
The localized scheduling algorithm is presented in Fig. 7, which runs at a central scheduler in each partition. The scheduler collects the topology information inside its partition and generates a collision-free schedule in each time slot. The algorithm is parameterized as follows: $\sigma = 10$, $\rho(n) = r(n)$, $\delta(n) = 3(1+\Delta)r(n)$ and $\xi(n) = 4(1+\Delta)r(n)$ (in $\mathbb{I}_{\text{prot}}$) or $\delta(n) = 3(1+\Delta_2)r(n)$ and $\xi(n) = 4(1+\Delta_2)r(n)$ (in $\mathbb{I}_{\text{phy}}$ and $\mathbb{I}_{\text{gen}}$). If there are broad/multi-cast communications, the links in the same group of broad/multi-cast branches are taken as one link for the scheduling purpose.

The next two theorems prove that LSA achieves order optimal capacity with constant scheduling complexity.

THEOREM 8. *Using LSA, $\lambda_l(n) = \Theta(\lambda_g(n))$.*

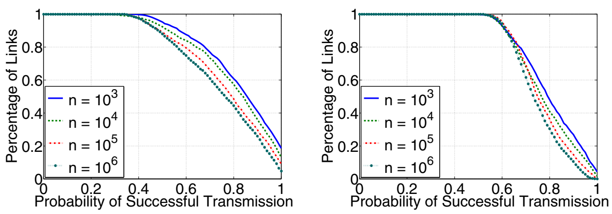
PROOF. It is clear that the configuration of LSA satisfies $\rho(n) = \Omega(r(n))$, $\delta(n) = O(r(n))$ and $\xi(n) = \Omega(r(n))$. Our task is then to prove that LSA satisfies **S1**, **S2** and **S3** so that we can use Theorem 1 to prove $\lambda_l(n) = \Theta(\lambda_g(n))$.

Requirement **S1** is obviously met by setting $\rho(n) = r(n)$ and $\sigma = 10$. Requirement **S2** is satisfied via line 6 of LSA, which excludes any link from being scheduled if its distance from a scheduled link is less than $\delta(n)$. Therefore, a minimum space $\delta(n)$ is guaranteed between any two active links in the same schedule. Lastly, we show that there exists at least one active link within radius $\xi(n)$ from any arbitrary location in a partition, to satisfy **S3**. We first consider the interference model $\mathbb{I}_{\text{prot}}$. Given an arbitrary point p in a partition, because $\xi(n) - \delta(n) = (1+\Delta)r(n)$, the transmitting links scheduled outside $D(p, \xi(n))$ do not suppress transmissions in the disk $D(p, (1+\Delta)r(n))$. According to LSA, if there exist any links in $D(p, (1+\Delta)r(n))$, at least one of them is scheduled. We next show that such links do exist. For random node locations, as $n \to \infty$, there exist nodes in $D(p, \Delta r(n))$ almost surely. For arbitrary node locations, given any $r(n) \geq \frac{\sqrt{2}}{\Delta}$, there is at least one node



(a) $\Delta = 1$ (b) $\Delta = 2$

Figure 8: $\rho(n) = 1$.



(a) $\Delta = 1$ (b) $\Delta = 2$

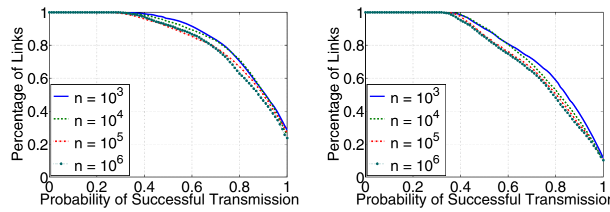
Figure 9: $\rho(n) = r^{\frac{1}{2}}(n)$.

in $D(p, \Delta r(n))$. Moreover, as the length of any link does not exceed the critical transmission radius $r(n)$, a node in $D(p, \Delta r(n))$ forms at least one link with other nodes in $D(p,(1+\Delta)r(n))$. This proves existence of at least one active link in $D(p, \xi(n))$. The proof is the same for interference models $\mathbb{I}_{\text{phy}}$ and $\mathbb{I}_{\text{gen}}$ by replacing Δ with Δ_2. $\square$

THEOREM 9. *LSA finishes in constant number of steps.*

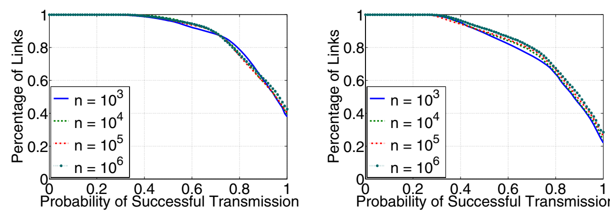
PROOF. As LSA selects the subset of transmitting links randomly, we model the number of steps as a random variable Z_1 and denote its maximum as $\hat{Z}_1$. Besides, we consider two random experiments of covering given regions with uniform disks of radius $\delta(n)$. The goal is to cover the specified regions seamlessly while keeping a minimum separation $\delta(n)$ between the centers of neighbor disks. In the first experiment we cover a partition. In the second experiment we cover a square region $2\sigma\rho(n) \times 2\sigma\rho(n)$ that is co-centric with the partition. We define random variables Z_2 and Z_3 as the number of disks used in the first and the second experiments, and denote their maxima as $\hat{Z}_2$ and $\hat{Z}_3$ respectively. It is not difficult to find the following relations: $Z_1 \le \hat{Z}_1 \le \hat{Z}_2 \le \hat{Z}_3$. Furthermore, the number of disks needed to cover the square region never exceeds $\frac{(2\sigma\rho(n))^2}{(\pi/4)(\delta(n)/2)^2} = \frac{64\sigma^2}{9\pi(1+\Delta)^2}$ (in $\mathbb{I}_{\text{prot}}$) or $\frac{64\sigma^2}{9\pi(1+\Delta_2)^2}$ (in $\mathbb{I}_{\text{phy}}$ and $\mathbb{I}_{\text{gen}}$). Therefore, $Z_1 \le \hat{Z}_3 \le \frac{64\sigma^2}{9\pi(1+\Delta)^2}$ or $Z_1 \le \hat{Z}_3 \le \frac{64\sigma^2}{9\pi(1+\Delta_2)^2}$, which proves that LSA finishes in constant number of steps regardless of network size n. $\square$

We have simulated the performance of LSA in random networks to verify its optimality in capacity scaling. We increase the network size n from 10^3 to 10^6 and set $r(n) = \sqrt{\frac{\ln n}{\pi}}$. The partition dimension is chosen from a range of values on different orders of $r(n)$. The traffic in the network is a random mixture of unicast, multicast and broadcast. As the key to optimal capacity scaling is the existence of a con-



(a) $\Delta = 1$ (b) $\Delta = 2$

Figure 10: $\rho(n) = r(n)$.



(a) $\Delta = 1$ (b) $\Delta = 2$

Figure 11: $\rho(n) = r^{\frac{3}{2}}(n)$.

stant lower bound on the probability of successful link transmissions, we plot the curves for the cumulative percentage of links versus their achieved transmission successful probability, as shown in Fig. 8–11. A point in these curves denotes the percentage of links that have higher successful probability than the corresponding x-axis value of the point. It is observed in Fig. 8 and Fig. 9 that the curves move left-ward as n increases when $\rho(n) = o(r(n))$. The moving trend shows that, given sufficiently large n, the curve will eventually be arbitrarily close to the y-axis. Hence, if $\rho(n) = o(r(n))$, all the links will have constant transmission failures as $n \to \infty$. In contrast, Fig. 10 and Fig. 11 display stationary curves as $n \to \infty$. When $\rho(n) = \Omega(r(n))$, there exists some constant $0 < p < 1$ such that the successful probability of every link is higher than p regardless of network size n. This constant p guarantees a constant factor in the capacity difference between the localized and the global scheduling schemes. The simulation results thus validate the sufficiency and the necessity of condition $\rho(n) = \Omega(r(n))$ for scheduling partition to achieve order optimal capacity scaling.

7. CONCLUSION

We have proposed in this paper the scheduling partition methodology for achieving order optimal capacity scaling in large wireless networks. It bridges the gap between the requirement for a globally collision-free transmission schedule assumed in the current study of wireless network capacity and the fact of inapplicability of the existing scheduling algorithms due to their excessive complexity in large networks. Our new approach divides a large network into small partitions and schedules transmissions in each partition independently. As a result, the scheduling complexity is reduced significantly. Meanwhile, it achieves the same order of capacity scaling as the theoretically derived capacity bounds. We have characterized the partition and scheduling approach by using three parameters and determined their design principles. Our results provide a practical solution for the maximum capacity scaling in large wireless networks.

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9. APPENDIX

9.1 Proof of Theorem 2

PROOF. Suppose the network $\mathcal{B}$ is divided into a set of disjoint partitions $\{\mathcal{C}_i\}$ ($\mathcal{B} = \cup_i \mathcal{C}_i$) and there is a local schedule $\mathcal{S}_{l,i}(t)$ in each partition $\mathcal{C}_i$. Let $\widetilde{\mathcal{S}}_{l,i}(t)$ denote the subset of failed links in $\mathcal{S}_{l,i}(t)$ due to cross-partition collisions. Note that $\widetilde{\mathcal{S}}_{l,i}(t) = \emptyset$ in the generalized physical interference model as there is no absolute link failure in this model. By defining a global scheduling scheme $\mathcal{S}_g(t) = \cup_i(\mathcal{S}_{l,i}(t) \backslash \widetilde{\mathcal{S}}_{l,i}(t))$, we see that $\mathcal{S}_g(t)$ supports the same $\lambda_l(n)$ as $\mathcal{S}_{l,i}(t)$ when the same routing protocol is used. □

9.2 Proof of Theorem 3

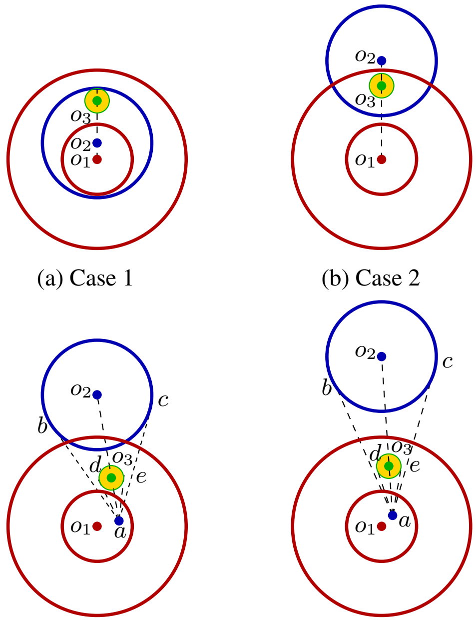
PROOF. We define an indicator function $I_{l_{ij}}(\lambda_s)$ such that $I_{l_{ij}}(\lambda_s) = 1$ if the rate λ_s from source v_s traverses l_{ij} and $I_{l_{ij}}(\lambda_s) = 0$ otherwise. Obviously $w_{ij}(n) = \sum_{s=1}^{n} \lambda_s I_{l_{ij}}(\lambda_s)$, which shows that the data transmission rate on a link is the sum of all the traversing source rates. If we scale every source rate simultaneously by a constant, the data rate on every link in the network is scaled by the same constant.

We first prove the sufficient condition. When $w_{l,ij}(n) = \Omega(w_{g,ij}(n))$, then there exists a constant c that satisfies $w_{l,ij}(n) \geq c w_{g,ij}(n)$ as $n \to \infty$. It indicates that if we scale each source rate from $\lambda_g(n)$ to $c\lambda_g(n)$, $w_{l,ij}(n)$ can accommodate the new source rate. Thus, $\lambda_l(n) = \Omega(\lambda_g(n))$.

We next prove the necessary condition. When $\lambda_l(n) = \Omega(\lambda_g(n))$, then there exists a constant c such that, given $\lambda_g(n)$, localized scheduling can support $c\lambda_g(n)$. As the routing protocol is fixed, it is then required on every link l_{ij} that localized scheduling can accommodate the link transmission rate $cw_{g,ij}(n)$. Hence $w_{l,ij}(n) = \Omega(w_{g,ij}(n))$. □

9.3 Proof of Lemma 1

PROOF. Since specification **S1** dictates that the partition diameter is not larger than $2\sigma\rho(n)$, any partition that overlaps with the disk must be contained in a larger co-centric



(a) Case 1 (b) Case 2

(c) Case 3 (d) Case 4

Figure 12: The relative locations of the co-centric disks and an overlapping partition. The co-centric disks are centered at o_1, denoted as $D(o_1, \kappa_1 r)$ and $D(o_1, \kappa_3 r)$ respectively. The annulus is denoted as $D(o_1, \kappa_3 r) \setminus D(o_1, \kappa_1 r)$. The partition is centered at o_2. For clarity, we have only shown the inner disk of radius ρ, denoted as $D(o_2, \rho)$, but ignored the partition boundary as well as the outer disk of radius $\sigma\rho$. Point a is inside $D(o_1, \kappa_1 r)$. Lines $\overline{ab}$ and $\overline{ac}$ are tangent to $D(o_2, \rho)$. Lines $\overline{ad}$ and $\overline{ae}$ are tangent to $D(o_3, \epsilon)$. $d = |\overline{o_1 o_2}|$.

disk of radius $\kappa_1 r(n) + 2\sigma\rho(n)$. Recall that the partitions are disjoint and each occupies at least an area of $\pi\rho^2(n)$. Hence, the number of partitions that can be contained in the larger disk is at most $\frac{\pi(\kappa_1 r(n)+2\sigma\rho(n))^2}{\pi\rho^2(n)} \leq (\frac{\kappa_1}{\kappa_2} + 2\sigma)^2$, which is a constant. $\square$

9.4 Proof of Lemma 2

PROOF. For simplicity reason, we abbreviate $r(n)$ and $\rho(n)$ as r and ρ respectively in this proof. Besides, we use $\mathcal{O}$ to denote the overlap area between a partition and the annulus. The idea of the proof is to find a disk in $\mathcal{O}$, denoted as $D(o_3, \epsilon)$ where o_3 is the center and ϵ is the radius, which covers an area of at least cr^2. By defining d as the Euclidean distance between the center of the two co-centric disks and the center of an overlapping partition, we discuss all the possible cases of d.

Case 1: $0 \leq d \leq \kappa_3 r - \kappa_2 r$. As shown in Fig. 12(a), we find point o_3 on the extension of line $\overline{o_1 o_2}$ such that $|\overline{o_2 o_3}| = \frac{\kappa_1 r + \kappa_2 r}{2}$ and define $\epsilon = \frac{\kappa_2 r - \kappa_1 r}{2}$. Given any point $p \in D(o_3, \epsilon)$, it is easy to see that $|\overline{o_2 p}| \leq |\overline{o_2 o_3}| + |\overline{o_3 p}| \leq \frac{\kappa_1 r + \kappa_2 r}{2} + \frac{\kappa_2 r - \kappa_1 r}{2} = \kappa_2 r \leq \rho$, indicating $p \in D(o_2, \rho)$. In addition, $|\overline{o_1 p}| \geq |\overline{o_1 o_3}| - |\overline{o_3 p}| \geq \frac{\kappa_1 r + \kappa_2 r}{2} - \frac{\kappa_2 r - \kappa_1 r}{2} = \kappa_1 r$ and $|\overline{o_1 p}| \leq |\overline{o_1 o_3}| + |\overline{o_3 p}| \leq \kappa_3 r - \kappa_2 r + \frac{\kappa_1 r + \kappa_2 r}{2} + \frac{\kappa_2 r - \kappa_1 r}{2} = \kappa_3 r$, showing $p \in D(o_1, \kappa_3 r) \setminus D(o_1, \kappa_1 r)$. Hence, $D(o_3, \epsilon) \subseteq \mathcal{O}$. Since the area of $D(o_3, \epsilon)$ is at least $\pi(\frac{\kappa_2 r - \kappa_1 r}{2})^2$, we have proven the lemma by choosing $c_1 = \frac{\pi}{4}(\kappa_2 - \kappa_1)^2$.

Case 2: $\kappa_3 r - \kappa_2 r < d \leq \rho + \frac{\kappa_1 r + \kappa_3 r}{2}$. As shown in Fig. 12(b), $D(o_2, \rho)$ overlaps with $D(o_1, \kappa_3 r)$ partially. We locate point o_3 on line $\overline{o_1 o_2}$ such that $|\overline{o_1 o_3}| = \frac{\kappa_1 r + 3\kappa_3 r}{4}$ and define $\epsilon = \frac{\kappa_3 r - \kappa_1 r}{4}$. It is obvious that $D(o_3, \epsilon) \subseteq D(o_1, \kappa_3 r) \setminus D(o_1, \kappa_1 r)$. Besides, we obtain $|d - \frac{\kappa_1 r + 3\kappa_3 r}{4}| \leq \rho - \frac{\kappa_3 r - \kappa_1 r}{4}$ from $\kappa_3 r - \kappa_2 r < d \leq \rho + \frac{\kappa_1 r + \kappa_3 r}{2}$. Given any point $p \in D(o_3, \epsilon)$, we have $|\overline{o_2 p}| \leq |\overline{o_2 o_3}| + |\overline{o_3 p}| \leq |d - \frac{\kappa_1 r + 3\kappa_3 r}{4}| + \frac{\kappa_3 r - \kappa_1 r}{4} \leq \rho - \frac{\kappa_3 r - \kappa_1 r}{4} + \frac{\kappa_3 r - \kappa_1 r}{4} = \rho$, indicating $D(o_3, \epsilon) \subseteq D(o_2, \rho)$. Thus, $D(o_3, \epsilon) \subseteq \mathcal{O}$ and $D(o_3, \epsilon)$ has an area $\pi(\frac{\kappa_3 r - \kappa_1 r}{4})^2$. The lemma is true by choosing $c_2 = \frac{\pi}{16}(\kappa_3 - \kappa_1)^2$.

Case 3: $\rho + \frac{\kappa_1 r + \kappa_3 r}{2} < d \leq \rho + \kappa_3 r$. As shown in Fig. 12(c), $D(o_2, \rho)$ overlaps with $D(o_1, \kappa_3 r)$ partially in this case too. Let a denote a point over which the partition overlaps with $D(o_1, \kappa_1 r)$, $\overline{ab}$ and $\overline{ac}$ denote the lines tangent to $D(o_2, \rho)$. Because the partition is convex, the area encompassed by $\overline{ab}$, $\overline{ac}$ and $D(o_2, \rho)$ must be part of the partition too. We choose point o_3 on line $\overline{o_2 a}$ such that $|\overline{o_1 o_3}| = \frac{3\kappa_1 r + \kappa_3 r}{4}$, which is possible because $|\overline{o_1 a}| < \kappa_1 r$, $|\overline{o_1 o_2}| > \frac{\kappa_1 r + \kappa_3 r}{2}$ and $\overline{o_2 a}$ is continuous. We then define $\epsilon = \frac{\rho(\kappa_3 r - \kappa_1 r)}{8(\rho + \kappa_1 r + \kappa_3 r)}$. It is obvious that $D(o_3, \epsilon) \subseteq D(o_1, \kappa_3 r) \setminus D(o_1, \kappa_1 r)$. To prove $D(o_3, \epsilon)$ is inside the partition, it is only necessary to show $\angle o_3 a d < \angle o_2 a b$, where $\overline{ad}$ is tangent to $D(o_3, \epsilon)$. Since $\sin(\angle o_3 a d) = |\overline{o_3 d}|/|\overline{o_3 a}| \leq \frac{\epsilon}{(3\kappa_1 r + \kappa_3 r)/4 - \kappa_1 r} < \frac{\rho}{\rho + \kappa_1 r + \kappa_3 r} \leq |\overline{o_2 b}|/|\overline{o_2 a}| = \sin(\angle o_2 a b)$, we obtain $\angle o_3 a d < \angle o_2 a b$. Hence, $D(o_3, \epsilon) \subseteq \mathcal{O}$. The area of $D(o_3, \epsilon)$ is at least $\pi(\frac{\kappa_2 r(\kappa_3 r - \kappa_1 r)}{8(\kappa_1 r + \kappa_2 r + \kappa_3 r)})^2$, so the lemma is true by choosing $c_3 = \frac{\pi}{64}(\frac{\kappa_2(\kappa_3 - \kappa_1)}{\kappa_1 + \kappa_2 + \kappa_3})^2$.

Case 4: $\rho + \kappa_3 r < d \leq \sigma\rho + \kappa_1 r$. Note that $\rho + \kappa_3 r < \sigma\rho + \kappa_1 r$ when the partition overlaps with $D(o_1, \kappa_1 r)$ and $d > \rho + \kappa_3 r$. The proof is similar to Case 3. As shown in Fig. 12(d), we locate point o_3 on line $\overline{o_2 a}$ such that $|\overline{o_1 o_3}| = \frac{\kappa_1 r + \kappa_3 r}{2}$ and define $\epsilon = \frac{\rho(\kappa_3 r - \kappa_1 r)}{4(\sigma\rho + 2\kappa_1 r)}$. It is obvious that $D(o_3, \epsilon) \subseteq D(o_1, \kappa_3 r) \setminus D(o_1, \kappa_1 r)$. Again, we define $\overline{ab}$ and $\overline{ac}$ to be tangent to $D(o_2, \rho)$, $\overline{ad}$ and $\overline{ae}$ to be tangent to $D(o_3, \epsilon)$. We have $\angle o_3 a d < \angle o_2 a b$ because $\sin(\angle o_3 a d) = |\overline{o_3 d}|/|\overline{o_3 a}| \leq \frac{\epsilon}{(\kappa_1 r + \kappa_3 r)/2 - \kappa_1 r} < \frac{\rho}{\sigma\rho + 2\kappa_1 r} \leq |\overline{o_2 b}|/|\overline{o_2 a}| = \sin(\angle o_2 a b)$. So, $D(o_3, \epsilon) \subseteq \mathcal{O}$. Proof follows using $c_4 = \frac{\pi}{16}(\frac{\kappa_2(\kappa_3 - \kappa_1)}{2\kappa_1 + \sigma\kappa_2})^2$.

For any $d > \sigma\rho + \kappa_1 r$, the partition does not overlap with $D(o_1, \kappa_1 r)$. We have therefore considered all the possible cases regarding d. In summary, the lemma is proven true by choosing $c = \min\{c_1, c_2, c_3, c_4\}$. $\square$

9.5 Proof of Lemma 3

PROOF. If there are no subsequences that satisfy $f(n_k) = o(g(n_k))$, there must exist a constant $c > 0$ such that $\frac{f(n)}{g(n)} \geq c$ for all n, which implies $f(n) = \Omega(g(n))$. Therefore, $f(n) \neq \Omega(g(n))$ must indicate $f(n_k) = o(g(n_k))$ for some subsequences $\{f(n_k)\}$ and $\{g(n_k)\}$.

If there exist subsequences $f(n_k) = o(g(n_k))$, then we have $\lim_{n_k \to \infty} \frac{f(n_k)}{g(n_k)} = 0$. Hence we cannot find any constant $c > 0$ such that $f(n) \geq cg(n)$ as $n \to \infty$, i.e., $f(n) \neq \Omega(g(n))$. $\square$

9.6 Proof of Lemma 4

PROOF. This statement is obviously true according to Lemma 3, since $f(n) \neq O(g(n))$ means $g(n) \neq \Omega(f(n))$ and $f(n_k) = \omega(g(n_k))$ means $g(n_k) = o(f(n_k))$. $\square$